Computability Assignment
Year 2012/13 - Number 1

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1 Question

Define a binary property $p(x, y)$ over natural numbers such that we have both

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$
2. $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

Write your answer here.
.
Property $p(x, y)$: every natural number has a successor
$\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.y = x + 1$
.
Claim 1.
$\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$
it holds by definition
.
Claim 2.
it holds. Let's prove it by contradiction.
If it's false, then
$\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y) \Longrightarrow$
$\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.y = x + 1 \Longrightarrow$
$\exists y \in \mathbb{N}.(\forall x \in \mathbb{N}) + 1 = y \Longrightarrow$ RZ: I see what you mean, but please avoid this notation
there exists a y that is the successor of every natural number.
If so, $y = x + 1$, and also $y = (x + 1) + 1$

so$y = y + 1 \implies$
$0 = 1$
Impossible