# Computability Assignment
# Year 2012/13 - Number 1

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## 1 Question

Define a binary property $p(x, y)$ over natural numbers such that we have both

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$
2. $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y)$

Provide a definition for $p$, and a proof for the above claims.

### 1.1 Answer

There are some possible way to define $p(x, y)$.

$$p(x, y) = \begin{cases} true & if\ x = y \\ false & otherwise \end{cases}$$

In this case I have defined that the function returns $true$ iff the two numbers are equal.
Proof:
1. The first statement says that for all $x$, exists a $y$ such that the property is $true$. So, given the $x$ value, always exists an element that is equal to it in $\mathbb{N}$, for instance $x$.
2. The second statement says that does not exist a $y$ such that for all $x$ the property is $true$. This is true because we cannot find an element in $\mathbb{N}$ such that it is equal to every other elements in $\mathbb{N}$. In particular, if such $y$ exists, it means that exists two (actually all but we can use only 2) different numbers that are equal to the same $y$, so take $x_1, x_2 \in \mathbb{N}$ with $x_1 \neq x_2$, $x_1 = y$, $x_2 = y$ but in this case it must be that $x_1 = x_2$ that is a contradiction.