Computability Assignment
Year 2012/13 - Number 1

Please keep this file anonymous: do not write your name inside this file.
More information about assignments at http://disi.unitn.it/∼zunino/teaching/computability/assignments

1 Question

Define a binary property $p(x, y)$ over natural numbers such that we have both

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$
2. $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

Write your answer here.

$p(x, y) \iff x < y$

1. $\forall x \in \mathbb{N}.\exists y = x + 1.p(x, y)$
2. if $\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y)$ then $\mathbb{N} \ni x = y{+}1.p(x, y)$ but $\begin{array}{ccc} p(x, y) & \Rightarrow & x < y \\ x = y + 1 & \Rightarrow & y + 1 > y \end{array}$
   so $\forall y \in \mathbb{N}.\exists x \in \mathbb{N}.\neg p(x, y)$ therefore $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y)$