RZ: Please edit the provided LYX file instead next time.

## Solution

Let $p$ be the property of 'successorship' over $\mathbb{N}$, given by $p(x)=x+1$, i.e. $p=\{(x, x+1) \mid x \in \mathbb{N}\}$

Now property 1 , which is $\forall x \in \mathbb{N}, \exists y \in \mathbb{N} . p(x, y)$, is trivially satisfied, since by definition of $p$, for all $x \in \mathbb{N}$, exists $y=x+1$ such that $p(x, y)$ holds.

Now property 2 , which is $\nexists y \in \mathbb{N} . \forall x \in \mathbb{N} . p(x, y)$ is satisfied. By contradition if property 2 is not satisfied then there exists $y \in \mathbb{N}$, such that for all $x \in \mathbb{N}$, $p(x, y)$ holds. This implies that if $x^{\prime}=x+1, p(x, y)$ and $p\left(x^{\prime}, y\right)$, which implies that $y=x+1=x+1+1$, which is a contradition

