Computability Assignment
Year 2012/13 - Number 1

September 21, 2012

1 Question

Define a binary property $p(x, y)$ over natural numbers such that we have both

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$
2. $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

Considering $p(x, y) \iff x < y$ as the binary property, we can prove the two claims:

a)$\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y) \iff \forall x \in \mathbb{N}.\exists y \in \mathbb{N}.x < y$ which in $\mathbb{N}$ is satisfied for each x assuming $y := x + 1$

b)$\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y) \iff \forall y \in \mathbb{N}.\exists x \in \mathbb{N}.\neg p(x, y) \iff \forall y \in \mathbb{N}.\exists x \in \mathbb{N}.(x \geqslant y)$ which in $\mathbb{N}$ is satisfied for each x assuming $y := x$