Computability Assignment
Year 2012/13 - Number 1

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1 Question

Define a binary property $p(x, y)$ over natural numbers such that we have both

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$
2. $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

Write your answer here.
The property could be $p(x, y) = y > x$

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.y > x$

Proof:
Take any $x \in \mathbb{N}$
let $y = x + 1$, such $y \in \mathbb{N}$ and
$y > x \Longrightarrow x + 1 > x$, which is true for all x.

1. $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.y > x$

Proof:
(by contradiction)
Let's assume that $\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.y > x$
this means that there exists at least one $y \in \mathbb{N}$ which is greater than all $x \in \mathbb{N}$
in other words, there is one natural number which is greater than all the other natural numbers,
since the natural numbers are infinite such y can not exist, therefore contradiction.