# Computability Assignment Year 2012/13 - Number 1 

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## 1 Question

Define a binary property $p(x, y)$ over natural numbers such that we have both

1. $\forall x \in \mathbb{N} . \exists y \in \mathbb{N} . p(x, y)$
2. $\neg \exists y \in \mathbb{N} . \forall x \in \mathbb{N} . p(x, y)$

Provide a definition for $p$, and a proof for the above claims.

### 1.1 Answer

$p(x, y)= \begin{cases}\text { true } & x=y \\ \text { false } & \text { otherwise }\end{cases}$
Proof:
Given $x$, a value $y$ that satisfies $p$ exists, and it is $x$, hence 1 is true
A single $y$ that satisfies $p$ for all the possible values of $x$ does not exist. Assuming its value is c , this would mean that $\mathrm{x}=\mathrm{c}$. But then $\mathrm{p}(\mathrm{c}+1, \mathrm{c})$ would be false, hence statement 2 is true.

