Computability Assignment Year 2012/13 - Number 1

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1 Question

Define a binary property p(x, y) over natural numbers such that we have both

- 1. $\forall x \in \mathbb{N} : \exists y \in \mathbb{N} : p(x, y)$
- 2. $\neg \exists y \in \mathbb{N}. \forall x \in \mathbb{N}. p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

$$p(x,y) = \begin{cases} true & x = y\\ false & otherwise \end{cases}$$

Proof:

Given x, a value y that satisfies p exists, and it is x, hence 1 is true

A single y that satisfies p for all the possible values of x does not exist. Assuming its value is c, this would mean that x = c. But then p(c + 1, c) would be false, hence statement 2 is true.