Computability Assignment
Year 2012/13 - Number 1

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1 Question

Define a binary property $p(x, y)$ over natural numbers such that we have both

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$
2. $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

I Start defining the property p as the greater function $>$ between y and x $(y > x)$.

Now I can verify the proposition 1 by induction.
First of all I define $y = succ(x)$ where $succ(x) = x + 1$.

RZ: you don't really need induction here, it's enough to say that $p(x, x + 1)$ is true since $x + 1 > x$.

Base case

Setting $x = 0$ we obtain that $y = succ(x) = 1$. The property $p(0, 1)$ is verified since $1 > 0$ is true.

Inductive Step

We consider valid the property till $x = n$. We have to prove that p is also valid for $x = n + 1$.

We get that $y = succ(x) = n + 1 + 1$, and so $p(n + 1 + 1, n + 1)$. Since $n+1+1 > n+1$, then the property p is verified for all x.
It follows that the first proposition is verified.

Now I have to verify the proposition 2, by induction again.

This proposition can be rewritten as follows: $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x,y) \equiv \forall y \in \mathbb{N}.\neg\forall x \in \mathbb{N}.p(x,y) \equiv \forall y \in \mathbb{N}.\exists x \in \mathbb{N}.\neg p(x,y)$
Now I set $\neg p(x,y) \equiv q(x,y)$ which can be interpreted as $x \leq y$.

RZ: as above, you don't need induction. It's enough to say that if you pick x=0, you have $q(x,y)$ for all y since $0 \leq y$.

Base case

Setting $y = 0$ and $x = 0$ the property $q(0,0)$ is verified since $0 \leq 0$ is true.

Inductive Step

We consider valid the property till $y = n$. We have to prove that q is also valid for $y = n + 1$.
We set $x = 0$, so we obtain $q(0, n + 1)$. Since $0 \leq n + 1$, then the property q is verified for all y.
Since the property q is always valid we can infer, moving in a reverse manner, that the property p is always true.
It follows that the second proposition is verified.