

Computability Assignment

Year 2012/13 - Number 2

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1 Question

Let A, B be two sets. Prove that the properties below are equivalent.

- $A = \emptyset \vee B = \emptyset$
- $A \times B = \emptyset$

1.1 Answer

Write your answer here.

2 Preliminaries

Given an infinite sequence of sets $(A_i)_{i \in \mathbb{N}}$, we define $\bigcup_{i=0}^{\infty} A_i = \bigcup \{A_i \mid i \in \mathbb{N}\}$ and $\bigcup_{i=0}^k A_i = \bigcup \{A_i \mid i \in \mathbb{N} \wedge i \leq k\} = A_0 \cup A_1 \cup \dots \cup A_k$.

3 Question

Assume $(A_i)_{i \in \mathbb{N}}$ to be an infinite sequence of sets of natural numbers, satisfying

$$A_0 \subseteq A_1 \subseteq A_2 \subseteq A_3 \cdots \subseteq \mathbb{N} (*)$$

For each property p_i shown below, state whether

- the hypothesis $(*)$ is sufficient to conclude that p_i holds; or
- the hypothesis $(*)$ is sufficient to conclude that p_i does not hold; or

- the hypothesis $(*)$ is not sufficient to conclude anything about the truth of p_i .

Justify your answers (briefly).

1. p_1 : $\forall k \in \mathbb{N}. A_k = \bigcup_{i=0}^k A_i$
2. p_2 : for all i , if A_i is infinite, then $A_i = A_{i+1}$
3. p_3 : if $\forall i \in \mathbb{N}. A_i \neq A_{i+1}$, then $\bigcup_{i=0}^{\infty} A_i = \mathbb{N}$
4. p_4 : if $\forall i \in \mathbb{N}. A_i$ is finite, then $\bigcup_{i=0}^{\infty} A_i$ is finite
5. p_5 : if $\forall i \in \mathbb{N}. A_i$ is finite, then $\bigcup_{i=0}^{\infty} A_i$ is infinite
6. p_6 : if $\forall i \in \mathbb{N}. A_i$ is infinite, then $\bigcup_{i=0}^{\infty} A_i$ is infinite

3.1 Answer

Write your answer here.