

# EXPLOITING SPARSE REPRESENTATIONS FOR ROBUST ANALYSIS OF NOISY COMPLEX VIDEO SCENES

## CONTRIBUTIONS

- We formalize the task of **extracting activity patterns** as a **non-negative matrix factorization (NMF)** problem, considering as reconstruction function the robust **Earth Mover's Distance (EMD)** and imposing **sarsity constraints**.
- We derive an **alternating optimization approach** to solve the proposed problem efficiently and we show that it is reduced to a sequence of linear programs (LP).
- With respect to previous work on EMD clustering [1, 6], our method is **more scalable** and produces **more interpretable results** due to the sparsity constraints.

## EFFICIENT EARTH MOVER'S DISTANCE (EMD- $L_1$ )

The  $\mathcal{D}_{EMD}(\mathbf{h}, \mathbf{p})$  is obtained as the solution of the transportation problem:

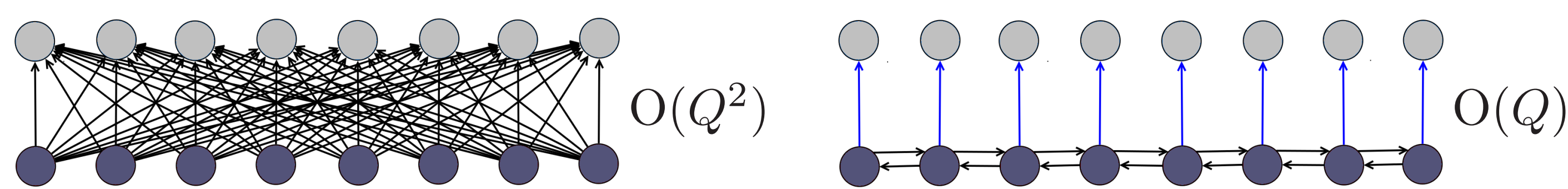
$$\min_{f_{qt} \geq 0} \sum_{t,q=1}^Q d_{qt} f_{qt} \quad \text{s.t.} \quad \sum_{q=1}^Q f_{qt} = h^t, \quad \sum_{t=1}^Q f_{qt} = p^q$$

By using **EMD with  $L_1$  as ground distance** [2], this simplifies as:

$$\min_{f_{qt} \geq 0} \sum_q \sum_{t \in \mathcal{N}(q)} f_{q,t}$$

$$\text{s.t.} \quad \sum_{t \in \mathcal{N}(q)} f_{q,t} - \sum_{t \in \mathcal{N}(q)} f_{t,q} = h^q - p^q \quad \forall q$$

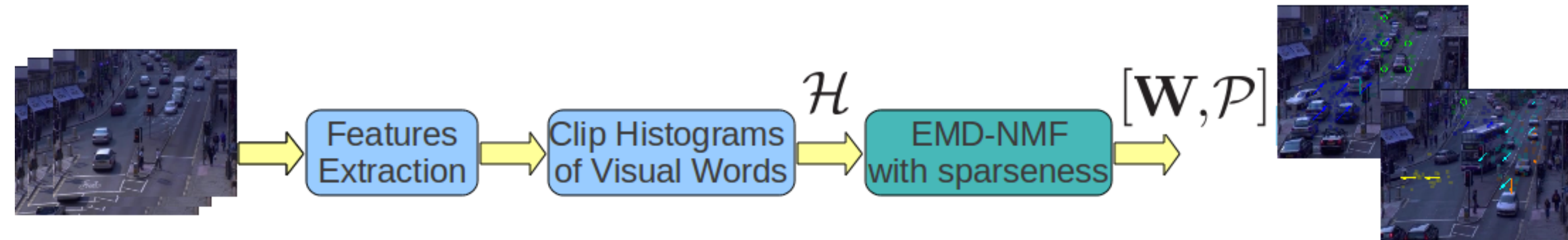
$\mathcal{N}(q)$  is represented by the adjacent bins.



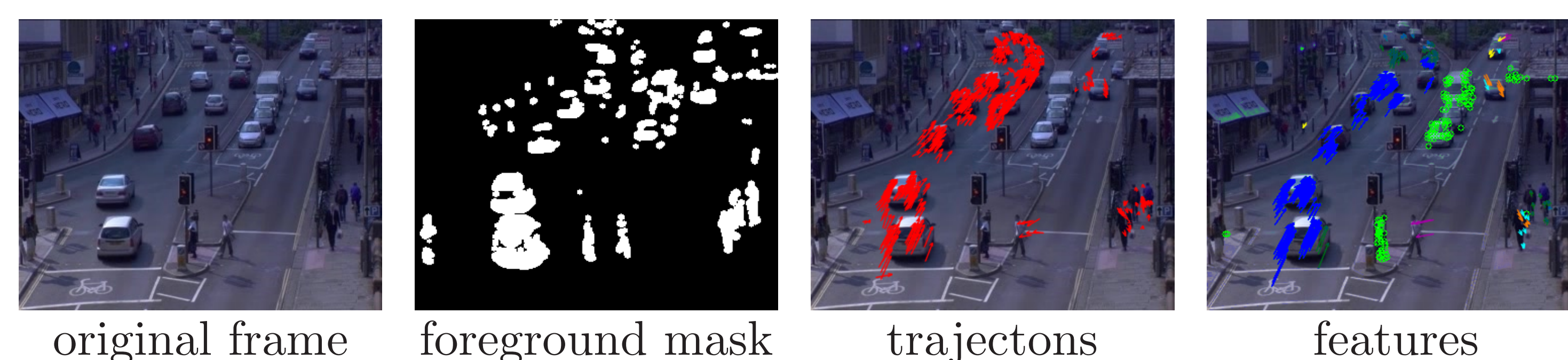
Complexity reduces from  $O(Q^2)$  to  $O(Q)$ .

## OUR APPROACH

Unsupervised learning approach for video scene analysis.



## FEATURES EXTRACTION



## EMD NMF WITH SPARSENESS CONSTRAINTS

Given a set of clip histograms  $\mathcal{H} = \{\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_N\}$ , discovering high-level activities is modeled as finding a set of bases  $\mathcal{P} = \{\mathbf{p}^1, \mathbf{p}^2, \dots, \mathbf{p}^K\}$ ,  $K \ll N$ , and the coefficients  $\mathbf{W} = [\mathbf{w}_1 \mathbf{w}_2 \dots \mathbf{w}_N]$ ,  $\mathbf{w}_i \in \mathbb{R}^K$ , such that:

$$\min_{\mathbf{p}^k, \mathbf{w} \geq 0} \sum_{i=1}^N \mathcal{D}_{EMD}(\mathbf{h}_i, \sum_k w_i^k \mathbf{p}^k) \quad (1)$$

$$\text{s.t.} \quad \omega_m \leq \Omega(\mathbf{p}^k) \leq \omega_M, \quad \forall k = 1 \dots K \quad (2)$$

Choosing  $\Omega(\mathbf{x}) = \frac{1}{\sqrt{n-1}}(\sqrt{n} - \|\mathbf{x}\|_1 / \|\mathbf{x}\|_2)$ ,  $\mathbf{x} \in \mathbb{R}^N$ , (2) enforces sparsity. The resulting optimization problem is:

$$\min_{p_q^k, w_i^k, f_{q,t}^i \geq 0} \sum_{i=1}^N \sum_q \sum_{t \in \mathcal{N}(q)} f_{q,t}^i \quad (3)$$

$$\text{s.t.} \quad \sum_{t \in \mathcal{N}(q)} f_{q,t}^i - \sum_{t \in \mathcal{N}(q)} f_{t,q}^i = h_i^q - \sum_k w_i^k p_q^k, \quad \sum_k w_i^k = 1, \quad \sum_q p_q^k = 1, \quad \forall q, \forall i, \forall k$$

$$\|\mathbf{p}^k\|_2 \leq \frac{1}{c_M} \mathbf{e}^T \mathbf{p}^k \quad \frac{1}{c_m} \mathbf{e}^T \mathbf{p}^k \leq \|\mathbf{p}^k\|_2 \quad \forall k \quad (4)$$

with  $c_x = \sqrt{Q} - \omega_x(\sqrt{Q} - 1)$ ,  $\mathbf{e} \in \mathbb{R}^Q$  is a vector of ones.

## ALTERNATING OPTIMIZATION

The EMD learning problem is solved using alternating optimization:

**Input:** Original clips histograms  $\mathcal{H} = \{\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_N\}$ .

Initialize  $\mathbf{W} = [\mathbf{w}_1 \mathbf{w}_2 \dots \mathbf{w}_N]$  with positive random values.

Normalize the columns of  $\mathbf{W}$  such that  $\sum_k w_i^k = 1, \forall i = 1, \dots, N$ .

**while** not converged

Solve (3) s.t. (4) with respect to  $\mathbf{p}^k, \mathbf{f}$  using TPC method.

Solve the LP (3) with respect to  $\mathbf{W}, \mathbf{f}$ .

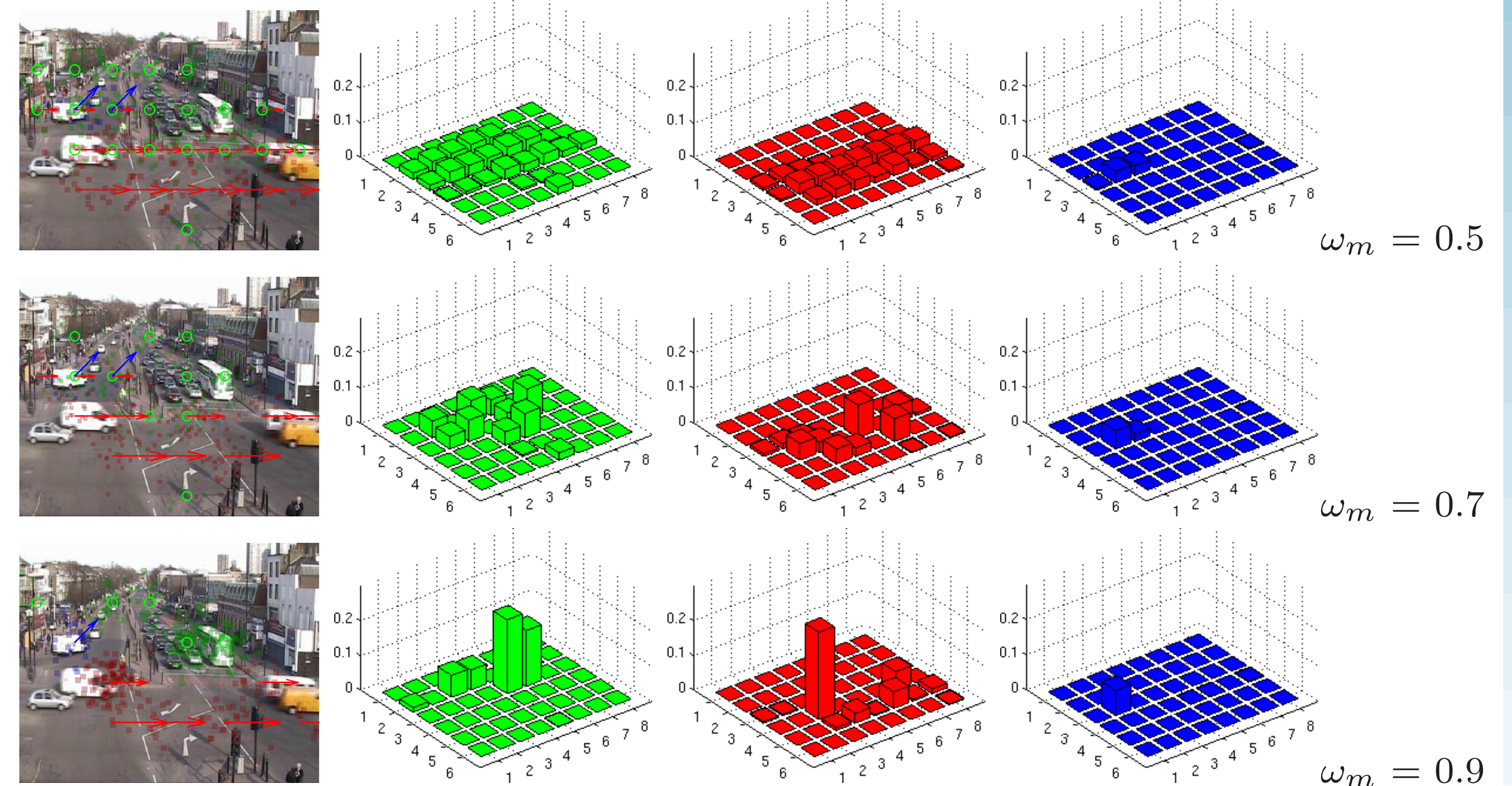
**end**

**Output:**  $\mathbf{W}, \mathcal{P} = \{\mathbf{p}^1, \mathbf{p}^2, \dots, \mathbf{p}^K\}$ .

The **Tangent Plane Constraint (TPC)** method [3] consists in approximating the non convex cone constraints by linear constraints.

## RESULTS - HIGH LEVEL ACTIVITIES EXTRACTION

Computed basis matrix at increasing level of sparsity  $\omega_m$ :



Nice grouping effect of combining EMD- $L_1$  with sparsity constraints!

## RESULTS - ANOMALY DETECTION



(Left) Anomaly score and temporal segmentation bar for a 40 clips video sequence. (Right) Representative frames are extracted from clips detected as anomalous.

## RESULTS - QUANTITATIVE EVALUATION

Clustering accuracy at varying level of sparsity  $\omega_m$ :

$\omega_m$	0	0.1	0.3	0.5	0.7	0.9
Junction2 (48 clips)	89.58%	89.58%	89.58%	89.58%	<b>91.67%</b>	89.58%
Roundabout (60 clips)	88.33%	88.33%	<b>90.00%</b>	<b>90.00%</b>	<b>90.00%</b>	<b>90.00%</b>
Junction (39 clips)	<b>89.74%</b>	<b>89.74%</b>	<b>89.74%</b>	<b>89.74%</b>	<b>89.74%</b>	84.62%

Comparison with previous approaches:

	std pLSA [4]	hrc pLSA [4]	DDP-HMM [5]	EMP [6]	our approach ( $\omega_m = 0.7$ )
Roundabout (60 clips)	81.67%	75.00%	85.00%	86.67%	<b>90.00%</b>
Junction	89.74%	76.92%	87.18%	<b>92.31%</b>	89.74%
Roundabout (148 clips)	84.46%	72.30%	85.14%	<b>86.40%</b>	85.81%

Datasets are publicly available<sup>1</sup>. Code is available at our website<sup>2</sup>.

## REFERENCES

<sup>1</sup> [http://www.eecs.qmul.ac.uk/~jianli/Dataset\\_List.html](http://www.eecs.qmul.ac.uk/~jianli/Dataset_List.html)

<sup>2</sup> [http://disi.unitn.it/~zen/sparse\\_emd.html](http://disi.unitn.it/~zen/sparse_emd.html)

- [1] E. Ricci, G. Zen, N. Sebe, S. Messelodi. A Prototype Learning Framework using EMD: Application to Complex Scenes Analysis In *PAMI*, 2012.
- [2] H. Ling, K. Okada. An efficient Earth Mover Distance algorithm for robust histogram comparison In *PAMI*, 2006.
- [3] H. Tuy. Convex programs with an additional reverse convex constraint *J. of Optim. Theory and Applic*, 1987
- [4] J. Li, S. Gong, T. Xiang. Global behaviour inference using probabilistic latent semantic analysis In *BMVC*, 2008.
- [5] D. Kuettel, M.D. Breitenstein, L.V. Gool, V. Ferrari. What's going on? Discovering spatio-temporal dependencies in dynamic scenes In *CVPR*, 2010.
- [6] G. Zen, E. Ricci. Earth mover's prototypes: a convex learning approach for discovering activity patterns in dynamic scenes. In *CVPR*, 2011.