## nuXMV: Exercises*

Patrick Trentin<br>patrick.trentin@unitn.it http://disi.unitn.it/trentin

# Formal Methods Lab Class, May 19, 2017 



Università degli Studi di
Trento
*These slides are derived from those by Stefano Tonetta, Alberto Griggio, Silvia Tomasi, Thi Thieu Hoa Le, Alessandra Giordani, Patrick Trentin for FM lab 2005/16

## Contents

(1) Exercises

- Dining Philosophers
- Insertion Sort
- Cleaning Robot
(2) Extra Exercises (from previous years)
- Elevator
- Odd/Even Counter
- Overflow Counter


## Exercise: Dining Philosophers [1/2]

Five philosophers sit around a circular table and spend their life alternatively thinking and eating. Each philosopher has a large plate of noodles and a fork on either side of the plate. The right fork of each philosopher is the left fork of his neighbor. Noodles are so slippery that a philosopher needs two forks to eat it. When a philosopher gets hungry, he tries to pick up his left and right fork, one at a time. If successful in acquiring two forks, he eats for a while (preventing both of his neighbors from eating), then puts down the forks, and continues to think.


## Exercise: Dining Philosophers [2/2]

## Exercise:

(1) Implement in SMV a system that encodes the philosophers problem. Assume that when a philosopher gets hungry, he tries to pick up his left fork first and then the right one.
Hint: you might consider an altruist philosopher, which can resign his fork in a deadlock situation.
(2) Verify the correctness of the system, by specifiying and checking the following properties:

- Never two neighboring philosophers eat at the same time.
- No more than two philosophers can eat at the same time.
- Somebody eats infinitely often.
- If every philosopher holds his left fork, sooner or later somebody will get the opportunity to eat.


## Contents

(1) Exercises

- Dining Philosophers
- Insertion Sort
- Cleaning Robot
(2) Extra Exercises (from previous years)
- Elevator
- Odd/Even Counter
- Overflow Counter


## Exercise: Insertion Sort [1/2]

## Exercise:

- encode the following code in NUXMV:

```
void isort(arr) {
        // init: i = 1, j = 1;
        while (i < 5) {
        j = i;
        while (j > 0 & array[j] < array[j-1]) {
                swap(array[j], array[j-1]);
                j--;
        }
    16: i++;
        }
    17: // done!
        }
```

- set arr equal to $\{9,7,5,3,1\}$
- verify the following properties:
- the algorithm always terminates
- eventually in the future, the array will be sorted forever
- the algorithm is not done ( $\mathrm{pc}=17$ ) until the array is sorted


## Exercise: Insertion Sort [2/2]

## Hints:

- use ' pc ' to keep track of the possible state values $\{11,12,13$, $14,15,16,17\}$
- declare ' $i$ ' in $1 . .5$, initialize 1
- declare ' $j$ ' in $0 . .4$, initialize 1
- ensure that the content of 'arr' does never change when 'pc $!=14$ '
- ensure that the content of 'arr' that is not involved in a 'swap' operation does not change even when ' $\mathrm{pc}=14$ '
- (easier?) encode the constraints over 'arr' with constrained-style modelling
- (easier?) encode the evolution of ' pc ', ' i ' and ' j ' with assignment-style modelling


## Contents

(1) Exercises

- Dining Philosophers
- Insertion Sort
- Cleaning Robot
(2) Extra Exercises (from previous years)
- Elevator
- Odd/Even Counter
- Overflow Counter


## Exercise: Cleaning Robot [1/3]

Exercise: model a rechargeable cleaning robot which task is to move around a $10 \times 10$ room and clean it.

The robot state is so composed:

- variables " $x$ " and " $y$ ", ranging from 0 to 9 , which keeps track of the robot's position
- variable "state", with values in \{ MOVE, CHECK, CHARGE, CLEAN, OFF \}, which keeps track of the next action taken by the robot
- variable "budget" in $\{0 . .100\}$ which signals the remaining power
- output variable "pos", defined to be equal $y \cdot 10+x$

At the beginning, the robot is in state "CHECK" and all other vars are 0.
The budget is decreased by a single unit each time the robot is in state "MOVE" or "CLEAN" (and budget $>0$ ), and restored to 100 if the robot is in "CHARGE" state. Otherwise, the budget doesn't change.

## Exercise: Cleaning Robot [2/3]

The robot changes state according to this ordered set of rules:

- if the robot is in "pos" 0 and the budget is smaller than 100, then the next state is "CHARGE"
- if the budget is 0 , then the next state is "OFF"
- if the robot is in state "CHARGE" or "MOVE", then the next state is "CHECK"
- if the robot is in state "CHECK", then the next state is either "CLEAN" or "MOVE"
- otherwise, the next state is "MOVE".

Encode, using the constraint-style (easier!), the following constraints:

- if the state is different than "MOVE", then the position of the robot never changes.
- if the state is equal to "MOVE", then the robot moves by a single square in one of the cardinal directions: it increases or decreases either " $x$ " or " $y$ ", but not both at the same time.


## Exercise: Cleaning Robot [3/3]

Encode and verify the following properties:

- in all possible executions, the robot changes position infinitely many times (false)
- it's definitely the case that sooner or later the robot exhausts its budget, turns OFF and stops moving (false)
- it is never the case that the robot's action is either "MOVE" or "CLEAN" and the available budget is zero (false)
- if the robot charges infinitely often, then it changes position infinitely many times (true)
- there exists an execution in which the robot cleans every cell that it visits (true)
- if the robot is in "pos" 0 , then it is necessarily always the case that in the future it will occupy a different position (true)
- the robot does not move along the diagonals (true)


## Contents

(1) Exercises

- Dining Philosophers
- Insertion Sort
- Cleaning Robot
(2) Extra Exercises (from previous years)
- Elevator
- Odd/Even Counter
- Overflow Counter


## Exercise: Elevator [1/5]

## Exercise:

- Given the model of an elevator system for a 4-floors building, including the complete description of:
- reservation buttons
- cabin
- door
- controller
- Enrich the model with properties encoding the requirements that must be met by each component of the system, and verify that such requirements are satisfied.


## Exercise: Elevator - Button [2/5]

For each floor there is a button to request service, that can be pressed. A pressed button stays pressed unless reset by the controller. A button that is not pressed can become pressed nondeterministically.

## Requirements:

- The controller must not reset a button that is not pressed.


## Exercise: Elevator - Cabin [3/5]

The cabin can be at any floor between 1 and 4. It is equipped with an engine that has a direction of motion, that can be either standing, up or down. The engine can receive one of the following commands: nop, in which case it does not change status; stop, in which case it becomes standing; up (down), in which case it goes up (down).

## Requirements:

- The cabin can receive a stop command only if the direction is up or down.
- The cabin can receive a move command only if the direction is standing.
- The cabin can move up only if the floor is not 4 .
- The cabin can move down only if the floor is not 1 .


## Exercise: Elevator - Door [4/5]

The cabin is also equipped with a door (kept in a separate module in the SMV program), that can be either open or closed. The door can receive either open, close or nop commands from the controller, and it responds opening, closing, or preserving the current state.

## Requirements:

- The door can receive an open command only if the door is closed.
- The door can receive a close command only if the door is open.


## Exercise: Elevator - Controller [5/5]

The controller takes in input (as sensory signals) the floor and the direction of motion of the cabin, the status of the door, and the status of the four buttons. It decides the controls to the engine, to the door and to the buttons.

## Requirements:

- no button can reach a state where it remains pressed forever.
- no pressed button can be reset until the cabin stops at the corresponding floor and opens the door.
- a button must be reset as soon as the cabin stops at the corresponding floor with the door open.
- the cabin can move only when the door is closed.
- if no button is pressed, the controller must issue no commands and the cabin must be standing.


## Contents

(1) Exercises

- Dining Philosophers
- Insertion Sort
- Cleaning Robot
(2) Extra Exercises (from previous years)
- Elevator
- Odd/Even Counter
- Overflow Counter


## Exercise: Odd/Even Counter [1/2]

Implement a 5-bit counter that alternates counting all odd numbers from 31 to 1 (e.g. 31, 29, 27, ... 3, 1) and counting all even numbers from 30 to 0 (e.g. 30, 28, 26, 2, 0). Use a variable "out" to represent the output of the counter. Use five Boolean variables "b0", "b1", "b2", "b3", "b4" to represent the bits of the counter, from the least-significative to the most-significative ones. Initially, all bits are set to TRUE. The transition relation is described as follows:

- "b0" changes value only when all other bits are FALSE
- "b1" changes value at each transition
- "b2" changes value only when "b1" is FALSE
- "b3" changes value only when both "b1" and "b2" are FALSE
- "b4" changes value only when "b1", "b2" and "b3" are all FALSE


## Exercise: Odd/Even Counter [2/2]

Model the 5-bit counter, express the following properties, and check with nuXmv that all properties are verified.

- it is necessarily always the case that, if out is 1 , then at the next step the value of the counter is 30
- it is necessarily always the case that if out $=31$ then in 5 iterations out will evaluate to 21
- it is always the case that b1 changes value at each iteration
- it is always the case that, if b1, b2 and b3 are all FALSE, then the next value of b4 is !b4
- infinitely often out is 0
- if out=30 then eventually in the future out $=20$


## Contents

(1) Exercises

- Dining Philosophers
- Insertion Sort
- Cleaning Robot
(2) Extra Exercises (from previous years)
- Elevator
- Odd/Even Counter
- Overflow Counter


## Exercise: Overflow Counter [1/3]

Implement a 3-bit counter which counts the number of times an input boolean variable "bin" changes value from FALSE to TRUE. Use three boolean variables "b0", "b1", "b2" to represent the bits of the counter, from the least-significant to the most-significant one. Use an output variable "out" to represent the value of the counter. Use a variable "overflow", with values in the set \{NO, YES \}, to keep track of a counter overflow event. Use a variable "obin" to keep track of the previous value of the input variable "bin", and an output variable "rise" to express the fact that "bin" changed value from FALSE to TRUE in the current step. Use an input boolean variable "reset" to reset the value of "b0", "b1", "b2" and "obin" to their initial value. Initially, "b0", "b1", "b2", "bin" and "obin" should be set to FALSE, while "overflow" should evaluate 'NO'.

## Exercise: Overflow Counter [2/3]

Implement, using the assign-syntax, the following transitions:

- "obin" is set to FALSE if "reset" is TRUE, and to "bin" otherwise
- "b0" is set to FALSE if "reset" is TRUE, it is set to "!b0" if "rise" is TRUE, and keeps its value otherwise
- "b1" is set to FALSE if "reset" is TRUE, it is set to "!b1" if "rise \& b0" is TRUE, and keeps its value otherwise
- "b2" is set to FALSE if "reset" is TRUE, it is set to "!b2" if "rise \& b0 \& b1" is TRUE, and keeps its value otherwise
- "overflow" is set to 'NO' if "reset" is TRUE, it is set to 'YES' if "rise \& b0 \& b1 \& b2" is TRUE, and keeps its value otherwise Manually verify that the simulation works as intended.


## Exercise: Overflow Counter [3/3]

Express the following properties, and have NUXmV verify that all properties are FALSE.

- CTL: it is necessarily always the case that infinitely often the counter is 0
- CTL: it is necessarily always the case that eventually the counter is always different than 0
- CTL: it is necessarily always the case that, if "overflow" is 'YES' in a given state then it also holds that "overflow" is 'YES' until "reset"
- CTL: it is necessarily always the case that when "b0", "b1" and "b2" are TRUE then from the next state eventually the value of counter will go back to 0
- LTL: if "rise" is TRUE infinitely often, then "overflow" is 'YES' infinitely often as well
- Bonus Point: explain why the latter formula is verified if CTL is used instead of LTL.


## Exercises Solutions

- will be uploaded on course website within a couple of days
- send me an email if you need help or you just want to propose your own solution for a review
- learning programming languages requires practice: try to come up with your own solutions first!

