The K-Medoids Clustering Method

- Find representative objects, called medoids, in clusters
- PAM (Partitioning Around Medoids, 1987)
	- starts from an initial set of medoids and iteratively replaces one of the medoids by one of the non-medoids if it improves the total distance of the resulting clustering
	- **PAM** works effectively for small data sets, but does not scale well for large data sets
- CLARA (Kaufmann & Rousseeuw, 1990)
- CLARANS (Ng & Han, 1994): Randomized sampling
- Focusing + spatial data structure (Ester et al., 1995)

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A Typical K-Medoids Algorithm (PAM)

PAM (Partitioning Around Medoids) (1987)

- **PAM (Kaufman and Rousseeuw, 1987), built in Splus**
- Use real object to represent the cluster
	- **Select** *k* representative objects arbitrarily
	- For each pair of non-selected object **h** and selected object **i**, calculate the total swapping cost **TCih**
	- For each pair of **i** and **h**,
		- If $TC_{ih} < 0$, *i* is replaced by **h**
		- Then assign each non-selected object to the most similar representative object
	- **repeat steps 2-3 until there is no change**

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What Is the Problem with PAM?

- **Pam is more robust than k-means in the presence of** noise and outliers because a medoid is less influenced by outliers or other extreme values than a mean
- Pam works efficiently for small data sets but does not **scale well** for large data sets.
	- \bullet O(k(n-k)²) for each iteration

where n is $#$ of data, k is $#$ of clusters

 \rightarrow Sampling based method,

CLARA(Clustering LARge Applications)

CLARA (Clustering Large Applications) (1990)

- CLARA (Kaufmann and Rousseeuw in 1990)
	- Built in statistical analysis packages, such as $S+$
- It draws *multiple samples* of the data set, applies *PAM* on each sample, and gives the best clustering as the output
- Strength: deals with larger data sets than PAM
- **Weakness:**
	- **Efficiency depends on the sample size**
	- A good clustering based on samples will not necessarily represent a good clustering of the whole data set if the sample is biased

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CLARANS ("Randomized" CLARA) (1994)

- CLARANS (A Clustering Algorithm based on Randomized Search) (Ng and Han'94)
- **CLARANS** draws sample of neighbors dynamically
- The clustering process can be presented as searching a graph where every node is a potential solution, that is, a set of k medoids
- If the local optimum is found, CLARANS starts with new randomly selected node in search for a new local optimum
- It is more efficient and scalable than both PAM and CLARA
- **Focusing techniques and spatial access structures may** further improve its performance (Ester et al.'95)

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Hierarchical Clustering

 Use distance matrix as clustering criteria. This method does not require the number of clusters **k** as an input, but needs a termination condition

AGNES (Agglomerative Nesting)

- Introduced in Kaufmann and Rousseeuw (1990)
- **Implemented in statistical analysis packages, e.g., Splus**
- Use the Single-Link method and the dissimilarity matrix.
- Merge nodes that have the least dissimilarity
- Go on in a non-descending fashion
- Eventually all nodes belong to the same cluster

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Dendrogram: **Shows How the Clusters are Merged**

DIANA (Divisive Analysis)

- Introduced in Kaufmann and Rousseeuw (1990)
- Implemented in statistical analysis packages, e.g., Splus
- **Inverse order of AGNES**
- Eventually each node forms a cluster on its own

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Recent Hierarchical Clustering Methods

- **Major weakness of agglomerative clustering methods**
	- **do not scale well:** time complexity of at least $O(n^2)$, where *n* is the number of total objects
	- can never undo what was done previously
- **Integration of hierarchical with distance-based clustering**
	- BIRCH (1996): uses CF-tree and incrementally adjusts the quality of sub-clusters
	- ROCK (1999): clustering categorical data by neighbor and link analysis
	- CHAMELEON (1999): hierarchical clustering using dynamic modeling

BIRCH (1996)

- **Birch: Balanced Iterative Reducing and Clustering using** Hierarchies (Zhang, Ramakrishnan & Livny, SIGMOD'96)
- **Incrementally construct a CF (Clustering Feature) tree, a** hierarchical data structure for multiphase clustering
	- **Phase 1: scan DB to build an initial in-memory CF tree (a multi-level)** compression of the data that tries to preserve the inherent clustering structure of the data)
	- **Phase 2: use an arbitrary clustering algorithm to cluster the leaf** nodes of the CF-tree
- Scales linearly: finds a good clustering with a single scan and improves the quality with a few additional scans
- Weakness: handles only numeric data, and sensitive to the order of the data record.

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Clustering Feature Vector in BIRCH

Clustering Feature: *CF = (N, LS, SS)*

N: **Number of data points**

$$
LS: \sum_{i=1}^{N} = \overrightarrow{X_i}
$$

SS:
$$
\sum_{i=1}^{N} = \overrightarrow{X_i^2}
$$

CF-Tree in BIRCH

- **Clustering feature:**
	- summary of the statistics for a given subcluster: the 0-th, 1st and 2nd moments of the subcluster from the statistical point of view.
	- registers crucial measurements for computing cluster and utilizes storage efficiently
- A CF tree is a height-balanced tree that stores the clustering features \blacksquare for a hierarchical clustering
	- A nonleaf node in a tree has descendants or "children"
	- The nonleaf nodes store sums of the CFs of their children
- A CF tree has two parameters
	- **Branching factor: specify the maximum number of children.**
	- **threshold: max diameter of sub-clusters stored at the leaf nodes**

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The CF Tree Structure Root $CF₁$ $CF₂$ $CF₃$ CF_6 $B = 7$ $child₆$ child₁ $child₃$ child₂ $L = 6$ Non-leaf node $CF₂$ $CF₅$ CF₁ $CF₃$ child₃ child $child_2$ child₅ Leaf node Leaf node prev $|CF_1|CF_2|$ "" $|CF_6|$ next $\boxed{}$ prev $|CF_1|CF_2|$ "" $|CF_4|$ prev CF_1CF_2 CF_4 next \mathbf{I} Data Mining for Knowledge Management 24

Clustering Categorical Data: The ROCK Algorithm

- **ROCK: RObust Clustering using linKs**
	- S. Guha, R. Rastogi & K. Shim, ICDE'99
- **Major ideas**
	- Not distance-based
	- **Use links to measure similarity/proximity**
	- **Measure similarity between points, as well as between their corresponding** neighborhoods
		- two points are closer together if they share some of their neighbors
- Algorithm: sampling-based clustering
	- Draw random sample
	- **Cluster with links**
	- **Label data in disk**
	- **Computational complexity:** $O(n^2 + nm_m m_a + n^2 \log n)$

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Similarity Measure in ROCK

- **Traditional measures for categorical data may not work well, e.g.,** Jaccard coefficient
- **Example: Two groups (clusters) of transactions**
	- C_1 , <a, b, c, d, e>: {a, b, c}, {a, b, d}, {a, b, e}, {a, c, d}, {a, c, e},
	- $\{a, d, e\}$, $\{b, c, d\}$, $\{b, c, e\}$, $\{b, d, e\}$, $\{c, d, e\}$
	- **c**₂. <a, b, f, g>: {a, b, f}, {a, b, g}, {a, f, g}, {b, f, g}

Similarity Measure in ROCK

- Traditional measures for categorical data may not work well, e.g., Jaccard coefficient
- **Example: Two groups (clusters) of transactions**
	- C_1 , <a, b, c, d, e>: {a, b, c}, {a, b, d}, {a, b, e}, {a, c, d}, {a, c, e},
	- {a, d, e}, {b, c, d}, {b, c, e}, {b, d, e}, {c, d, e}
	- **.** C_2 . <a, b, f, g>: {a, b, f}, {a, b, g}, {a, f, g}, {b, f, g}
- **Jaccard co-efficient may lead to wrong clustering result**
	- C_1 : 0.2 ({a, b, c}, {b, d, e}} to 0.5 ({a, b, c}, {a, b, d})
	- **c**₁ & C₂: could be as high as 0.5 ({a, b, c}, {a, b, f})

1 Jaccard co-efficient-based similarity function:
$$
Sim(T_1, T_2) = \frac{|T_1 \cap T_2|}{|T_1 \cup T_2|}
$$

Ex. Let $T_1 = \{a, b, c\}$, $T_2 = \{c, d, e\}$

$$
Sim(T_1, T_2) = \frac{|\{c\}|}{|\{a, b, c, d, e\}|} = \frac{1}{5} = 0.2
$$

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Link Measure in ROCK

- **Links:** $#$ of common neighbors
	- **c**₁ <a, b, c, d, e>: {a, b, c}, {a, b, d}, {a, b, e}, {a, c, d}, {a, c, e}, {a, d, e}, {b, c, d}, {b, c, e}, {b, d, e}, {c, d, e}
	- C_2 <a, b, f, g>: {a, b, f}, {a, b, g}, {a, f, g}, {b, f, g}

Link Measure in ROCK

- **Links:** $#$ of common neighbors
	- C₁ <a, b, c, d, e>: {a, b, c}, {a, b, d}, {a, b, e}, {a, c, d}, {a, c, e}, {a, d, e}, $\{b, c, d\}$, $\{b, c, e\}$, $\{b, d, e\}$, $\{c, d, e\}$
	- C₂ < a, b, f, g > : {a, b, f}, {a, b, g}, {a, f, g}, {b, f, g}
- Let $T_1 = \{a, b, c\}$, $T_2 = \{c, d, e\}$, $T_3 = \{a, b, f\}$

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Link Measure in ROCK

- **Links:** $#$ of common neighbors
	- **c**₁ <a, b, c, d, e>: {a, b, c}, {a, b, d}, {a, b, e}, {a, c, d}, {a, c, e}, {a, d, e}, {b, c, d}, {b, c, e}, {b, d, e}, {c, d, e}
	- C₂ < a, b, f, g > : {a, b, f}, {a, b, g}, {a, f, g}, {b, f, g}
- Let $T_1 = \{a, b, c\}$, $T_2 = \{c, d, e\}$, $T_3 = \{a, b, f\}$
	- **IDE** link(T_1, T_2) = 4, since they have 4 common neighbors
		- $\{a, c, d\}, \{a, c, e\}, \{b, c, d\}, \{b, c, e\}$

Link Measure in ROCK

- Links: # of common neighbors
	- **c**₁ <a, b, c, d, e>: {a, b, c}, {a, b, d}, {a, b, e}, {a, c, d}, {a, c, e}, {a, d, e}, $\{b, c, d\}$, $\{b, c, e\}$, $\{b, d, e\}$, $\{c, d, e\}$
	- C₂ <a, b, f, g>: $\{a, b, f\}$, $\{a, b, g\}$, $\{a, f, g\}$, $\{b, f, g\}$
- Let $T_1 = \{a, b, c\}$, $T_2 = \{c, d, e\}$, $T_3 = \{a, b, f\}$
	- **u** link(T_1 , T_2) = 4, since they have 4 common neighbors
		- $\{a, c, d\}, \{a, c, e\}, \{b, c, d\}, \{b, c, e\}$
	- **If** link(T_1 , T_3) = 3, since they have 3 common neighbors
		- $\{a, b, d\}, \{a, b, e\}, \{a, b, g\}$
- Thus, link is a better measure than Jaccard coefficient

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CHAMELEON: Hierarchical Clustering Using Dynamic Modeling (1999)

- CHAMELEON: by G. Karypis, E.H. Han, and V. Kumar'99
- Measures the similarity based on a dynamic model
	- Two clusters are merged only if the *interconnectivity* and *closeness* (*proximity*) between two clusters are high *relative to* the internal interconnectivity of the clusters and closeness of items within the clusters
	- **Cure** ignores information about **interconnectivity** of the objects, **Rock** ignores information about the **closeness** of two clusters
- A two-phase algorithm
	- 1. Use a graph partitioning algorithm: cluster objects into a large number of relatively small sub-clusters
	- 2. Use an agglomerative hierarchical clustering algorithm: find the genuine clusters by repeatedly combining these sub-clusters

CHAMELEON (Clustering Complex Objects)

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Density-Based Clustering Methods

- **Clustering based on density (local cluster criterion), such** as density-connected points
- **Major features:**
	- Discover clusters of arbitrary shape
	- **Handle noise**
	- One scan
	- **Need density parameters as termination condition**
- **Several interesting studies:**
	- DBSCAN: Ester, et al. (KDD'96)
	- OPTICS: Ankerst, et al (SIGMOD'99).
	- DENCLUE: Hinneburg & D. Keim (KDD'98)
	- CLIQUE: Agrawal, et al. (SIGMOD'98) (more grid-based)

Density-Based Clustering: Basic Concepts

- \blacksquare Two parameters:
	- **Eps:** Maximum radius of the neighbourhood
	- MinPts: Minimum number of points in an Eps-neighbourhood of that point
- $N_{FpS}(p)$: {*q* belongs to *D* | dist(p, q) <= Eps}
- Directly density-reachable: A point ρ is directly densityreachable from a point q w.r.t. Eps, MinPts if
	- p belongs to $N_{Eps}(q)$

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Density-Reachable and Density-Connected

- Density-reachable:
	- A point ρ is density-reachable from a point q w.r.t. Eps, MinPts if there is a chain of points $p_1, ..., p_m$, $p_1 = q$, $p_n = p$ such that ρ_{i+1} is directly density-reachable from p_i
- **Density-connected**
	- A point ρ is density-connected to a point q w.r.t. *Eps, MinPts* if there is a point q such that both, ρ and q are densityreachable from o w.r.t. Eps and MinPts

DBSCAN: Density Based Spatial Clustering of Applications with Noise

- Relies on a *density-based* notion of cluster: A *cluster* is defined as a maximal set of density-connected points
- Discovers clusters of arbitrary shape in spatial databases with noise

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DBSCAN: The Algorithm

- Arbitrary select a point p
- Retrieve all points density-reachable from p w.r.t. Eps and MinPts.
- If p is a core point, a cluster is formed.
- If ρ is a border point, no points are density-reachable from p and DBSCAN visits the next point of the database.
- Continue the process until all of the points have been processed.

DBSCAN: Sensitive to Parameters

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Model-Based Clustering

- What is model-based clustering?
	- **Attempt to optimize the fit between the given data and some** mathematical model
	- Based on the assumption: Data are generated by a mixture of underlying probability distribution
- **Typical methods**
	- **Statistical approach**
		- EM (Expectation maximization), AutoClass
	- **Machine learning approach**
		- COBWEB, CLASSIT
	- **Neural network approach**
		- SOM (Self-Organizing Feature Map)

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EM — Expectation Maximization

- \blacksquare EM \blacksquare A popular iterative refinement algorithm
- **An extension to k-means**
	- Assign each object to a cluster according to a weight (prob. distribution)
	- New means are computed based on weighted measures
- General idea
	- **Starts with an initial estimate of the parameter vector**
	- **Iteratively rescores the patterns against the mixture density produced by** the parameter vector
	- The rescored patterns are used to update the parameter updates
	- **Patterns belonging to the same cluster, if they are placed by their scores in** a particular component
- Algorithm converges fast but may not be in global optima

The EM (Expectation Maximization) Algorithm

- **Initially, randomly assign k cluster centers**
- **Iteratively refine the clusters based on two steps**
	- **Expectation step:** assign each data point X_i to cluster C_i with the following probability $(2) 2222$

$$
P(X_i \in C_k) = p(C_k | X_i) = \frac{p(C_k)p(X_i|C_k)}{p(X_i)},
$$

- **Maximization step:**
	- **Estimation of model parameters**

$$
m_k = \frac{1}{N} \sum_{i=1}^{N} \frac{X_i P(X_i \in C_k)}{\sum_j P(X_i \in C_j)}
$$

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Clustering High-Dimensional Data

- **Clustering high-dimensional data**
	- **Many applications: text documents, DNA micro-array data**
	- **Major challenges:**
		- Many irrelevant dimensions may mask clusters
		- Distance measure becomes meaningless—due to equi-distance
		- Clusters may exist only in some subspaces
- Methods
	- **Feature transformation: only effective if most dimensions are relevant**
		- PCA & SVD useful only when features are highly correlated/redundant
	- **Feature selection: wrapper or filter approaches**
		- useful to find a subspace where the data have nice clusters
	- Subspace-clustering: find clusters in all the possible subspaces
		- CLIQUE, ProClus, and frequent pattern-based clustering

CLIQUE (Clustering In QUEst)

- Agrawal, Gehrke, Gunopulos, Raghavan (SIGMOD'98)
- Automatically identifying subspaces of a high dimensional data space that allow better clustering than original space
- CLIQUE can be considered as both density-based and grid-based
	- It partitions each dimension into the same number of equal length interval
	- It partitions an m-dimensional data space into non-overlapping rectangular units
	- A unit is dense if the fraction of total data points contained in the unit exceeds the input model parameter
	- A cluster is a maximal set of connected dense units within a subspace

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CLIQUE: The Major Steps

- Partition the data space and find the number of points that lie inside each cell of the partition.
- I dentify the subspaces that contain clusters using the Apriori principle
- **Identify clusters**
	- Determine dense units in all subspaces of interests
	- Determine connected dense units in all subspaces of interests.
- Generate minimal description for the clusters
	- Determine maximal regions that cover a cluster of connected dense units for each cluster
	- Determination of minimal cover for each cluster

Strength and Weakness of CLIQUE

- **Strength**
	- automatically finds subspaces of the highest dimensionality such that high density clusters exist in those subspaces
	- insensitive to the order of records in input and does not presume some canonical data distribution
	- scales *linearly* with the size of input and has good scalability as the number of dimensions in the data increases

Neakness

The accuracy of the clustering result may be degraded at the expense of simplicity of the method

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Summary

- **Cluster analysis groups objects based on their similarity** and has wide applications
- **EXEC** Measure of similarity can be computed for various types of data
- Clustering algorithms can be categorized into partitioning methods, hierarchical methods, density-based methods, grid-based methods, and model-based methods
- **Dutlier detection and analysis are very useful for fraud** detection, etc. and can be performed by statistical, distance-based or deviation-based approaches
- There are still lots of research issues on cluster analysis

Problems and Challenges

- Considerable progress has been made in scalable clustering methods
	- **Partitioning: k-means, k-medoids, CLARANS**
	- Hierarchical: BIRCH, ROCK, CHAMELEON
	- Density-based: DBSCAN, OPTICS, DenClue
	- Grid-based: STING, WaveCluster, CLIQUE
	- Model-based: EM, Cobweb, SOM
	- Frequent pattern-based: pCluster
	- Constraint-based: COD, constrained-clustering
- **Current clustering techniques do not address all the** requirements adequately, still an active area of research

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