Roadmap

- Frequent Patterns
- A-Priori Algorithm
- Improvements to A-Priori
  - Park-Chen-Yu Algorithm
  - Multistage Algorithm
  - Approximate Algorithms
  - Compacting Results

PCY Algorithm

- Hash-based improvement to A-Priori.
- During Pass 1 of A-priori, most memory is idle.
- Use that memory to keep counts of buckets into which pairs of items are hashed.
  - Just the count, not the pairs themselves.
- Gives extra condition that candidate pairs must satisfy on Pass 2.

- J. Park, M. Chen, and P. Yu. An effective hash-based algorithm for mining association rules. In *SIGMOD'95*

PCY Algorithm --- Before Pass 1 Organize Main Memory

- Space to count each item.
  - One (typically) 4-byte integer per item.
- Use the rest of the space for as many integers, representing buckets, as we can.

PCY Algorithm --- Pass 1

```
FOR (each basket) {
    FOR (each item)
      add 1 to item's count;
    FOR (each pair of items) {
      hash the pair to a bucket;
      add 1 to the count for that
    bucket
    }
}
```

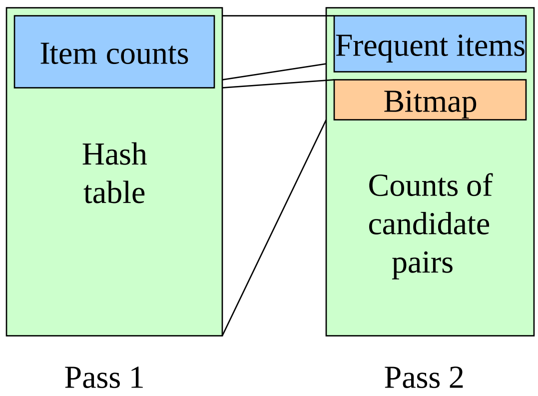
Observations About Buckets

1. If a bucket contains a frequent pair, then the bucket is surely frequent.
   - We cannot use the hash table to eliminate any member of this bucket.
2. Even without any frequent pair, a bucket can be frequent.
   - Again, nothing in the bucket can be eliminated.
3. But in the best case, the count for a bucket is less than the support s.
   - Now, all pairs that hash to this bucket can be eliminated as candidates, even if the pair consists of two frequent items.

PCY Algorithm --- Between Passes

- Replace the buckets by a bit-vector:
  - 1 means the bucket count exceeds the support *s* (frequent bucket); 0 means it did not.
- Integers are replaced by bits, so the bit-vector requires little second-pass space.
- Also, decide which items are frequent and list them for the second pass.

Picture of PCY



PCY Algorithm --- Pass 2

- Count all pairs {*i,j*} that meet the conditions:
  1. Both *i* and *j* are frequent items.
  2. The pair {*i,j*}, hashes to a bucket number whose bit in the bit vector is 1.
- Notice all these conditions are necessary for the pair to have a chance of being frequent.

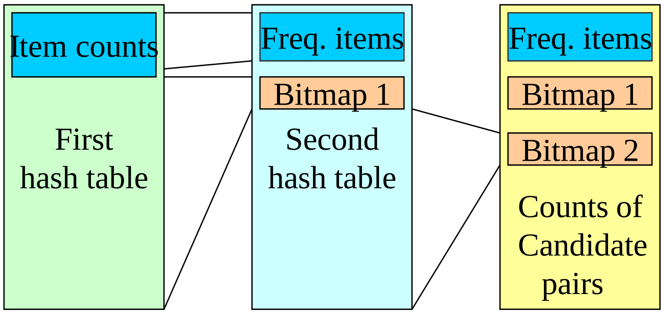
Memory Details

- Hash table requires buckets of 2-4 bytes.
  - Number of buckets thus almost 1/4-1/2 of the number of bytes of main memory.
- On second pass, a table of (item, item, count) triples is essential.
  - Thus, hash table must eliminate 2/3 of the candidate pairs to beat a-priori.

Multistage Algorithm

- Key idea: After Pass 1 of PCY, rehash only those pairs that qualify for Pass 2 of PCY.
- On middle pass, fewer pairs contribute to buckets, so fewer *false positives* --- frequent buckets with no frequent pair.

Multistage Picture



Multistage --- Pass 3

- Count only those pairs $\{i,j\}$ that satisfy:
  1. Both i and j are frequent items.
  2. Using the first hash function, the pair hashes to a bucket whose bit in the first bit-vector is 1.
  3. Using the second hash function, the pair hashes to a bucket whose bit in the second bit-vector is 1.

Important Points

1. The two hash functions have to be independent.

2. We need to check both hashes on the third pass.
   - If not, we would wind up counting pairs of frequent items that hashed first to an infrequent bucket but happened to hash second to a frequent bucket.

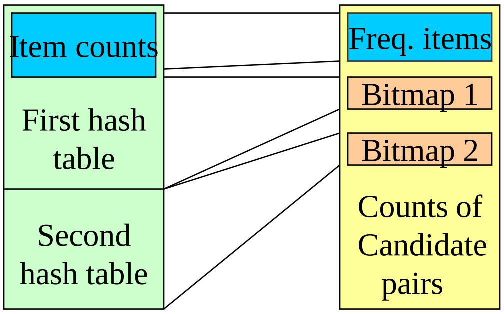
Multihash

- Key idea: use several independent hash tables on the first pass.

- Risk: halving the number of buckets doubles the average count. We have to be sure most buckets will still not reach count *s*.

- If so, we can get a benefit like multistage, but in only 2 passes.

Multihash Picture



Extensions

- Either multistage or multihash can use more than two hash functions.
- In multistage, there is a point of diminishing returns, since the bit-vectors eventually consume all of main memory.
- For multihash, the bit-vectors total exactly what one PCY bitmap does, but too many hash functions makes all counts $\geq s$.

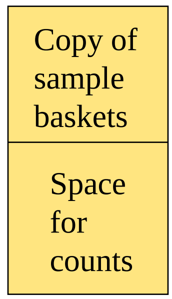
All (Or Most) Frequent Itemsets In $\leq$ 2 Passes

- Simple algorithm.
- SON (Savasere, Omiecinski, and Navathe).
- Toivonen.

Simple Algorithm --- (1)

- Take a main-memory-sized random sample of the market baskets.
- Run a-priori or one of its improvements (for sets of all sizes, not just pairs) in main memory, so you don't pay for disk I/O each time you increase the size of itemsets.
  - Be sure you leave enough space for counts.

The Picture



Simple Algorithm --- (2)

- Use as your support threshold a suitable, scaled-back number.
  - E.g., if your sample is 1/100 of the baskets, use *s* /100 as your support threshold instead of *s* .

Simple Algorithm --- Option

- Optionally, verify that your guesses are truly frequent in the entire data set by a second pass.
- But you don't catch sets frequent in the whole but not in the sample.
  - Smaller threshold, e.g., $s/125$, helps.

SON Algorithm --- (1)

- Repeatedly read small subsets of the baskets into main memory and perform the first pass of the simple algorithm on each subset.
- An itemset becomes a candidate if it is found to be frequent in *any* one or more subsets of the baskets.
- A. Savasere, E. Omiecinski, and S. Navathe. An efficient algorithm for mining association in large databases. In VLDB'95

SON Algorithm --- (2)

- On a second pass, count all the candidate itemsets and determine which are frequent in the entire set.
- Key "monotonicity" idea: an itemset cannot be frequent in the entire set of baskets unless it is frequent in at least one subset.

Toivonen's Algorithm --- (1)

- Start as in the simple algorithm, but lower the threshold slightly for the sample.
  - Example: if the sample is 1% of the baskets, use *s* /125 as the support threshold rather than *s* /100.
  - Goal is to avoid missing any itemset that is frequent in the full set of baskets.
- H. Toivonen. Sampling large databases for association rules. In *VLDB'96*

Toivonen's Algorithm --- (2)

- Add to the itemsets that are frequent in the sample the *negative border* of these itemsets.
- An itemset is in the negative border if it is not deemed frequent in the sample, but *all* its immediate subsets are.

Example: Negative Border

- *ABCD* is in the negative border if and only if it is not frequent, but all of *ABC*, *BCD*, *ACD*, and *ABD* are.

Toivonen's Algorithm --- (3)

- In a second pass, count all candidate frequent itemsets from the first pass, and also count the negative border.
- If no itemset from the negative border turns out to be frequent, then the candidates found to be frequent in the whole data are *exactly* the frequent itemsets.

Toivonen's Algorithm --- (4)

- What if we find something in the negative border is actually frequent?
- We must start over again!
- Try to choose the support threshold so the probability of failure is low, while the number of itemsets checked on the second pass fits in main-memory.

Theorem:

- If there is an itemset frequent in the whole, but not frequent in the sample, then there is a member of the negative border frequent in the whole.

Proof:

- Suppose not; i.e., there is an itemset *S* frequent in the whole, but not frequent or in the negative border in the sample.
- Let *T* be a smallest subset of *S* that is not frequent in the sample.
- *T* is frequent in the whole (monotonicity).
- *T* is in the negative border (else not "smallest").

Compacting the Output

- A long pattern contains a combinatorial number of sub-patterns, e.g., $\{a_1, \ldots, a_{100}\}$ contains $({}^{100}{}_{1}) + ({}^{100}{}_{2}) + \ldots + ({}^{1}{}_{1}{}^{0}{}_{0}{}^{0}{}_{0}) = 2^{100} - 1 = 1.27*10^{30}$ sub-patterns!

- Solution: *Mine closed patterns and max-patterns instead*

1. *Maximal Frequent itemsets* : no immediate superset is frequent.

2. *Closed itemsets* : no immediate superset has the same count.
   - Stores not only frequent information, but exact counts.

Closed Patterns and Max-Patterns

- An itemset X is closed if X is *frequent* and there exists *no super-pattern* Y X, *with the same support* as X (proposed by Pasquier, et al. @ ICDT'99)

- An itemset X is a max-pattern if X is frequent and there exists no frequent super-pattern Y X (proposed by Bayardo @ SIGMOD'98)

- Closed pattern is a lossless compression of freq. patterns
  - Reducing the # of patterns and rules

Example: Maximal/Closed

	Count	Maximal s=3	Closed
A	4	No	No
B	5	No	Yes
C	3	No	No
AB	4	Yes	Yes
AC	2	No	No
BC	3	Yes	Yes
ABC	2	No	Yes

Closed Patterns and Max-Patterns

- Exercise. DB = $\{<a_1, \ldots, a_{100}>, < a_1, \ldots, a_{50}>\}$
  - Min_sup = 1.
- What is the set of closed itemsets?

Closed Patterns and Max-Patterns

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  - $<a_1, \ldots, a_{100}>: 1$
  - $< a_1, \ldots, a_{50}>: 2$

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Closed Patterns and Max-Patterns

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  - Min_sup = 1.
- What is the set of closed itemsets?
  - $<a_1, \ldots, a_{100}>$: 1
  - $< a_1, \ldots, a_{50}>$: 2
- What is the set of max-patterns?
  - $<a_1, \ldots, a_{100}>$: 1
- What is the set of all patterns?

Ref: Basic Concepts of Frequent Pattern Mining

- (Association Rules) R. Agrawal, T. Imielinski, and A. Swami. Mining association rules between sets of items in large databases. SIGMOD'93.
- (Max-pattern) R. J. Bayardo. Efficiently mining long patterns from databases. SIGMOD'98.
- (Closed-pattern) N. Pasquier, Y. Bastide, R. Taouil, and L. Lakhal. Discovering frequent closed itemsets for association rules. ICDT'99.
- (Sequential pattern) R. Agrawal and R. Srikant. Mining sequential patterns. ICDE'95

Ref: Apriori and Its Improvements

- R. Agrawal and R. Srikant. Fast algorithms for mining association rules. VLDB'94.
- H. Mannila, H. Toivonen, and A. I. Verkamo. Efficient algorithms for discovering association rules. KDD'94.
- A. Savasere, E. Omiecinski, and S. Navathe. An efficient algorithm for mining association rules in large databases. VLDB'95.
- J. S. Park, M. S. Chen, and P. S. Yu. An effective hash-based algorithm for mining association rules. SIGMOD'95.
- H. Toivonen. Sampling large databases for association rules. VLDB'96.
- S. Brin, R. Motwani, J. D. Ullman, and S. Tsur. Dynamic itemset counting and implication rules for market basket analysis. SIGMOD'97.
- S. Sarawagi, S. Thomas, and R. Agrawal. Integrating association rule mining with relational database systems: Alternatives and implications. SIGMOD'98.