

# A Scalable Noise Reduction Technique for Large Case-Based Systems

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# Outline of the talk

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- Noise Reduction for IBL and CBR

- Noise Reduction for IBL: state-of-the-art

- Noise Reduction with Local SVM

- The Issue of Scalability for Competence Enhancement

## 2 Noise reduction for large datasets: FaLKNR

- Adopting the Cover Tree Data-structure

- Introducing the Assignment Neighbourhood

- Local Model Selection for FaLKNR

- Computational Complexity Analysis

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- Experimental Procedure and Datasets

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# Noise Reduction for Case-Based Reasoning (CBR)

- All (supervised) learning systems must deal with noise
- CBR is instance-based and it is thus particularly affected by noise
- Considering more instance for local learning ( $k$ -NN with  $k > 1$ ) can alleviate noise problems but with large  $k$  the locality assumption is violated and the accuracy decreases
- Noisy examples can prevent case-based explanation

## Noise Reduction (NR) for Instance-Based Learning (IBL)

Noise reduction is an important pre-processing step that allows higher accuracy performances for IBL and CBR systems.

**N.B.** noise reduction can be important in other machine learning fields:

- improve the quality of data in medical domains
- data cleansing in bioinformatics and high-throughput techniques
- simplify machine learning models and training phases

# State-of-the-art noise reduction techniques for IBL

Following [Wilson & Martinez, 2000] the most effective NR techniques are:

**Edited NN (ENN) Rule** removes from the training set examples that do not agree with the majority of their  $k$ -NN [Wilson,1972]

**Repeated Edited NN (RENN) Rule** repeats the ENN algorithm until no further eliminations can be made [Tomek,1976]

**All- $k$ NN (AkNN) rule** repeat the ENN rule for each example using all neighbourhood size between 1 and  $k$  [Tomek,1976]

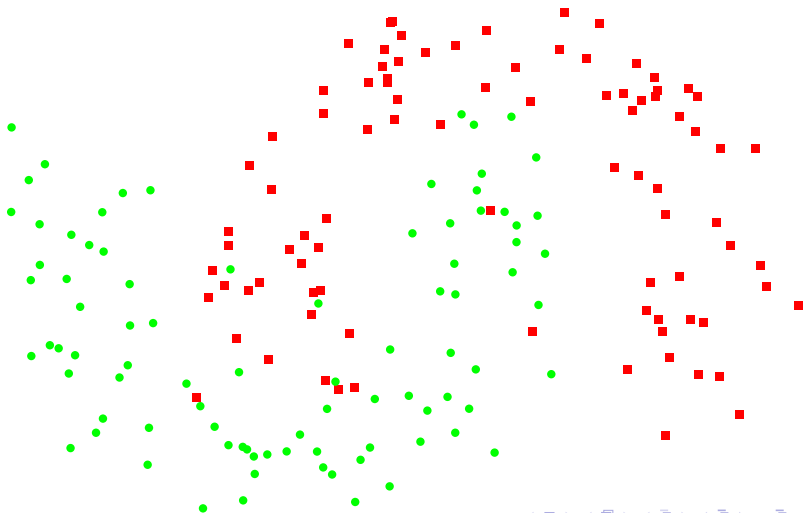
## State-of-the-art for Noise Reduction

Despite their simplicity, ENN, RENN and AkNN are not overcome by more recent approaches for general CBR and IBL problems

**N.B.:** For specific field some NR approaches can perform particularly well (e.g. BBNR [Delany & Cunningham,2007] for spam classification and [Malossini, Blanzieri, Ng,2004] for microarray mislabelling detection)

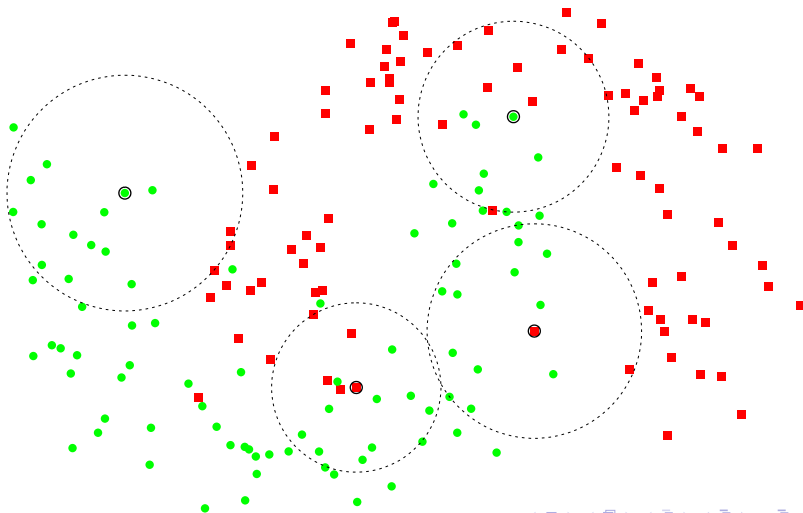
# Noise Reduction with Local Support Vector Machines

① for each training example...



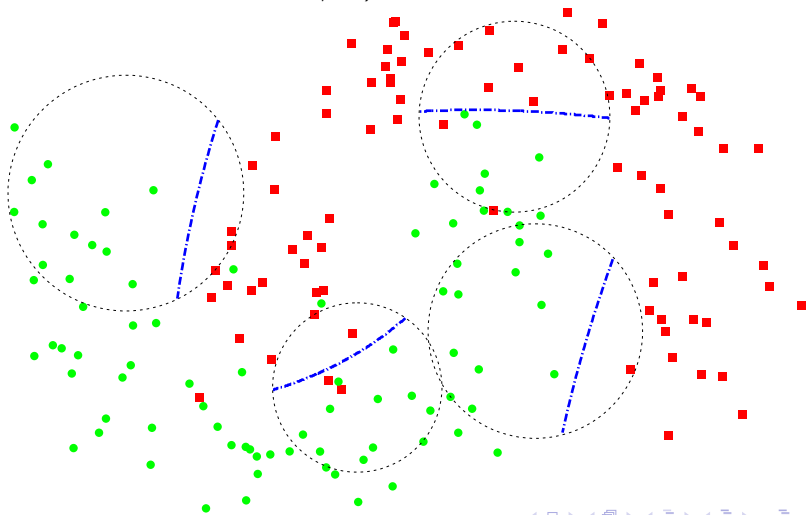
# Noise Reduction with Local Support Vector Machines

② ... retrieve its neighbourhood ( $k = 15$ )



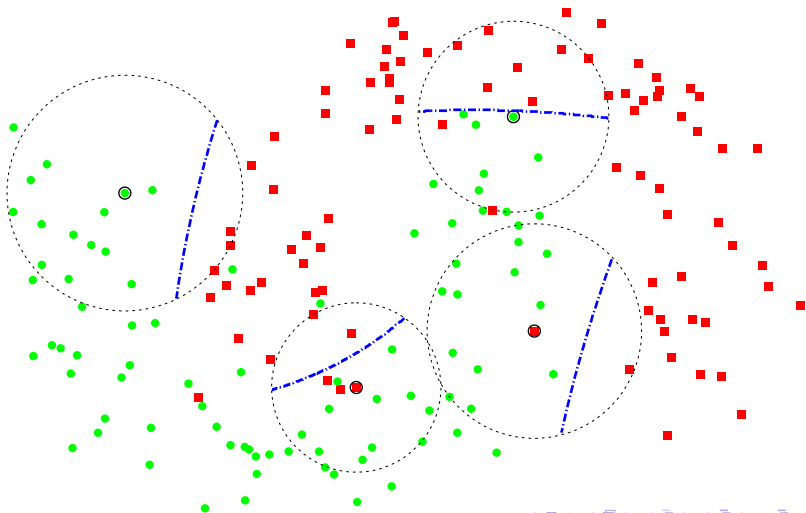
# Noise Reduction with Local Support Vector Machines

- 3 train a Local SVM model for each neighbourhood ( $C = 10$ , RBF kernel with  $\sigma = 1/10$ )



# Noise Reduction with Local Support Vector Machines

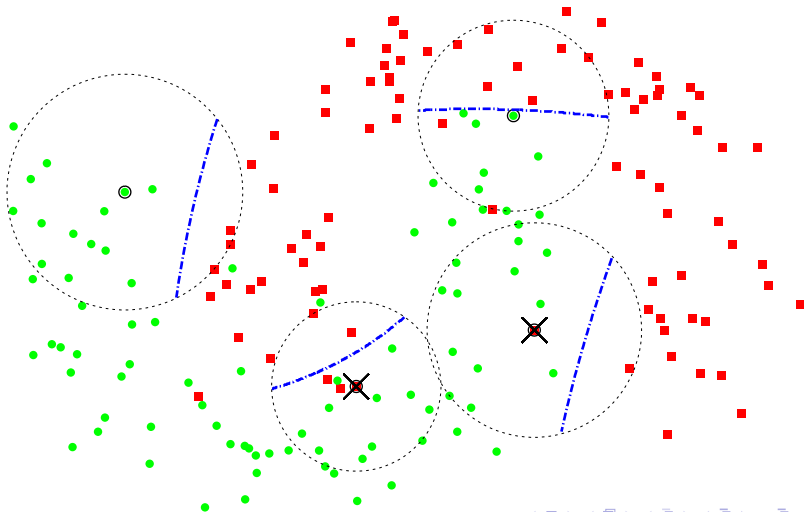
- predict the labels of the central examples of the neighbourhoods





# Noise Reduction with Local Support Vector Machines

- remove the examples that are misclassified by the Local SVM model



# Noise Reduction with Local Support Vector Machines

The LSVM-nr algorithm [Segata, Blanzieri, Delany, Cunningham, 2008] for noise reduction with Local SVM uses the probabilistic estimate of Local SVM to remove noisy training examples (in accordance to a threshold)

LSVM-nr is more accurate than ENN, RENN and AkNN

LSVM-nr showed statistically better results on the induced  $k$ -NN accuracies on 15 real datasets.

LSVM-nr is particularly effective also for:

- spam filtering (like BBNR)
- Gaussian noise in feature values
- uneven class densities

LSVM-nr is computationally less efficient than ENN and it is not suitable for large and very large CBR systems. . .

# The Issue of Scalability for Noise Reduction

Practical motivations:

- Modern CBR systems can be very large
- Datasets in medical and bioinformatics can be huge
- Accuracies of IBL are higher when the training set density is rather high (and thus the data is abundant)

Theoretical motivations:

method	learning bound	condition
NN	$2 \times$ Bayes Error	$n \mapsto \infty$
$k$ -NN	Bayes Error	$n \mapsto \infty, k \mapsto \infty, k/n \mapsto 0$
edited NN	Bayes Error	$n \mapsto \infty$

## How to improving classification accuracy of IBL and CBR

- ① carefully remove noisy examples from the Case-Base: LSVM-nr
- ② scale the noise reduction system in order to use as many examples as possible for NN classification: the topic of the present work

# Noise reduction for large datasets: FaLKNR

**F**ast **L**ocal **K**ernel **N**oise **R**eduction (FaLKNR) is a noise reduction technique based on Local SVM noise reduction scalable for large datasets.

The main ideas contained in FaLKNR are:

- Adopting the Cover Tree data-structure
- Developing a set of strategies to lower the number of neighbourhoods that need to be retrieved and the number of local SVM that need to be trained
- Developing a local model selection approach to efficiently select the hyper-parameters of the technique

# The Cover Tree Data-structure [Beygelzimer, Kakade, Langford, 2006]

Cover Trees (CT) are indexed trees satisfying the following invariants:

**Nesting**  $C_i \subset C_{i-1}$

**Covering**  $\forall p \in C_{i-1}$  there exists a  $q \in C_i$  such that  $dist(p, q) \leq 2^i$  and  $\exists! q$  that is a parent of  $p$

**Separation**  $\forall p, q \in C_i, dist(p, q) > 2^i$

they permits excellent performances:

space requirements:  $\mathcal{O}(n)$

construction time:  $\mathcal{O}(n \log n)$

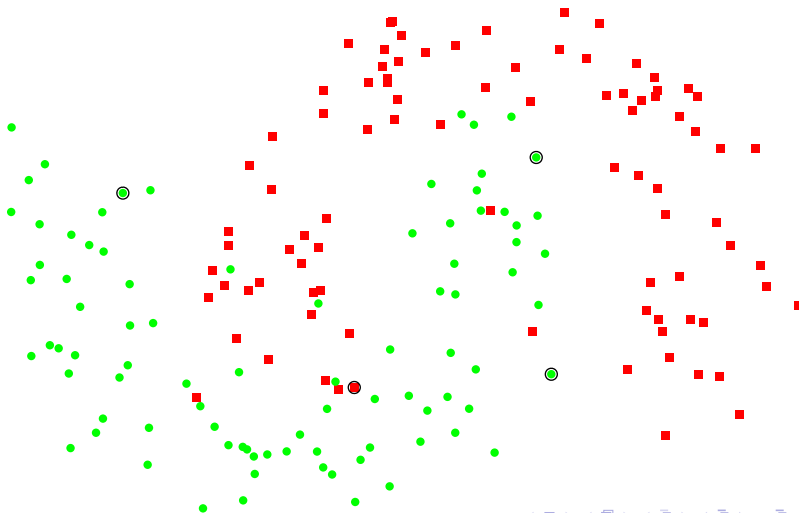
single insertion/removal/query:  $\mathcal{O}(\log n)$

CT are applicable in metric spaces, thus in Hilbert spaces

$$\|\Phi(x) - \Phi(x')\|^2 = K(x, x) + K(x', x') - 2K(x, x')$$

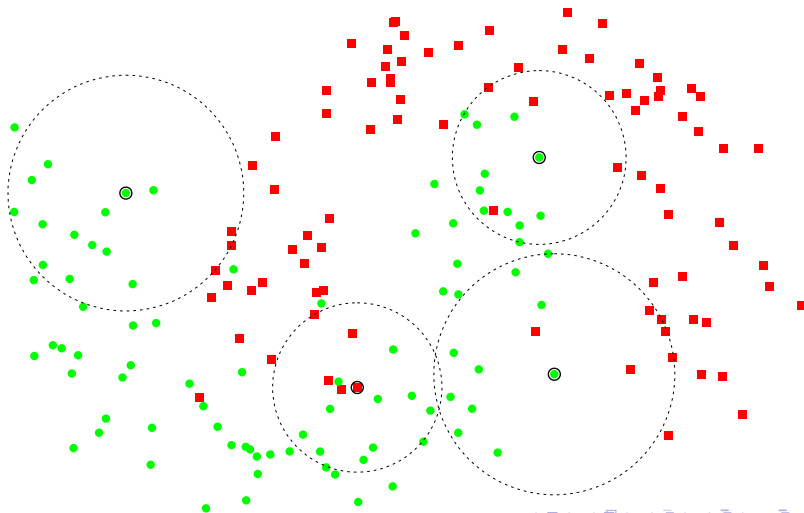
# Lowering the Number of Local SVM Trained

① for *some* training examples...



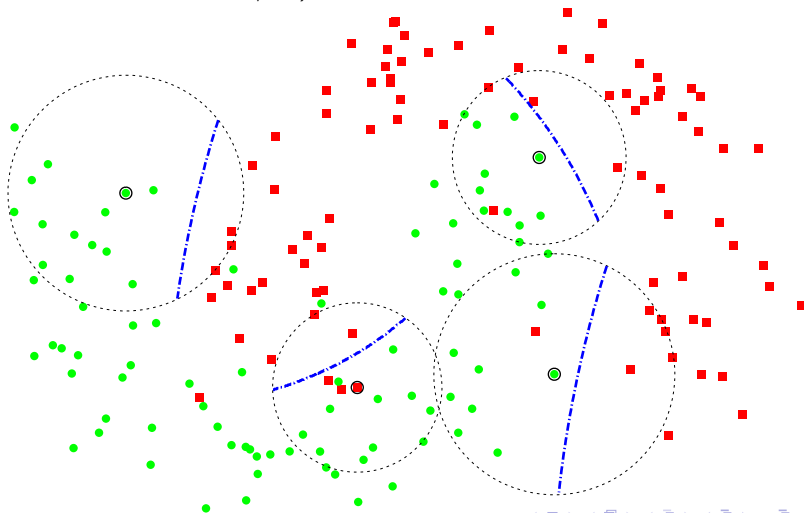
# Lowering the Number of Local SVM Trained

② ... retrieve their neighbourhood ( $k = 15$ )



# Lowering the Number of Local SVM Trained

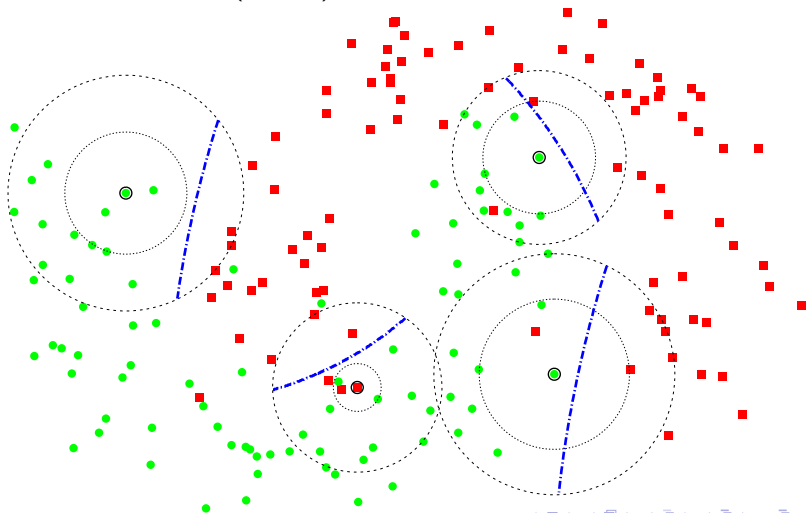
- train a Local SVM on each neighbourhood ( $C = 10$ , RBF kernel with  $\sigma = 1/10$ )





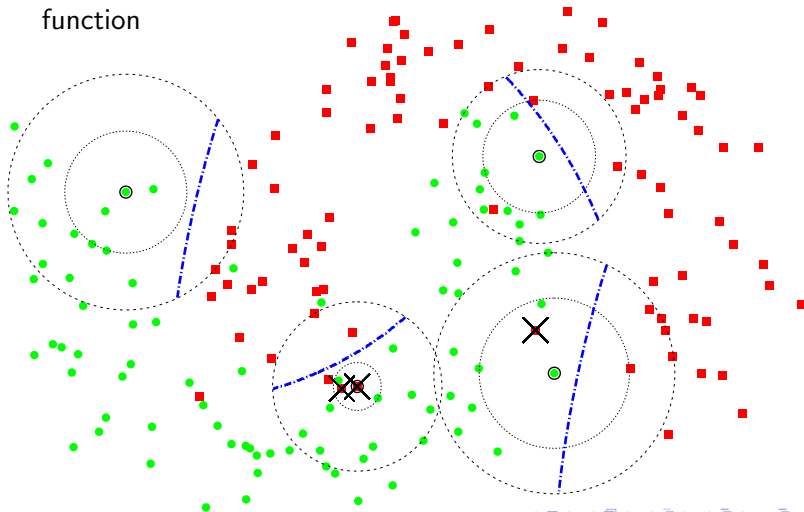
# Lowering the Number of Local SVM Trained

- ④ consider the *assignment  $k'$ -neighbourhood* of each neighbourhood ( $k' = 4$ )



# Lowering the Number of Local SVM Trained

- 5 remove the examples in the assignment neighbourhoods that are misclassified by the corresponding Local SVM decision function



# FaLKNR: the choice of the assignment $k'$ -neighbourhoods

## Definition ( $k'$ -neighbourhood covering set)

Given  $k' \in \mathbb{N}$ , a  $k'$ -neighbourhood covering set of centers  $\mathcal{C}_{k'} \subseteq X$  is a subset of the training set such that the following holds:

$$\bigcup_{c \in \mathcal{C}_{k'}} \{x_{r_c(i)} \mid i = 1, \dots, k'\} = X.$$

- finding the *minimal*  $k'$ -neighbourhood covering set is NP-HARD!
- for FaLKNR is not so crucial to find the very minimal  $\mathcal{C}$  (minimality vs redundancy)
- greedy approximated approaches for the related problems of *Set Cover Problem* and *Minimum Sphere Set Covering Problem* have been proposed
- a point can be in the  $k'$ -neighbourhood of multiple centers

# A greedy approach to approximate the minimal $\mathcal{C}$

The idea of the greedy approach for approximated minimal  $\mathcal{C}$

Recursively take as centers those points which are not  $k'$ -neighbours of any point that has already been taken as center

The set of  $c_i \in \mathcal{C}$  with  $i = 1, \dots, |\mathcal{C}|$  can be detected as:

$$c_i = x_j \in X \quad \text{with } j = \min \left( l \in \{1, \dots, n\} \mid x_l \in X \setminus X_{c_i} \right) \\ \text{where } X_{c_i} = \bigcup_{l < i} \left\{ x_{r_{c_i}(h)} \mid h = 1, \dots, k' \right\}.$$

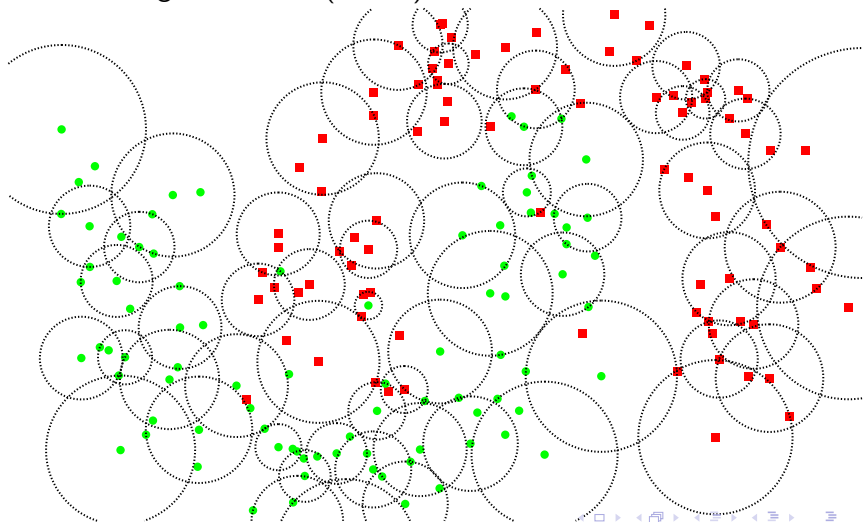
The function  $\text{cnt}(t) : X \rightarrow \mathcal{C}$  assigns each training example to a center:

$$\text{cnt}(x_i) = x_j \in \mathcal{C} \quad \text{with } j = \min \left( l \in \{1, \dots, n\} \mid x_l \in \mathcal{C} \text{ and } x_i \in X_{x_l} \right) \\ \text{where } X_{x_l} = \left\{ x_{r_{x_l}(h)} \mid h = 1, \dots, k' \right\}.$$

The separation invariant of Cover Trees can help in selecting the  $(i + 1)$ -th center...

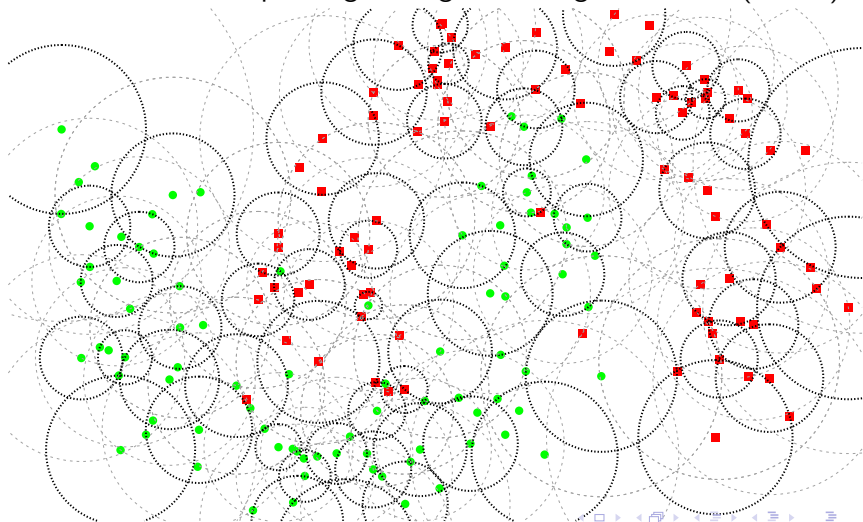
# FaLKNR: the overall picture

- the training set covered by the  $\mathcal{C}$  assignment  $k'$ -neighbourhoods ( $k' = 4$ )



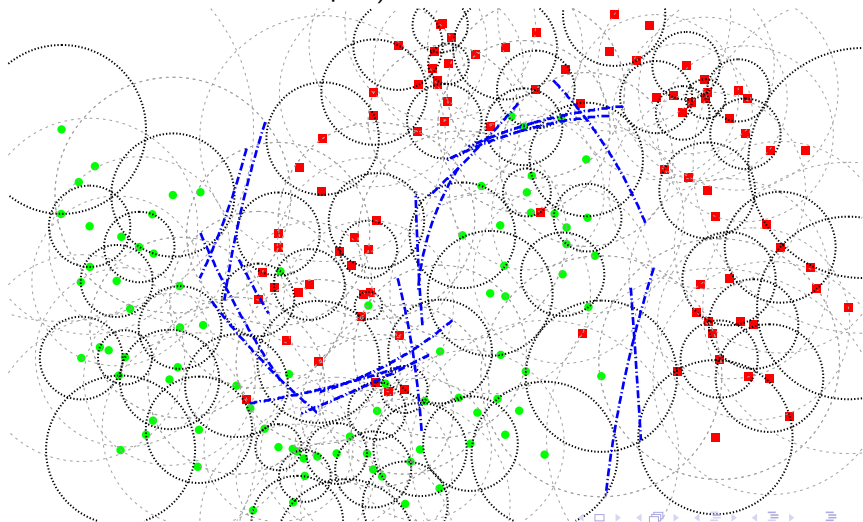
# FaLKNR: the overall picture

- the training set covered by the  $\mathcal{C}$  neighbourhoods ( $k = 15$ ) and the corresponding  $\mathcal{C}$  assignment neighbourhoods ( $k' = 4$ )



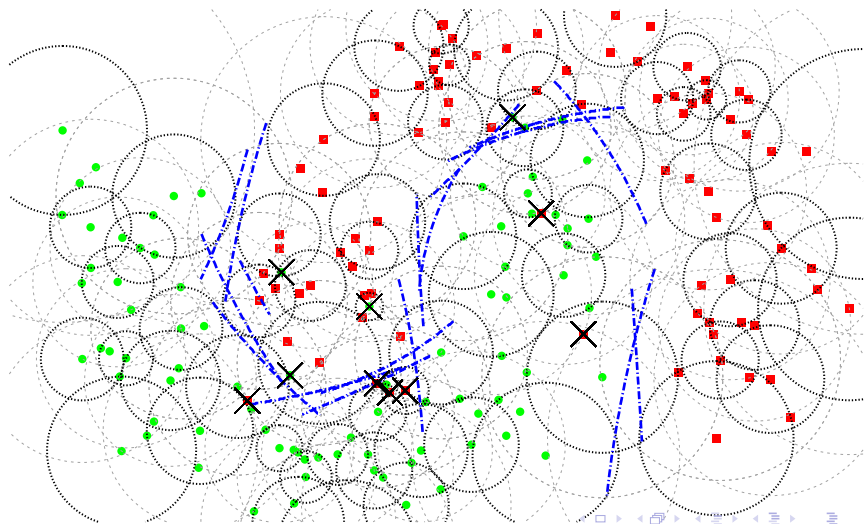
# FaLKNR: the overall picture

- the Local SVM decision functions of FaLKNR (only 17 Local SVMs for 185 examples)



# FaLKNR: the overall picture

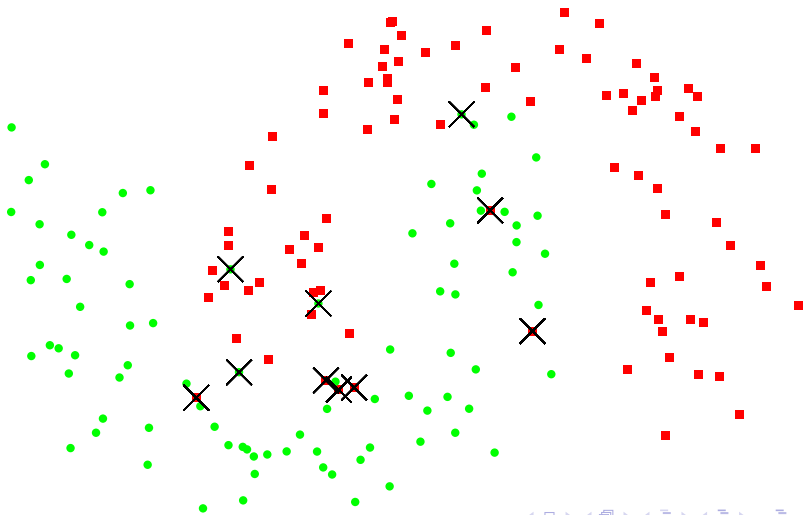
- the examples removed by FaLKNR





# FaLKNR: the overall picture

- the dataset edited with FaLKNR



# Local Model Selection for FaLKNR Parameters

## The idea of local model selection

Use a subset of the local neighbourhoods to select  $k$  and the parameters for all the local models that need to be trained

The Local Model Selection Procedure:

- for a random (small) subset of training examples and each parameter choice  $P$ 
  - ① separate the  $k' < k$  nearest neighbours of  $x$ , called  $S$ , from the  $k$  nearest neighbours of  $x$ , called  $S^E$ ;
  - ② randomly split  $S$  in  $\kappa$  disjoint internal validation sets  $S_i$  with  $0 < i < \kappa$ ;
  - ③ for each fold  $i$  train a model with the  $S^E \cup (S \setminus S_i)$  using the  $P$  parameters set evaluating it on the correspondent  $S_i$  set, taking the mean of the accuracies on the  $\kappa$  folds.
- select the  $P$  parameter set giving the best average  $\kappa$  fold accuracy

# Computational Complexity Analysis of FaLKNR

- ① construct the Cover Tree<sup>1</sup>
- ② retrieve the  $|\mathcal{C}|$  neighbourhoods
- ③ train the  $|\mathcal{C}|$  local SVMs (and perform local model selection)<sup>1</sup>
- ④ univocally assign each example to a  $k'$ -neighbourhood
- ⑤ predict the each training label with the corresponding model<sup>1</sup>
- ⑥ remove the misclassified examples<sup>1</sup>


## FaLKNR computational complexity

Time Complexity:  $\mathcal{O}(n \log n + |\mathcal{C}| \cdot k \log n + |\mathcal{C}| \cdot k^3 + n + n \cdot k + n) =$   
 $= \mathcal{O}(n \log n + |\mathcal{C}| \cdot k \log n + |\mathcal{C}| \cdot k^3) \stackrel{a}{=} \mathcal{O}(n \log n)$

Space Complexity:  $\mathcal{O}(n + |\mathcal{C}| \cdot k^2) \stackrel{a}{=} \mathcal{O}(n)$

<sup>a</sup> Assuming  $k$  relatively small and fixed, and the **worst case** in which  $k' = 1$  and thus  $|\mathcal{C}| = n$


Note that the training of the  $|\mathcal{C}|$  models can very easily be parallelised...

<sup>1</sup>The mathematical formulation of these steps is not discussed here. 

# The Experimental Setting

- Comparison between FaLKNR ENN RENN AkNN and AkNNc (a conservative variant of AkNN) on the basis of the induced NN generalization accuracies
- Neighbourhood sizes:  $k = 3$  for ENN RENN AkNN and AkNNc,  $k = 1000$ ,  $k' = 250$  for FaLKNR
- $C$  of FaLKNR estimated with local model selection,  $\sigma$  of RBF kernel estimated locally as the median distance in the neighbourhood
- Comparison performed on 9 datasets with large training sets (from 50k to more than 500k training examples)<sup>1</sup>

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<sup>1</sup>Details of the used datasets can be found in the paper. 

## Accuracy and Computational Results of FaLKNR

dataset	NN accuracies (in %)						comput. performances (s)		
	uned.	FaLKNR	ENN	RENN	AkNN	AkNNc	FaLKNR <sup>2</sup>	ENN <sup>3</sup>	speed-up
ijcnn1	96.6	<b>96.7</b>	96.3	<u>96.0</u>	<u>96.0</u>	96.2	39	61	1.6
connect-4	<u>66.2</u>	<b>69.8</b>	69.3	68.3	69.3	69.4	455	1244	2.7
seismic	<u>65.3</u>	<b>73.3</b>	71.9	72.6	72.2	71.8	950	3025	3.2
acoustic	<u>67.4</u>	<b>75.3</b>	73.7	74.2	74.0	73.8	331	2641	8.0
2-spirals	<u>83.2</u>	<b>88.6</b>	87.6	88.1	87.9	87.7	97	44	0.5
census-inc	<u>92.6</u>	<b>94.5</b>	94.2	94.3	94.4	94.3	771	6965	9.0
poker-hand	<u>56.6</u>	<b>60.7</b>	57.8	58.3	58.3	58.0	2230	16905	7.6
rna	<b>96.3</b>	95.8	<u>94.0</u>	<u>94.0</u>	94.3	94.3	550	3340	6.1
cover-type	<b>95.8</b>	95.4	95.2	<u>95.0</u>	95.1	95.2	993	1538	1.5

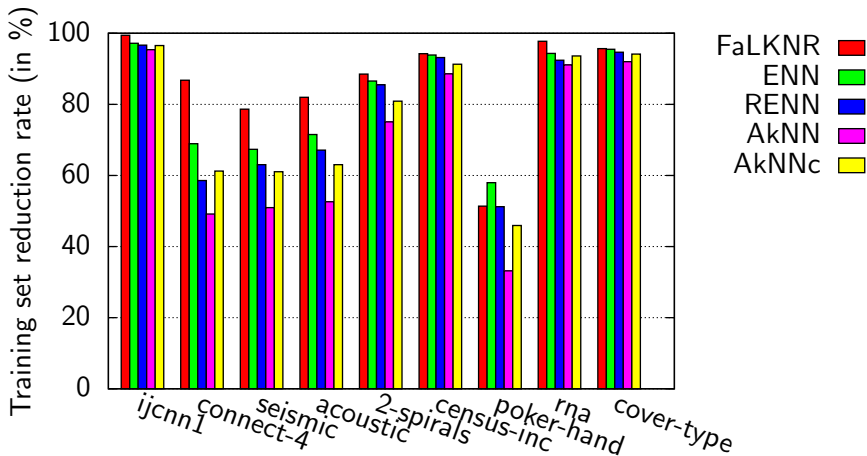
Wilcoxon Signed Rank Test ( $\alpha = 0.05$ )

NN accuracies using FaLKNR are better than unedited NN accuracies and accuracies of NN edited with ENN RENN AkNN AkNNc.

<sup>2</sup>The computational time of FaLKNR includes the local model selection.

<sup>3</sup>ENN is implemented using Cover Trees and it is faster than RENN, AkNN and AkNNc.

# Reduction Rates of the Edited Training Sets



# Final Remarks and Future Works

- The maximal margin principle is effective for noise reduction both for small [Segata, Blanzieri, Delany, Cunningham, 2008] and large CBR systems
- FaLKNR overcome state-of-the-art noise reduction techniques with statistical significance for large CBR systems
- FaLKNR is generally faster than traditional approaches based on locality and CBR rules

## Possible further developments

- Other principles different from the maximal margin principle can be exploited locally with the same framework
- FaLKNR is promising also for cleansing bioinformatics datasets and as preprocessing step for other machine learning approaches
- FaLKNR can integrate a competence preserving step to decrease the size of the case base without decreasing NN accuracies

# A SW Library for Fast Local Kernel Machines: FaLKM-lib

FaLKM-lib v1.0 [Segata, 2009] is a software library for fast local kernel machine implemented in C++. It contains the following modules:

**FkNN** a (kernel-space)  $k$ NN implementation using Cover Trees

**FkNNSVM** the  $k$ NNSVM algorithm for Local SVM

**FkNNSVM-nr** a noise reduction algorithm based on  $k$ NNSVM

**FaLK-SVM** very fast and scalable learning with local kernel machines

**FaLKNR** the fast and scalable noise reduction algorithm

The modules share also tools for model selection, efficient local model selection, performance assessment. . .

**FaLKM-lib is freely available for research purposes**

You can download the code, datasets, benchmark, additional infos and papers, and examples at <http://disi.unitn.it/~segata/FaLKM-lib>

**Any comments/suggestions are welcome!**



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