# Genetic-Algorithm Assisted Maximum-Likelihood Detection of OFDM Symbols in the Presence of Nonlinear Distortions

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# Abstract

This letter<sup>1</sup> aims at proposing the utilization of evolutionary computation methodologies (i.e.,

Genetic Algorithms – GAs) in order to solve the problem of the Maximum Likelihood estimation of

OFDM symbols in the presence of nonlinear distortions. Experimental results can prove the

effectiveness of the proposed detection algorithm achieved with a reasonable computational load.

**<u>INDEX TERMS</u>**: Multicarrier Modulation (MCM), Orthogonal Frequency Division Multiplexing (OFDM), Genetic Algorithms, Nonlinear distortion, Maximum-Likelihood (ML) detection.

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#### **<u>1. Introduction</u>**

Multicarrier modulations [1] are regarded as emerging technologies for new-generation networking applications. In particular, Orthogonal Frequency Division Multiplexing (OFDM) can profit by full-digital FFT-based implementation and is intrinsically resilient against frequency-selective channel distortions [1]. At present, two open problems limit the efficiency of OFDM techniques when employed in real-world application testbeds: non-linear distortions involved by power amplifiers [2] and channel estimation errors occurring in time-varying multipath fading channels [3]. In this letter, we are focusing on the problem of the optimum OFDM symbol recovery in the presence of non-linear distortions.

In wireless networks deployment, tight requirements in terms of power efficiency generally impose the utilization of saturating RF power amplifiers. It has been shown (see, e.g., [2], [4], and [5]) that a saturating non-linearity produces a sort of self-interference (namely: *clipping noise*) depending on the transmitted symbols and the in-out characteristics of nonlinear blocks. Various methodologies have been proposed in literature in order to reduce the negative impact of clipping noise on OFDM performances. All such methodologies are sub-optimum or quasi-optimum (in fact, the theoretically-optimum reception of nonlinearly distorted OFDM symbols based on Maximum-Likelihood (ML) estimation is computationally unsustainable). In [4] and [5], decision-directed iterative algorithms have been adopted in order to sharply reduce the computational burden of OFDM receivers to a polynomial order with respect to the subcarrier number. Such methodologies are very attractive from a computational point of view, but they are not designed for severe clipping. An alternative methodology for clipped OFDM symbol recovery has been proposed in [6] that is based on the recursive application of Bayesian inference. As compared with decision-directed approaches, the algorithm of [6] is characterized by a consistently increased computational complexity, but results in terms of Symbol-Error-Rate seem to be better in case of severe clipping. A completely different approach aimed at reducing the effects of clipping on OFDM signals has been proposed by Li and Cimini in [7] and Dinis and Gusmão in [8]. A deliberate clipping is introduced in order to obtain an OFDM signal characterized by an "almost constant" envelope. In such a way, linear amplifiers might be still employed without decrease of power efficiency. Unfortunately, the deliberate clipping is itself a nonlinear distortion that may involve a spectral regrowth of the transmitted signal. For this reason, it is necessary to filter the OFDM signal in the frequency domain. The impact of "clipping and

filtering" (C&F) operation is relevant on OFDM performances, as shown in [7]. For this reason, efficient symbol recovery techniques are also required when "clipping and filtering" is adopted. In this specific context, iterative methodologies based on decision feedback have been proposed in [9] for residual clipping noise removal, and in [10] for sub-optimum iterative ML symbol estimation. Algorithms described in [9] and [10] are rather general in their mathematical formulations. So, they can be applied not only to the C&F case, but also to recover OFDM symbols clipped by non-linear amplifiers.

In such a framework, the exploitation of Genetic Algorithms (GAs) [11] may represent an interesting alternative solution to OFDM symbol recovery in the presence of non-linear distortions. The exploitation of GAs in telecommunications, electromagnetism and signal processing is dated since about ten years. Applications are currently ranging (among the others) from antenna array optimization [12], DS/CDMA multi-user detection (MUD) [13], allocation of power resources in cellular DS/CDMA networks [14], interference cancellation in MC-CDMA systems [15], etc. More in general, GAs can provide reliable and affordable solutions to optimization problems that cannot be solved by full-space search due to computational reasons. Recently, GAs have recently found some interesting applications also in OFDM transmissions. Alias, Chen and Hanzo proposed in [16] a GA-based approach in order to find the optimal weight vector of the Minimum-BER MUD receiver in the context of a multiple-antenna aided multi-user OFDM system. Hong, Dong and Yuan employed in [17] a GA in order to derive the optimal distance spectrum (i.e., the codeword difference matrix considering all possible event error paths) in space-time trellis-coded OFDM. Finally, in [18], a GA-based approach is proposed in order to search for low Peak Average to Power Ratio (PAPR) near-optimum training sequences for OFDM systems.

In the present work, we propose the adoption of a GA-based approach in order to find a near-optimum solution to the ML estimation of OFDM symbols distorted in a nonlinear way. We shall demonstrate that the proposed GA-based approach clearly outperforms state-of-the-art solutions even in the presence of heavy nonlinear distortions. Moreover, such good results are achieved by spending an affordable computational effort. The test case considered here is related to an OFDM signal distorted by a nonlinear power amplifier. Nevertheless, the proposed analysis might be extended without conceptual difficulties also to the C&F case. The letter is structured as follows: Section 2 is aimed at describing the system model. Section 3 details the proposed GA-based symbol estimation methodology. Section 4 presents some selected experimental results. Finally, in Section 5, the letter conclusions are drawn.

#### 2. System model

The analytical expression for a generic multi-carrier OFDM symbol transmitted during the generic signaling interval i of duration T is given as follows [1]:

$$S_{i}(t) = A_{s} \sum_{k=0}^{N-1} s_{k,i} \exp(j2\pi kt/T) \Pi(t-iT)$$
(1)

where  $A_s$  is the carrier amplitude,  $\underline{s}_i \triangleq \{s_{k,i}, k = 0, ..., N - 1\}$  is the vector of the *M*-level complex symbols transmitted over the *N* subcarriers (in the present letter, we consider a 16-QAM modulation; therefore, M=16), and  $\Pi(t)$  is the rectangular waveform of unit amplitude. As known from the literature (see, e.g., [1]), the practical realization of OFDM modulation is feasible in the digital domain by applying an Inverse Fast Fourier Transform (IFFT) to the symbol vector  $\underline{s}_i$  and, therefore, by passing the discrete-time signal achieved to a Digital-to-Analog converter. The discrete sequence produced by the IFFT block (sample duration equal to T/N) can be expressed by the following equation:

$$w_{n,i} = A_s \sum_{k=0}^{N-1} s_{k,i} \exp(j2\pi kn/N) \ n = 0, 1, ..., N-1$$
(2)

Let us now introduce a nonlinear memoriless block into the transmission system. In particular, Solid State Power Amplifier (SSPA) nonlinearity is considered that introduces an amplitude distortion whose mathematical expression is given below (the normalized Rapp model [19] has been chosen):

$$g(x) = |x| / \left\{ 1 + \left( |x| / \alpha \right)^{2\beta} \right\}^{\frac{1}{2\beta}} x \in C \quad \alpha, \beta \in \Re$$
(3)

The amount of distortion can be measured (in dB) in terms of *Clip Level* (CL) defined as:  $CL \triangleq 20\log_{10}(\alpha/\sigma_x)$  where  $\sigma_x^2$  is the variance of the input signal. The precise mathematical dealing of [4] allows us to express the baseband output of the memoriless nonlinearity in useful compact form:

$$w_{n,i}^{D} \stackrel{\circ}{=} g\left(w_{n,i}\right) = \lambda^{g} w_{n,i} + \Omega_{n}(\underline{s}_{i},g) \quad n = 0,..,N-1$$

$$\tag{4}$$

where:  $g(\bullet)$  is the nonlinear distortion function, and  $\lambda^g$  is a constant chosen in order to minimize the MSE between  $g(w_i(n))$  and  $\lambda^g w_{n,i}$ . Therefore, the discrete sequence  $\Omega_n(\underline{s}_i, g)$  is the minimum distortion energy sequence [4]. It has been shown in [20] that, in case of SSPA nonlinearities,  $\lambda^g \cong 1$  for *CL*>7dB, and  $0.9 < \lambda^g < 0.99$  for  $2.5dB < CL \le 7dB$ . The exact calculation of the term  $\lambda^g$  is theoretically allowed (see e.g. [8]). Nevertheless, for a wide range of meaningful *CL* values, the approximation  $\lambda^{g} \cong 1$  holds well. This assumption, already considered in [4], allows us to rewrite Eq.4 in the conveniently approximated form:

$$w_{n,i}^{D} \cong w_{n,i} + \Omega_n(\underline{s}_i, g) \qquad n = 0, \dots, N-1$$
(5)

The coherent OFDM demodulator performs an FFT over the received baseband discrete-time sequence. Such an operation, applied to the distorted input sequence of Eq.5, provides the following output:

$$r_{k,i}^{D} \triangleq FFT(w_{n,i}^{D}) = s_{k,i} + \Phi_k(\underline{s}_i, g) \qquad k = 0,..,N-1$$
 (6)

where  $\Phi_k(\underline{s}_i, g) \triangleq FFT(\Omega_n(\underline{s}_i, g))$  is the k-th sample of the *out-of-band distortion* sequence or *clipping noise* resulting at the output of the OFDM demodulator [4]. In this work, we adopt the idea of estimating the symbol vector  $\underline{s}_i$  in the presence of clipping noise, as illustrated in the following section.

## 3. The proposed GA-assisted ML symbol estimation

The optimum symbol estimation in the presence of nonlinear distortion and additive Gaussian noise is the Maximum Likelihood (ML) estimation. It consists in the computation of the symbol vector  $\underline{\hat{s}}_{i}^{opt}$  minimizing the following metric:

$$\Lambda(\underline{\hat{s}}_{i}) = \left\| \underline{y}_{i} - [\underline{\hat{s}}_{i} + \underline{\Phi}(\underline{\hat{s}}_{i}, g)] \right\|^{2}$$

$$\tag{7}$$

where  $\underline{y}_i \triangleq \underline{r}_i^D + \underline{\eta}$  is the received signal sample vector,  $\underline{\eta}$  being the AWGN noise sample vector. The nonlinear distortion is deterministic and completely known; therefore, the ML-based computation of  $\underline{\hat{s}}_i^{opt}$  is theoretically feasible. The price to be paid is a computational load exponentially growing with the dimension of the symbol vector *N*. The number of subcarrier employed in OFDM commercial systems ranges from 64 (e.g. HYPERLAN 2 system [21]) to 256 (ADSL-DMT system shown in [4]), up to 2048 (VDSL-DMT standard [4]). For this reason, theoretical ML detection cannot be adopted in real-world applications and sub-optimum detection strategies should be investigated. A feasible solution has been proposed by Tellado, Loo and Cioffi in [4]: it is based on the iterative estimation of the symbol vector  $\underline{\hat{s}}_i^{(\nu)}$ ( $\nu$  is the number of the iteration) obtained by the metric of Eq.7. The term  $\underline{\Phi}(\underline{\hat{s}}_i, g)$  is replaced by its iterative estimation  $\underline{\Phi}^{\nu}(\underline{\hat{s}}_i^{(\nu-1)}, g)$ , computed on the basis of the symbol vector estimation obtained at the previous iteration. The first iteration of the algorithm is the hard decision made by the conventional OFDM demodulation. Other sub-optimum algorithms have been proposed in literature. The decision-aided reconstruction (DAR) iterative approach shown in [5] assumes that, at the first iteration, the frequencydomain sample  $\hat{Z}_{n,i}^{(0)} \triangleq w_{n,i}^{D} + N_{n,i}$  is an estimation of the clipped signal  $w_{n,i}^{D}$  (being  $N_{n,i}$  the additive Gaussian noise sample in the frequency domain). At generic iteration v, a symbol decision  $\underline{\tilde{s}}_{i}^{(v)}$  is taken in the frequency domain on the basis of the estimation of the clipped signal  $\hat{Z}_{n,i}^{(\nu)}$  by minimizing an absolute error metric [5]. The clipped sequence is then reconstructed in the time domain using the symbol decision  $\underline{\widetilde{s}}_{i}^{(\nu)}$ . Finally, the IFFT-transformed sequence  $\hat{w}_{n,i}^{D(\nu)}$  becomes the estimation of the clipped signal at the successive iteration, i.e.:  $\hat{Z}_{n,i}^{(\nu+1)}$ . The Bayesian inference has been proposed by Declercq and Giannakis in [6] in order to iteratively recover clipped OFDM symbols. At the generic iteration v, the fullconditional posterior distribution function of the information symbols,  $\underline{s}_i^{(\nu)}$ , is computed on the basis of the estimated symbols at the previous iteration,  $\hat{\underline{s}}_i^{(\nu-1)}$ , and of the received signal samples  $\underline{y}_i$ . The symbol estimation is made on the basis of a recursive MAP criterion. Practically  $\hat{s}_i^{(\nu)}$  is the symbol vector that maximizes the full-conditional posterior distribution [6]. In [9], a decision feedback-based interference cancellation procedure is shown for OFDM symbols affected by clipping. The iterative symbol decision  $\underline{\widetilde{s}}_{i}^{(\nu-1)}$  is employed here to estimate the out-of-band distortion in the frequency domain  $\Omega_{n}^{(\nu-1)}(\underline{\widetilde{s}}_{i}^{(\nu-1)},g)$ and to remove it from the received signal. Finally, Ochiai [10] analysed the performances of optimum and sub-optimum detection for clipped OFDM signals. The sub-optimum iterative ML detection proposed in [10] initially considers the vector of bit decision provided by the conventional OFDM demodulator  $\hat{\underline{b}}_i^{(0)}$ . At the iteration v, a list of candidate bit vectors is generated by the XOR operation  $\hat{\underline{b}}_i^{(v-1)} \oplus \underline{e}$ , where  $\underline{e}$  is an error pattern with Hamming weight ranging from 1 to  $I_{max}$ . The candidate bit vectors are therefore turned on candidate symbol vectors  $\hat{\underline{s}}_{i}^{(\nu)}$  and the best one is selected by minimizing the metric of Eq.7.

Our solution is based on the use of *Genetic Algorithms*. Genetic Algorithms are robust, stochastic search methods modelled on the principles of natural selection and evolution [11]. GAs differ from conventional optimisation techniques in that:

- a) They operate on a group (namely: *population*) of trial solutions (namely: *individuals*) in parallel. A positive number, namely: *fitness*, is assigned to each individual representing a measure of goodness;
- b) They normally operate on a coding of the function parameters (namely: *chromosome*) rather than on the parameter themselves;
- c) They use stochastic operators (*selection*, *crossover*, and *mutation*) to explore the solution domain.

The metric  $\Lambda(\underline{\hat{s}}_i)$  is regarded as the fitness of the GA. A set of individuals is encoded with chromosomelike bit strings (in our case the vector  $\underline{\hat{s}}_i$ ). The cardinality of the set of individuals is called *population size* [11]. At each iteration, called *generation*, the genetic operators of crossover and mutation are applied to selected chromosomes with probability  $P_C$  and  $P_M$ , respectively, in order to generate new solutions belonging to the search space. The population generation process terminates when a satisfactory solution is reached or when a fixed number of iterations (namely: *generation number*) are completed.

Genetic algorithms have been successfully applied for a wide range of problems (see Section 1) characterized by a large number of unknown parameters and highly non-linear behavior [11]. The major advantages of the GAs with respect to the other optimization algorithms, such as gradient conjugate-based methods, are mainly related to their independence from the initialization and their ability to prevent local minima. Moreover it is well known from the scientific literature that it is possible to enhance the convergence ratio making a good choice of the algorithm parameters [11, 22]. In particular a proper choice of the population size and generation number is mandatory in order to avoid too high computational burden and to keep performances good. These characteristics make the GAs particularly attractive for the proposed application with respect the other methods proposed in literature. For the sake of comparisons the method proposed in [4], and other similar decision-aided recursive methods [5, 9, 10], are strongly dependent from the initial choice of the symbol vector that is based on the hard decision made by the conventional demodulator. If the initial choice is considerably affected by decision errors (this happens in case of severe clipping), these errors propagate iteration after iteration, leading to a nasty "floor" in the BER curve. The only method for OFDM clipped symbol recovery, which seems to be less sensitive to the effects of the

initial hard decision, is the Bayesian inference proposed in [6]. Nevertheless, the computational burden required by this algorithm is very high, as compared with recursive algorithms (except than [10]), and also with the GA-based receiver (issues concerning computational complexity will be detailed in next section). The initialisation of the GA has been performed in random modality. In particular, at each generation, the population is initialised by individuals consisting of vectors collecting complex random symbols. Such an initialisation procedure is appropriate for the specific problem addressed in this letter. In fact the symbol source can be regarded, without losing generality, as a random process generating equiprobable and

statistically independent complex numbers (i.e.: the 16-QAM symbols).

To conclude this section, it should be said that the proposed GA-assisted ML symbol recovery could be applied, without any conceptual difficulty, also to the C&F case. In fact, a metric very similar to the one shown in Eq.7 can be computed for "clipped and filtered" OFDM signals as proven in [10].

# 4. Experimental results

In order to assess the performances of the proposed GA-based ML estimation approach, some intensive simulation trials have been performed. An OFDM transmission configuration has been considered with a bit-rate of 4Mb/s and number of subcarriers *N* equal to 32 and 64. The parameter setting of the SSPA distortion has been done by fixing  $\beta$ =2 and choosing two different values of  $\alpha$  in order to achieve *CL* values equal to 5dB and 7dB respectively. As far as the parameterization of the genetic algorithm is concerned, we firstly selected crossover probability  $P_c$  and mutation probability  $P_M$  equal to 0.9 and 0.01, respectively. This setting is reasonable because  $P_c$  is the index of the "evolutionary capability" of the GA, whereas a high value of  $P_M$  would turn the GA into a kind of random search [11]. In the absence of specific analytical selection criteria [11] [22], the generation number  $\delta_{gen}$  and the population size  $\Gamma_{size}$  of the GA optimizer have been chosen by means of preliminary experimental trials explicitly devoted to. Results have been summarized in Fig. 1. We have considered in these simulations the heuristic selection criteria enunciated in [22]: a) the population size should be sufficiently large in order to have a conveniently- dimensioned space search, b) the number of generations should be appropriately assigned in dependence of the population size. In fact, in case of large population, too strict limit for the search time can force algorithm to stop without having enough time to realize its search possibility [22]. The test was

performed in the case of the heavier nonlinear distortion (*CL*=5dB) and for the highest number of subcarriers (*N*=64), considering a per-symbol SNR equal to 15dB. On the basis of the BER curves vs. population size for fixed generation numbers reported in Fig. 1, a reasonable choice considering the tradeoff between computational complexity and achieved performances is  $\delta_{gen}$ =250 and  $\Gamma_{size}$ =200. In Figs. 2-3-4, curves drawing BER results vs. per-symbol SNR are shown for different *CL* and *N*, and compared with results yielded by:

- A conventional FFT-based OFDM demodulator in the presence of nonlinear distortion [1].
- Iterative decision-directed algorithms: iterative decoding proposed in [4], decision-aided reconstruction of [5], decision-directed clipping removal of [9]. The number of iterations *X* has been limited to 3 for this class of algorithms, because no significant performance improvement has been noted by increasing it. These algorithms are characterized by quite analogous theoretical concepts. Their computational burden is almost the same.
- Clipped symbol recovery of [6] obtained by means of the Bayesian inference. We fixed  $\chi$ =5 as a reasonable tradeoff between the heavy computational load required by the algorithm and the performance improvement achievable by increasing  $\chi$ .
- Iterative sub-optimum ML detection of [10]. In this case, we fixed  $\chi = 10$  as a good compromise between computational demand and reliability of the symbol estimation. Following the suggestions of [10], we set the maximum Hamming weight of the error pattern  $I_{max}=1$ .
- A conventional FFT-based demodulator of an undistorted OFDM signal transmitted over a purely additive Gaussian channel. This last curve actually draws the lower bound on the achievable performances. In fact, the ML criterion becomes equivalent to the conventional detector in the case of AWGN without any distortion. The clipping noise added by a nonlinear distortion substantially reduces the average Euclidean distance between the OFDM signal generated by the correct bit sequence and other ones generated by error patterns [10]. Therefore, we can say that the pairwise error probability computed for the ML receiver in the presence of nonlinear distortion will be higher (or at least equal) to the error probability computed for the distortionless conventional receiver.

The first series of simulation results, obtained for N=32 and CL=5dB (see Fig. 2), prove that the GA-assisted ML estimation provides a BER characteristic fairly close to ideal one, whereas conventional

detection, iterative decoding [4], decision-aided reconstruction [5], and decision-feedback clipping removal [9] are very far from optimal performances. Bayesian inference [6] and iterative sub-optimum ML detection [10] works slightly better (their curves are almost coincident), but they both perform worse than GA-assisted ML detection especially for high SNRs. Fig. 3 shows another series of simulation results obtained by increasing the number of subcarriers (N=64 instead of 32) and keeping CL unaltered (CL=5dB). The SNR range has been increased up to 30dB to clearly prove the error-floor affecting the BER performances both of the conventional OFDM demodulation and of all the iterative decision-directed procedures ([4], [5], and [9]). In this case, also the iterative sub-optimum ML detection of [10] exhibits a severe error-floor. On the other hand, Bayesian inference seems to perform better than decision-directed iterative approaches, thus confirming its improved robustness for low CL values. But, the proposed GAassisted ML estimation provides much better results than all state-of-the-art algorithms used for comparison, with a dramatic BER decrease for high SNRs. Finally, the results given in Fig. 4 have been obtained by increasing CL up to 7dB and keeping the value of N unaltered with respect to the simulation in Fig. 3 (N=64). In this case, all receiver schemes can profit by the out-of-band distortion reduction and improve their performances. The BER curve of the GA-assisted ML estimation is very close the ideal one. All iterative detection algorithms work better (in particular Tellado's iterative decoding [4]), though remaining a bit far from ideal performances. One can note from Fig.4 that Bayesian inference does not provide any significant performance improvement with respect to iterative decision-directed approaches. This confirms the considerations made in [6] about the opportunity of using highly complex Bayesian inference when clipping effects are reduced.

Concerning computational issues, Tab.1 shows the order of computational complexity for each symboldetection algorithm assessed (second column), the number of elementary operations required by each OFDM symbol during the signaling period T in the selected simulation scenarios (third column – this is actually the number of elementary operations required to derive a solution to the considered problem), and finally the average number of elementary operations per data symbol (obtained by dividing per N the content of the third column). The fundamentals for deriving mathematical expressions in Tab.1 have been taken by Hanzo's book about single carrier and multicarrier modulations [1], by the referenced papers dealing with the algorithms tested for comparison ([4], [5], [6], [9], and [10]), and finally by Goldberg's book about Genetic Algorithms [11]. The reader can note that the computational burden of the GA-assisted ML detection increases only by one order of magnitude with respect to iterative decision-directed algorithms ([4], [5], and [9]), while providing much better results in terms of BER reduction. On the other hand, the computational burden of GA-assisted ML detection is consistently reduced with respect to computationally demanding Bayesian inference [6] and iterative sub-optimum ML detection [10]. In addition, these last two algorithms perform worse than GA in terms of BER, as shown in Figs. 2-4. They have also been reported in Tab.1 both the computationally unsustainable ML symbol recovery and the conventional FFT-based OFDM receiver, regarded here as indicative upper and lower bounds on the computational complexity.

## **5.** Conclusion

In this letter, a novel GA-assisted approach to ML symbol estimation of nonlinearly distorted OFDM symbols has been proposed and discussed. The obtained simulation results have proved a fair near-optimum behavior of the proposed algorithm, clearly outperforming state-of-the-art methodologies for multicarrier symbol estimation based on iterative decision-directed reconstruction, iterative sub-optimum ML estimation, and Bayesian inference. The computational load required by the GA-based estimator slightly increases with respect to the most computationally-efficient iterative algorithms; nevertheless, it is still acceptable as compared with the unaffordable burden of theoretically-optimum ML detection. It should be said that other algorithms tested for comparison, like e.g. Bayesian inference and iterative ML estimation are more computationally demanding than the proposed GA-assisted methodology.

Future work will consider other aspects, like the assessment of the proposed GA-based ML estimator in the presence of both nonlinear and linear distortions (e.g. multipath fading). For this last purpose, the impact of channel estimation errors on the symbol estimation accuracy should be carefully studied.

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# **Figure captions**

**Figure 1.** BER values provided by the proposed GA-assisted ML detection algorithm plotted vs. population size ( $\Gamma_{size}$ ) for a fixed per-symbol SNR (15dB) and different values of the generation number ( $\delta_{gen}$ ).

**Figure 2.** BER results vs. SNR provided by the different symbol estimation algorithms assessed (GA-assisted ML detection, iterative algorithms, conventional OFDM detection, lower bound: ideal detection without distortions): *CL*=5dB, *N*=32,  $\delta_{gen}$ =250,  $\Gamma_{size}$ =200.

**Figure 3.** BER results vs. SNR provided by the different symbol estimation algorithms assessed (GA-assisted ML detection, iterative algorithms, conventional OFDM detection, lower bound: ideal detection without distortions): *CL*=5dB, *N*=64,  $\delta_{gen}$ =250,  $\Gamma_{size}$ =200.

**Figure 4.** BER results vs. SNR provided by the different symbol estimation algorithms assessed (GA-assisted ML detection, iterative algorithms, conventional OFDM detection, lower bound: ideal detection without distortions): CL=7dB, N=64,  $\delta_{gen}=250$ ,  $\Gamma_{size}=200$ .

# **Table captions**

Table 1. Analysis of computational complexity of the different OFDM symbol estimation algorithms.



Figure 1



Figure 2



Figure 3



Figure 4

SYMBOL ESTIMATION	ORDER OF	# OF ELEMENTARY OPERATIONS	# OF ELEMENTARY
ALGORITHM	COMPUTATIONAL COMPLEXITY	PER OFDM SYMBOL	<b>OPERATIONS PER</b>
			DATA SYMBOL
GA-assisted ML	$(P_{C} + P_{M})\delta_{am}\Gamma_{am}$	$4.1 \times 10^4$ (N=64, $\delta_{aan}$ =250,	641
estimation (proposed)	C M' gen size	<b>F</b> -200)	
[11]		$\Gamma_{size} = 200)$	
Conventional OFDM	$N \log_2 N$	$3.84 \times 10^2 (N=64)$	6
demodulator [1]	62		
Iterative decoding [4]	$(N+2N\log_2 N)\chi$	$2.5 \times 10^3 (N=64, \chi=3)$	39
Decision-aided	$(2N+2N\log_2 N)\chi$	$2.7 \times 10^3 (N=64, \chi=3)$	56
reconstruction [5]			
Bayesian inference [6]	$MN^{2}\chi$	$3.27 \times 10^5 (N=64, \chi=5)$	5109
Decision-directed	$(N+2N\log_2 N)\chi$	$2.5 \times 10^3 (N=64, \chi=3)$	39
clipping removal [9]		~	
Sub-optimum iterative	$\begin{bmatrix} I_{\text{max}} & (N \log_2 M) \end{bmatrix}$	$2.95 \times 10^5 (N=64, \chi=3, I_{max}=1)$	4609
ML detection [10]	$\left\{1+\sum_{i=1}^{N}\chi_{i}\right\} N \log_{2} N$		
Theoretical ML	$2^{(\log_2 M)N}$	$1.2 \times 10^{77} (M=16, N=64)$	$1.875 \text{x} 10^{75}$
estimation [10]	-		

Table 1