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# A GENETIC ALGORITHM-BASED MMSE RECEIVER FOR MC-CDMA SYSTEMS TRANSMITTING OVER TIME-VARYING MOBILE CHANNELS

Claudio Sacchi, Massimo Donelli, Leandro D'Orazio, Riccardo Fedrizzi, and Francesco G.B. De Natale

***Abstract*** – In this letter[1], a novel multi-user detector, based on a Genetic Algorithm-assisted per-carrier MMSE criterion, is proposed for MC-CDMA systems transmitting over time-varying multipath fading channels. The analyzed multi-user detector outperforms state-of-the-art adaptive receivers based on deterministic gradient algorithms, particularly for an increasing number of users.

***Introduction*** – Despite their intrinsic sub-optimality, multi-user detection (MUD) algorithms based on Minimum-Mean-Squared Error (MMSE) criterion are often preferred in practical applications of Multi Carrier-CDMA (MC-CDMA) due to their reduced computational load and because they can easily support adaptive implementations. In [1] and [2] different MMSE-MUD adaptive receivers for MC-CDMA systems are shown. They are based on Least-Mean-Square (LMS) [1], Recursive-Least-Squared (RLS) [1], and Normalized-Least-Mean-Square (NLMS) [2] optimization algorithms. All these approaches rely on the concept of deterministic gradient [3]. They are very efficient from a computational point of view. On the other hand, their performances and convergence rates are strongly influenced by the choice of the LMS/RLS updating parameters. This drawback can hinder the employment of adaptive MMSE-MUD in time varying fading channels, making them more suitable for static channels (see, e.g., [1]). In the present letter, we are going to discuss a genetic algorithm (GA)-assisted approach for per-carrier MMSE-MUD applied to MC-CDMA communication systems working over time-varying mobile channels. The proposed GA-assisted MMSE-MUD works in two steps: a trained step and a decision-directed step. The trained step relies on the periodic transmission of a short, known training, sequence. The period is equal to the coherence time of the channel. During the decision-directed step, the GA optimizer is re-parameterized and the receiver weights are dynamically updated on the

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basis of the estimated data symbols. The objective of the proposed analysis is to develop a multi-user receiver structure characterized by improved adaptation capability with respect to channel conditions, reduced sensitivity to parameterization, and reasonable computational load.

***2. GA-assisted MMSE receiver structure –*** Let us consider a per-carrier MMSE multi-user detector [1]. The optimization criterion is to find an *M*-element weight vector (*M* is the number of subcarriers) such that:

$$q_m^O(i) = \arg\min_{q_m(i)} E\left\{\left|\left(\sum_{k=1}^{K} c_m^k a_i^k\right) - q_m(i) y_m(i)\right|^2\right\} \quad m = 0,1,..,M-1 \tag{1}$$

where: $y_m(i)$ is the signal received over the m-th subcarrier during the i-th signalling period of duration *T*, *K* is the number of active users, $c_m^k$ is the m-th chip of the k-th spreading code, and $a_i^k$ is the complex symbol transmitted by the k-th user. The explicit solution of MMSE-MUD problem can be computed as a function of the per-carrier channel coefficients [1]. Full-adaptive MMSE implementations (based on LMS and RLS criteria) are considered in [1] and [2] in order to avoid explicit channel estimation. Such solutions are very attractive from a computational point of view. Nevertheless, their efficiency is strongly limited by the sensitivity to the updating parameters, i.e.: the step size for LMS and the forgetting factor for RLS. Moreover, when fast channel variations occur, deterministic gradient-based solutions suffer from effects of lag errors [3] that make very difficult an effective tracking of the optimal weight $q_m^O(i)$. In this framework, we are proposing a semi-adaptive genetic algorithm-assisted MMSE-MUD algorithm. Genetic algorithms are stochastic-gradient based optimization tools whose basic features are [4]: a) the convergence to the optimal solution is theoretically guaranteed (provided that a suitable parameterization of the GA procedure is adopted) avoiding that solution be trapped in local minima, b) the GA-based procedure can dynamically adapt itself to time-varying system conditions, because a new population of individuals is computed at each new generation. The expected outcome of the proposed approach is the implementation of an MMSE-MUD algorithm that is efficient in the presence of time-varying channels. In order to accomplish this goal, we studied a GA-assisted MMSE strategy articulated into two steps:

1) *Training-aided step*. During this step, a *B* bit-length binary training sequence $\underline{\tilde{a}}^k = \left[\tilde{a}_1^k,...,\tilde{a}_B^k\right]$ is transmitted for each user *k*. The training step is repeated with a period approximately equal to the coherence time of the channel. The GA works with a selected parameterisation in terms of generation number $G_{Tr}$, population size $P_{Tr}$, crossover and mutation probabilities $\alpha_{Tr}$ and $\gamma_{Tr}$ respectively. The task of GA is to compute the weight vector $\left\{\hat{q}_m^{TR}, m = 0,..,M-1\right\}$ that minimizes the following metric:

$$\Lambda(\hat{q}_m) = \left\langle \left| \left( \sum_{k=1}^{K} c_m^k \tilde{a}_i^k \right) - \hat{q}_m(i) y_m(i) \right|^2 \right\rangle \quad m = 0,\ldots,M-1 \qquad (2)$$

The GA-based computation of the optimal weights is performed after having buffered *B* samples of the received signal $y_m(i)$ (see fig.1). Note that the ensemble average of eq.1 has been replaced by the sample average of eq.2 (bracketed notation), made on the entire duration of the training sequence.

2) *Decision-directed step*. It is known that, during a coherence period, the stochastic values assumed by the channel coefficients acting over each subcarrier are correlated. This means that time variations of the channel impulse response are reasonably small and a decision-directed updating step can proceed. In the proposed algorithm, the decision-directed step is performed by the GA, working with a different parameterisation and a different fitness function. The GA-based updating procedure is carried on symbol after symbol and it is initialised by the solution computed during the training-aided step, i.e.: $\underline{\hat{q}}^{TR}$. During each symbol period, a single generation of individuals is produced. The new population is generated starting from the solution computed at the previous signalling period $\underline{\hat{q}}^{DD}(h-1)$ and imposing to the Gaussian generator an updating standard deviation $\sigma_{up}$. Such a last parameter is conceptually linked to the Doppler spread and to the signal-to-noise ratio. Among the new population, the individual $\underline{\hat{q}}^{DD}(h)$ is chosen that minimizes the metric:

$$\Omega\left(\hat{q}_m^{DD}(h)\right) = \left| \left( \sum_{k=1}^{K} c_m^k \hat{a}_h^k \right) - \hat{q}_m^{DD}(h) y_m(h) \right|^2 \quad m = 0,..,M-1 \qquad (3)$$

In such a step, crossover and mutation operators don't work, because only a single GA generation runs. Note that the value $\hat{a}_h^k$ is related here to the estimated data symbol. When the decision-directed step ends, the GA is re-initialised with the weights computed at the end of the coherence time-window, i.e.: $\underline{\hat{q}}^{DD}(W_{coh})$ and re-parameterised in order to start again with the training-aided step.

***3. Experimental results*** – The semi-adaptive GA-assisted per-carrier MMSE-MUD algorithm has been tested by means of intensive simulations, considering the following fixed parameters: number of subcarriers *M*=32, symbol rate *r* =1024Kbaud/s, coherence bandwidth equal to 2MHz, Doppler spread equal to 100Hz. The following parameterization of the GA-based optimizer has been selected: a) training-aided step: generation number $G_{Tr}$=10 population size, $P_{Tr}$=10, crossover probability $\alpha_{Tr}$ =0.9, mutation probability $\gamma_{Tr}$=0.01, training sequence length *B*=32; b) decision-directed step: generation number $G_{DD}$=1, population size, $P_{DD}$=10. Finally, in overall simulation, we have considered $\sigma_{up}$ equal to 0.025. Such a choice was proven as the most effective for the largest amount of simulation configurations adopted. Simulation results in terms of measured bit-error-rate are shown in fig.2 and fig.3. In fig.2, BER results are plotted versus signal-to-noise ratio (*SNR*) for a fixed number of users (*K*=9). One can note that the BER curve related to the proposed GA-assisted MMSE-MUD algorithm is almost coincident with the curve related to ideal MMSE-MUD for all SNR values. On the other hand, BER performances of LMS MMSE are strongly influenced by the choice of the step size $\mu$. In any case, LMS performs worse than GA when SNR is high and the impact on system performances of multi-user interference (MUI) is dominant. A similar behavior can be noted for RLS. The setting of $\lambda$ parameter (equal to 0.75) has been performed on the basis of simulation results presented in [1]. Other settings of $\lambda$ provide worse BER results, not reported here. In fig.3, BER results are drawn vs. user number for *SNR*=20dB. We can see that the BER curve provided by the GA-assisted MMSE-MUD is very close to the ideal MMSE curve, also for a number of users almost equal to the maximum allowable (*K*=30). On the other hand, LMS performances substantially degrade with respect to ideal MMSE as the number of user increases. RLS curve is quite closed to GA-assisted MMSE-MUD and ideal MMSE curve when the number of users is small, but it trends to fairly degrade for larger values of *K*. About computational complexity issues, we can briefly mention that, on the basis of considerations made in [1], [2] and [4], the computational burden of GA-assisted MMSE-MUD in terms of elementary operations is about one order of magnitude higher than LMS-based MUD and about of the same order of RLS-based MUD.

***4. Conclusion –*** In this paper we proposed a novel semi-adaptive GA-based approach for MMSE-MUD in MC-CDMA systems transmitting information over time-varying fading channels. The proposed algorithm evidenced some advantages with respect to best-known state-of-the-art solutions. Simulation results achieved in terms of BER evidenced a near-ideal behavior of the proposed algorithm, outperforming LMS and RLS-based approaches especially when the impact of MUI becomes predominant in limiting transmission capacity.

***References***

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**Authors' affiliation:**

University of Trento, Department of Information and Telecommunication Technology (DIT), Via Sommarive 14 I-38050, Trento, Italy, e-mail: sacchi@dit.unitn.it

## FIGURE CAPTIONS

Figure 1. Block diagram of the semi-adaptive MMSE-MUD receiver: training-aided step

Figure 2. BER performances vs. *SNR* for the simulated MC-CDMA receivers (*K*=9)

Figure 3. BER performances vs. *K* for the simulated MC-CDMA receivers (*SNR*=20dB)

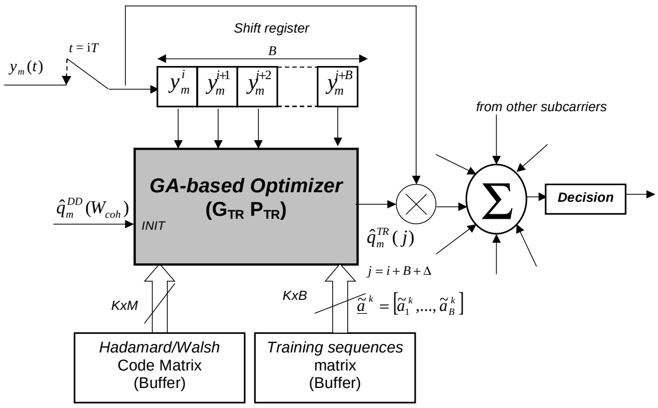



**Figure 1**

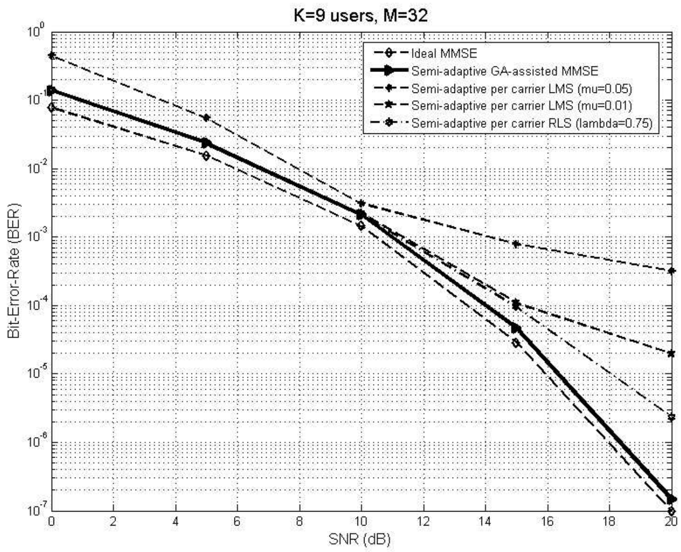
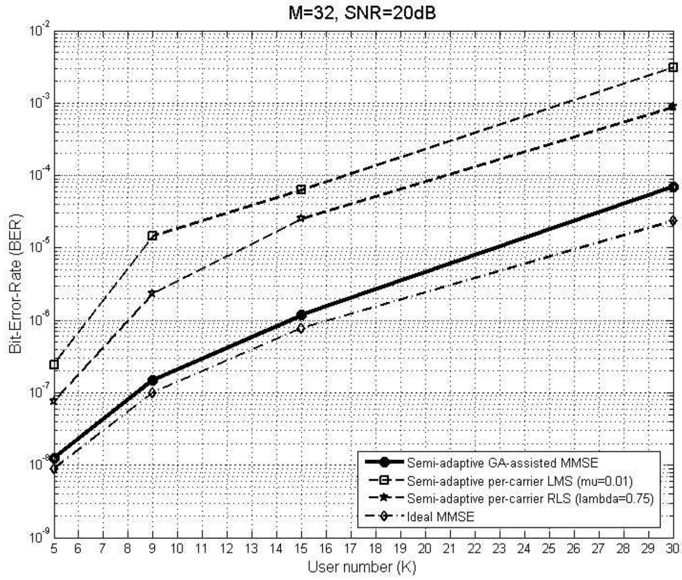



**Figure 2**



M=32, SNR=20dB
Bit-Error-Rate (BER)
User number (K)
Semi-adaptive GA-assisted MMSE
Semi-adaptive per-carrier LMS (mu=0.01)
Semi-adaptive per-carrier RLS (lambda=0.75)
Ideal MMSE


**Figure 3**