# Combining Instance Generation and Resolution

Christopher Lynch and Ralph Eric McGregor

Department of Mathematics & Computer Science
Clarkson University

# Theorem Proving

- Given a set of clauses $\Gamma$ and a clause C (a conclusion), our goal is to prove
  - $\Gamma \rightarrow C$ is valid
  - or equivalently, ($\Gamma \wedge \neg$C) is unsatisfiable

# Motivation

- Ordered Resolution and Instance Generation with semantic selection (SInst-Gen)
  - Each uses a unique proof procedure
  - Each has individual strengths
  - Both competitive in practice
- SIG-Res
  - hybrid inference system combining Ordered Resolution and SInst-Gen

# Outline

- Preliminaries
- Ordered Resolution
- SInst-Gen
- SIG-Res
- Spectrum
- Future Work

# Setting

- Standard first-order logic without equality
- Formula in conjunctive normal form

# Substitutions and Unifiers

- a substitution is a map from variables to terms
  - $\delta: V \to T$

$\delta:\{x \to a, y \to z\}$

- a unifier of atoms $P$ and $Q$ is a substitution $\delta$ such that $P\delta = Q\delta$

$R(x) \quad R(y)$

$\delta:\{x \to y\}$

$R(y) = R(y)$

# Most General Unifier

- the most general unifier of P and Q is a unifier *of P and Q, $\sigma$, such that for all unifiers of P and Q, $\delta$, there exists a substitution, $\tau$, such that $\sigma\tau=\delta$*

*R(x)* *R(y)*

$\delta$:{$x \rightarrow a, y \rightarrow a$}

*R(a) = R(a)*

$\sigma$:{$x \rightarrow y$}

$\tau$:{$y \rightarrow a$}

$\sigma\tau=\delta$

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# Orderings

- A (strict) partial ordering, >, is a transitive and irreflexive binary relation.
- A strict ordering is well founded if there is no infinite descending chain of elements.
- An ordering is stable under substitution if when $s > t$ then $s\sigma > t\sigma$.

# Maximal Literals

- Let > be a strict partial ordering on terms which is well founded and stable under substitutions. We say literal $L$ is maximal in clause $C$ if $L \in C$ and there is no $K \in C$ such that $K > L$.

# Ordered Resolution and Factoring

## Inference Rules

- Ordered Resolution

$$\frac{\Gamma \vee P \quad \Delta \vee \neg P'}{(\Gamma \vee \Delta)\sigma}$$

where $\sigma$ = mgu(P,P′)

and $P \in \max(\Gamma \vee P)$

and $P' \in \max(\Delta \vee \neg P')$

- Factoring

$$\frac{\Gamma \vee P \vee P'}{(\Gamma \vee P)\sigma}$$

where $\sigma$ = mgu(P,P′)

# Safe Factoring

- If C is a factor of D then clearly, D$\rightarrow$C. If C$\rightarrow$D, then we may delete D.
- Safe-Factoring applies factoring only when the factor implies the premise – allowing deletion of the premise.
- Ordered Resolution with Safe-Factoring is complete.

# Ordered Resolution Procedure

- Repeatedly apply ordered resolution and factoring inference rules in a fair manner.
  - Refutationally complete
  - If empty clause ($\perp$) is generated, then set of clauses is unsatisfiable

$$\frac{P \quad \neg P'}{\perp}$$

where $P$ and $\neg P'$ are unifiable

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# Interpretations as Models

- $\perp$ is also used to denote a distinguished constant and the substitution that maps all variables to $\perp$.
- Given a set of clauses,P, $P\perp$ can be viewed as a set of propositional clauses.
- A Herbrand Interpretation, I, is a consistent set of ground literals.
- If $P\perp$ is satisfiable then we denote a model for $P\perp$ by $I_\perp$.

# Selection

- Given a model $I_{\perp}$ for $P\perp$ we define a selection function, $sel(C, I_{\perp})$, which maps each clause $C \in P$ to a singleton set $\{L\}$ such that $L \in C$ and $L\perp$ is true in $I_{\perp}$.
- If $sel(C, I_{\perp}) = \{L\}$ then $L$ is referred to as a selected literal.

# SInst-Gen
## Inference Rule

$$\frac{\Gamma \vee P \qquad \Delta \vee \neg P'}{(\Gamma \vee P)\sigma \quad (\Delta \vee \neg P')\sigma}$$

*where* $P \in sel(\Gamma \vee P, I_{\perp})$,

$P' \in sel(\Delta \vee \neg P', I_{\perp})$

*and* $\sigma = mgu(P,P')$

# SInst-Gen Procedure

Given a set of first order clauses P

1. Construct $P\bot$
2. Run SAT on $P\bot$ (viewed as propositional clauses)

- If $P\bot$ is unsatisfiable, P is unsatisfiable and we are done.
- Else if $P\bot$ is satisfiable by $I_\bot$, determine selected literals.
    - If no Sinst-Gen inferences can be made, P is satisfiable and we are done.
    - Else, add Sinst-Gen conclusions to P and goto 1.

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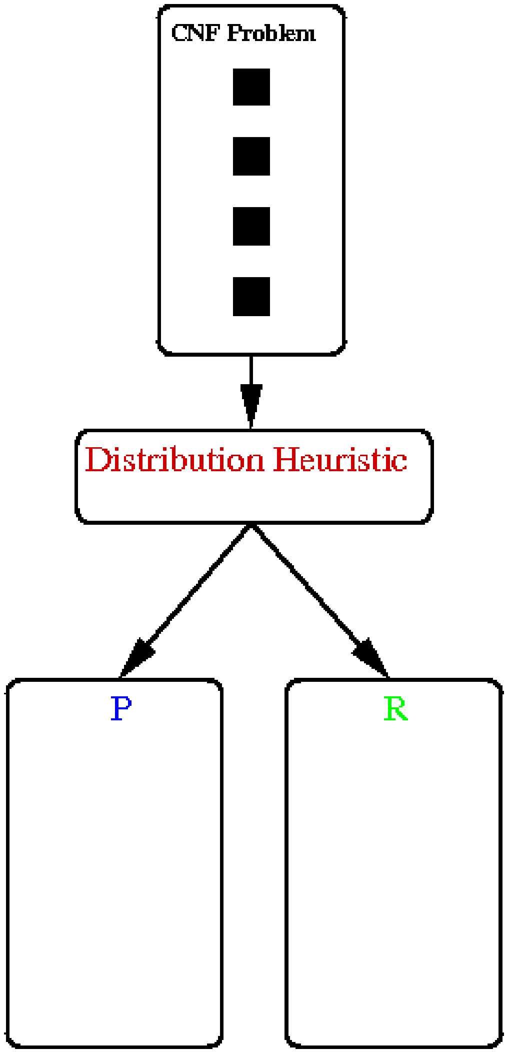
# Selection Redefined

If $C \in P$, $sel(C, I_\perp) = \{L\}$ such that $L \in C$ and $L^\perp \in I_\perp$

If $C \in R$, $sel(C, I_\perp) = max(C)$

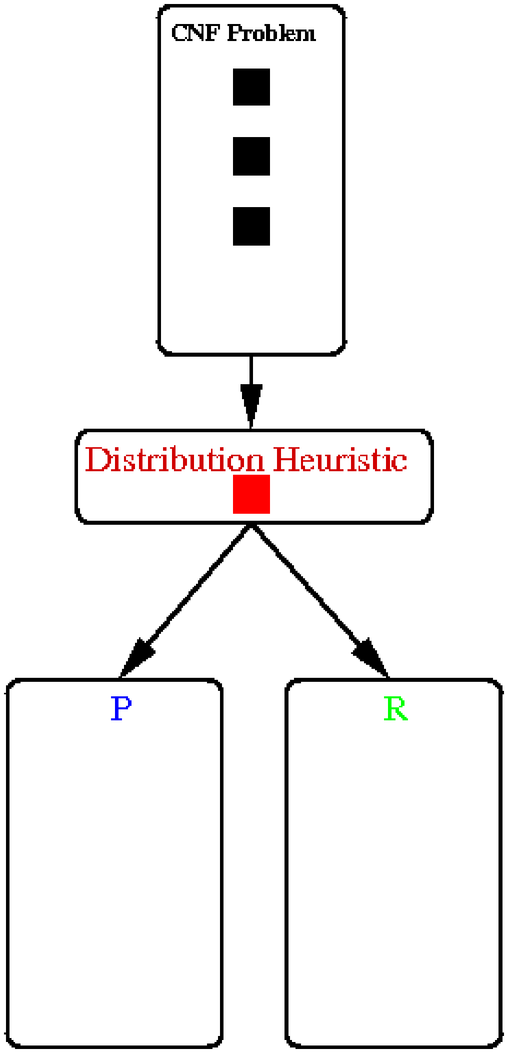
# Distribution Heuristic

- Distribute clauses into two sets P and R using a distribution heuristic



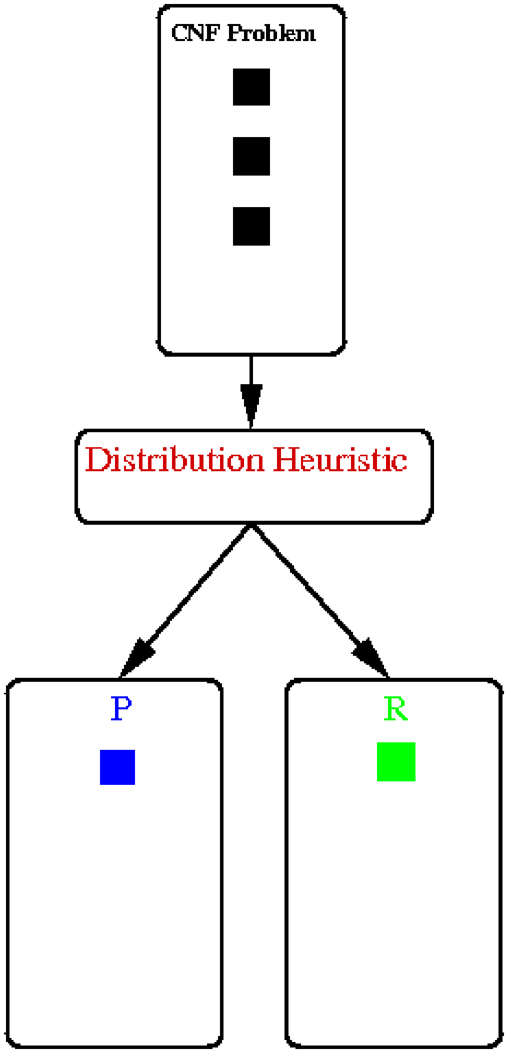
# Distribution Heuristic

- Clauses in R are treated as resolution clauses
- Clauses in P are treated as instance generation clauses



# Distribution Heuristic

- P and R are not necessarily disjoint



# Ground/Single Max Heuristic: GSM

- For each clause
  - if ground, put in P.
  - find the maximal literals (KBO)
    - if the number of maximal literals is 1 insert clause in R, otherwise insert clause in P
- Reasoning
  - Ground clauses on PI do not generate new clauses
  - Clauses containing a single maximal literal tend to produce smaller clauses with Res

# SInst-Gen

$$\Gamma \vee P \qquad \Delta \vee \neg P'$$

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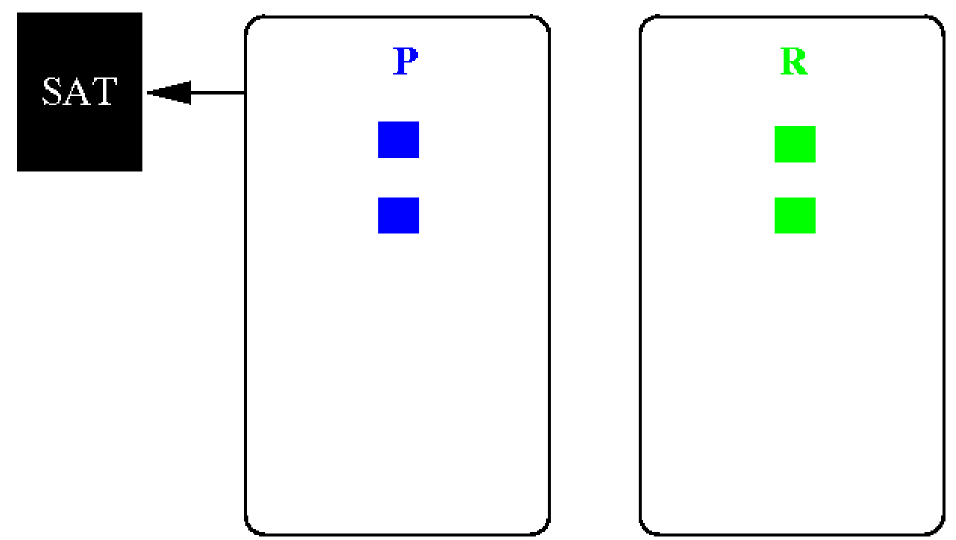
$$(\Gamma \vee P)\sigma \quad (\Delta \vee \neg P')\sigma$$




*where*

i) $\Gamma \vee$ P$\in$ P and $\Delta \vee \neg$P$\in$ P

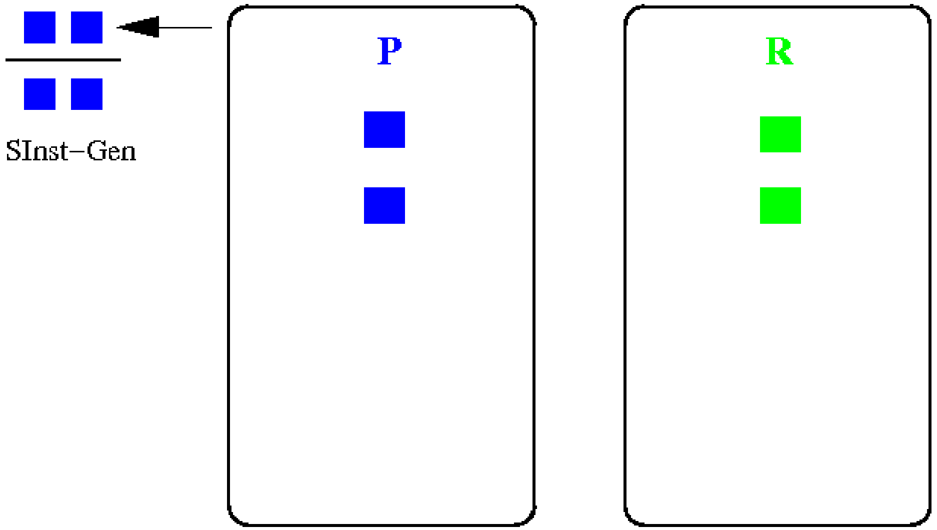
ii) P$\in$sel($\Gamma \vee$ P, $I_\bot$) and P$'\in$sel($\Delta \vee \neg$P$'$, $I_\bot$)

iii) $\sigma$ = mgu(P,P$'$)

iv) ($\Gamma \vee$ P)$\sigma$ $\in$ P and ($\Delta \vee \neg$P)$\sigma$ $\in$ P

# SInst-Gen

$$\frac{\Gamma \vee P \qquad \Delta \vee \neg P'}{(\Gamma \vee P)\sigma \quad (\Delta \vee \neg P')\sigma}$$




*where*

i) $\Gamma \vee P \in$ P and $\Delta \vee \neg P \in$ P

ii) $P \in sel(\Gamma \vee P, I_{\perp})$ and $P' \in sel(\Delta \vee \neg P', I_{\perp})$

iii) $\sigma$ = mgu(P,P′)

iv) $(\Gamma \vee P)\sigma \in$ P and $(\Delta \vee \neg P)\sigma \in$ P

# SInst-Gen

$$\frac{\Gamma \vee P \qquad \Delta \vee \neg P'}{(\Gamma \vee P)\sigma \quad (\Delta \vee \neg P')\sigma}$$

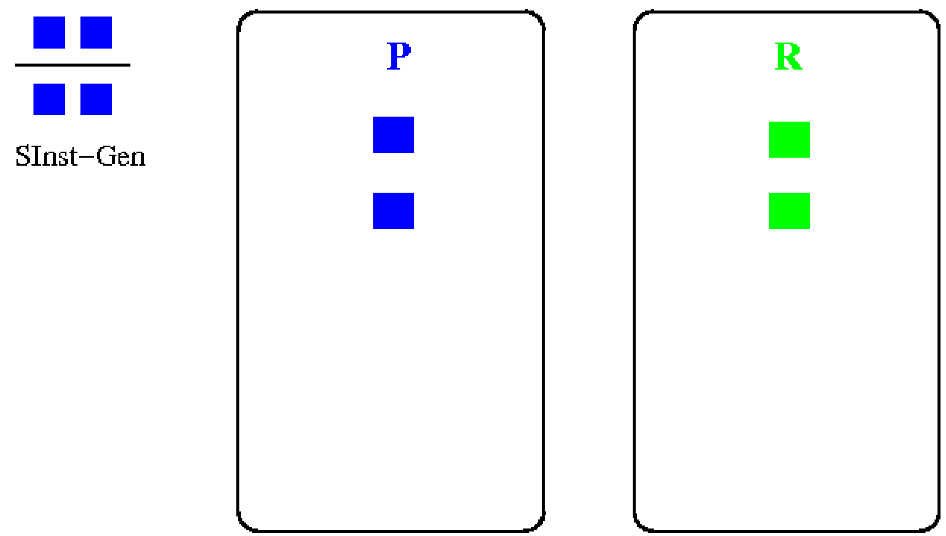
*where*

i) $\Gamma \vee P \in \mathrm{P}$ and $\Delta \vee \neg P \in \mathrm{P}$

ii) $P \in sel(\Gamma \vee P, I_{\perp})$ and $P' \in sel(\Delta \vee \neg P', I_{\perp})$

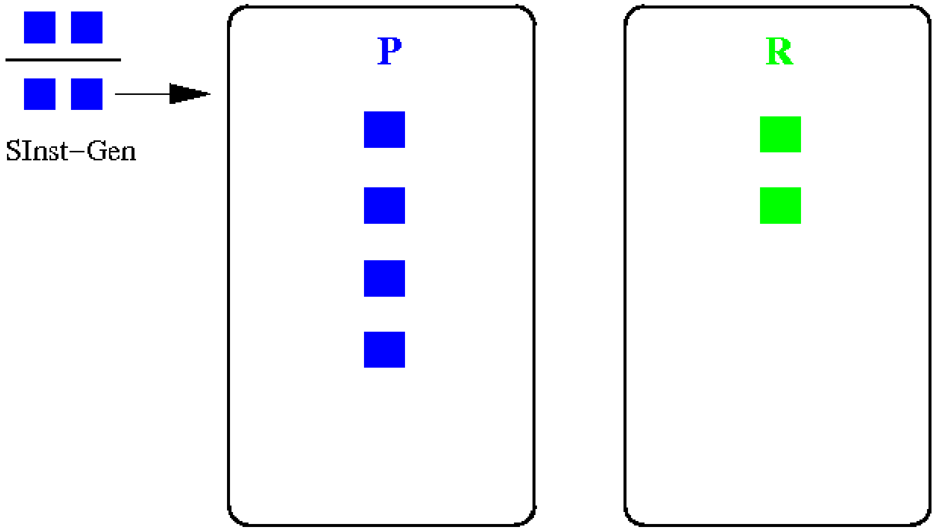
iii) $\sigma = mgu(P, P')$

iv) $(\Gamma \vee P)\sigma \in \mathrm{P}$ and $(\Delta \vee \neg P)\sigma \in \mathrm{P}$



# SInst-Gen

$$\frac{\Gamma \vee P \qquad \Delta \vee \neg P'}{(\Gamma \vee P)\sigma \quad (\Delta \vee \neg P')\sigma}$$




*where*

i) $\Gamma \vee P \in$ P and $\Delta \vee \neg P \in$ P

ii) $P \in sel(\Gamma \vee P, I_\perp)$ and $P' \in sel(\Delta \vee \neg P', I_\perp)$

iii) $\sigma = mgu(P,P')$

iv) $(\Gamma \vee P)\sigma \in$ P and $(\Delta \vee \neg P)\sigma \in$ P

# Resolution

$$\frac{\Gamma \vee P \quad \Delta \vee \neg P'}{(\Gamma \vee \Delta)\sigma}$$

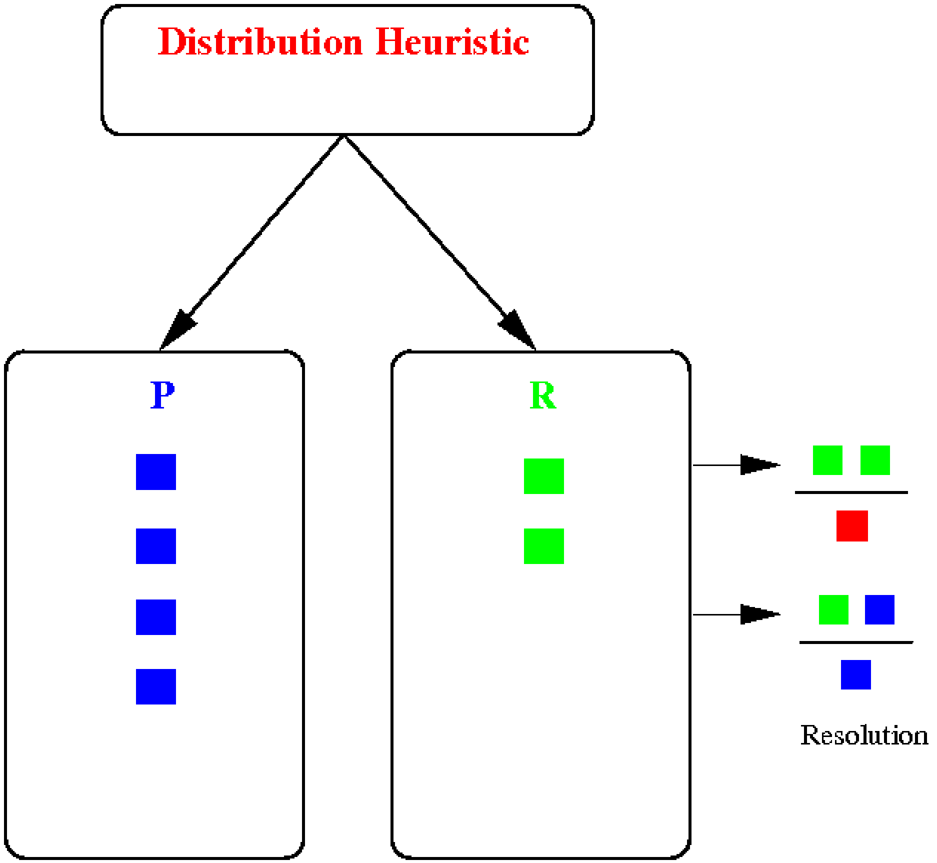
where

i) $\Gamma \vee P \in R$ or $\Delta \vee \neg P \in R$

i) $P \in sel(\Gamma \vee P, I_{\perp})$ and $P' \in sel(\Delta \vee \neg P', I_{\perp})$

iii) $\sigma = mgu(P,P')$

iv) $(\Gamma \vee P)\sigma \in P$ if $\Gamma \vee P \notin R$ or $\Delta \vee \neg P \notin R$



# Resolution

$$\frac{\Gamma \vee P \quad \Delta \vee \neg P'}{(\Gamma \vee \Delta)\sigma}$$

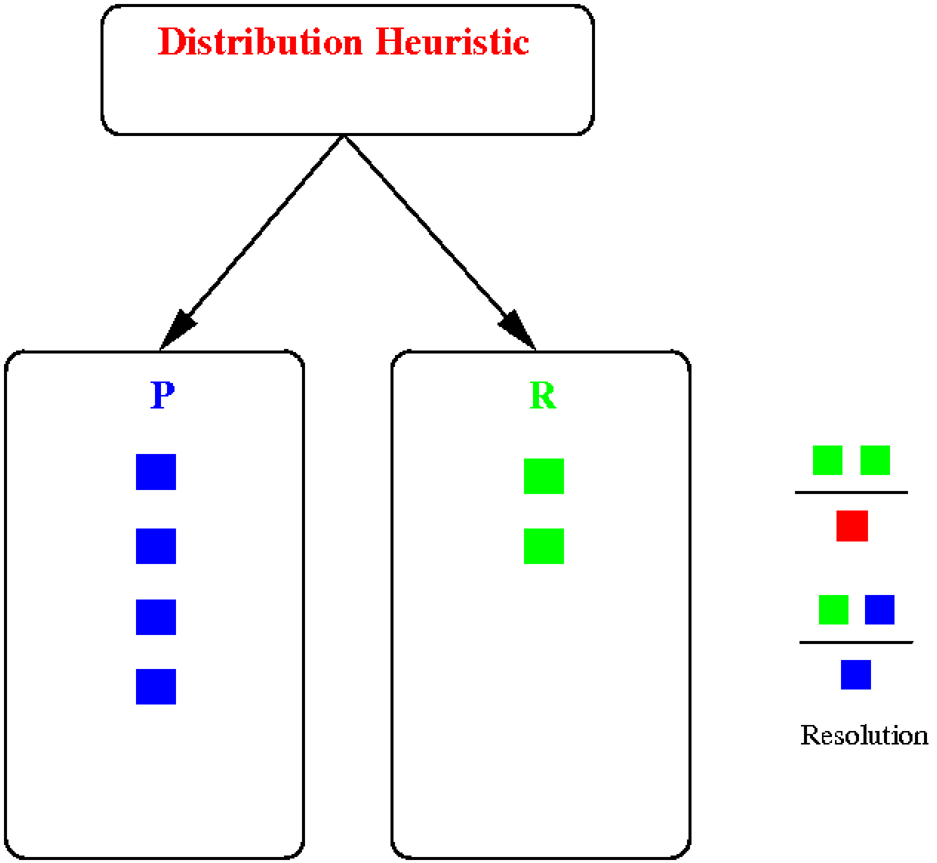
where

i) $\Gamma \vee P \in R$ or $\Delta \vee \neg P \in R$

i) $P \in sel(\Gamma \vee P, I_{\perp})$ and $P' \in sel(\Delta \vee \neg P', I_{\perp})$

iii) $\sigma = mgu(P,P')$

iv) $(\Gamma \vee P)\sigma \in P$ if $\Gamma \vee P \notin R$ or $\Delta \vee \neg P \notin R$



# Resolution

$$\Gamma \vee P \quad \Delta \vee \neg P'$$

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$$(\Gamma \vee \Delta)\sigma$$

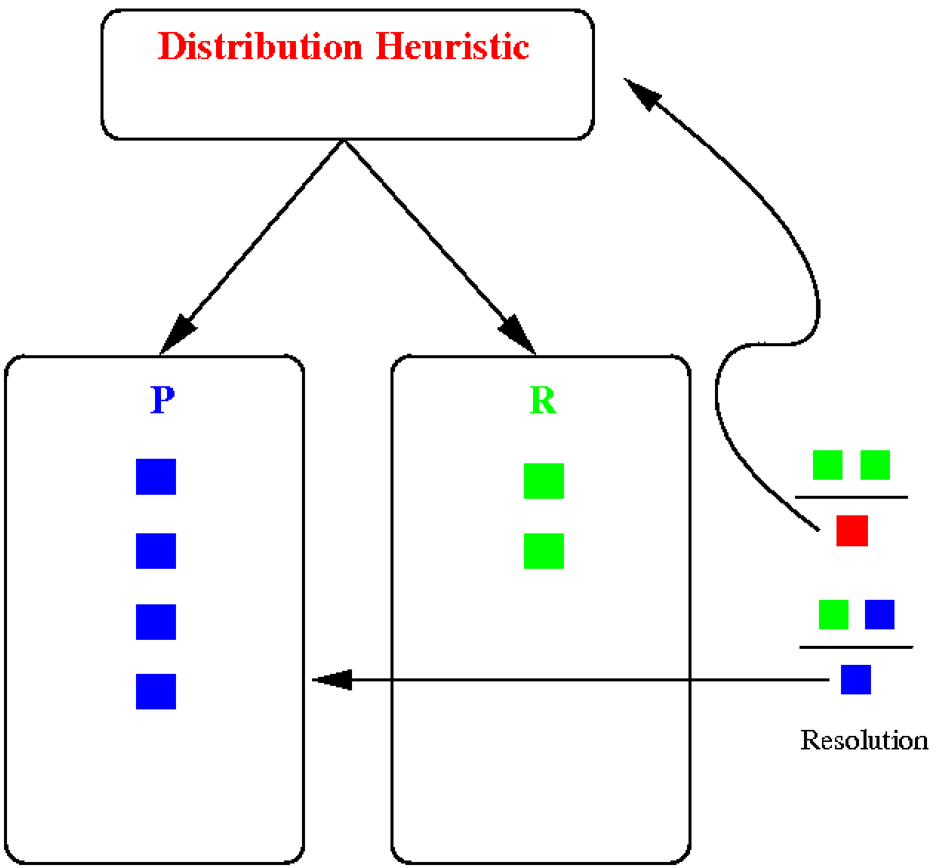
where

i) $\Gamma \vee P \in R$ or $\Delta \vee \neg P \in R$

i) $P \in sel(\Gamma \vee P, I_\perp)$ and $P' \in sel(\Delta \vee \neg P', I_\perp)$

iii) $\sigma = mgu(P,P')$

iv) $(\Gamma \vee P)\sigma \in P$ if $\Gamma \vee P \notin R$ or $\Delta \vee \neg P' \notin R$



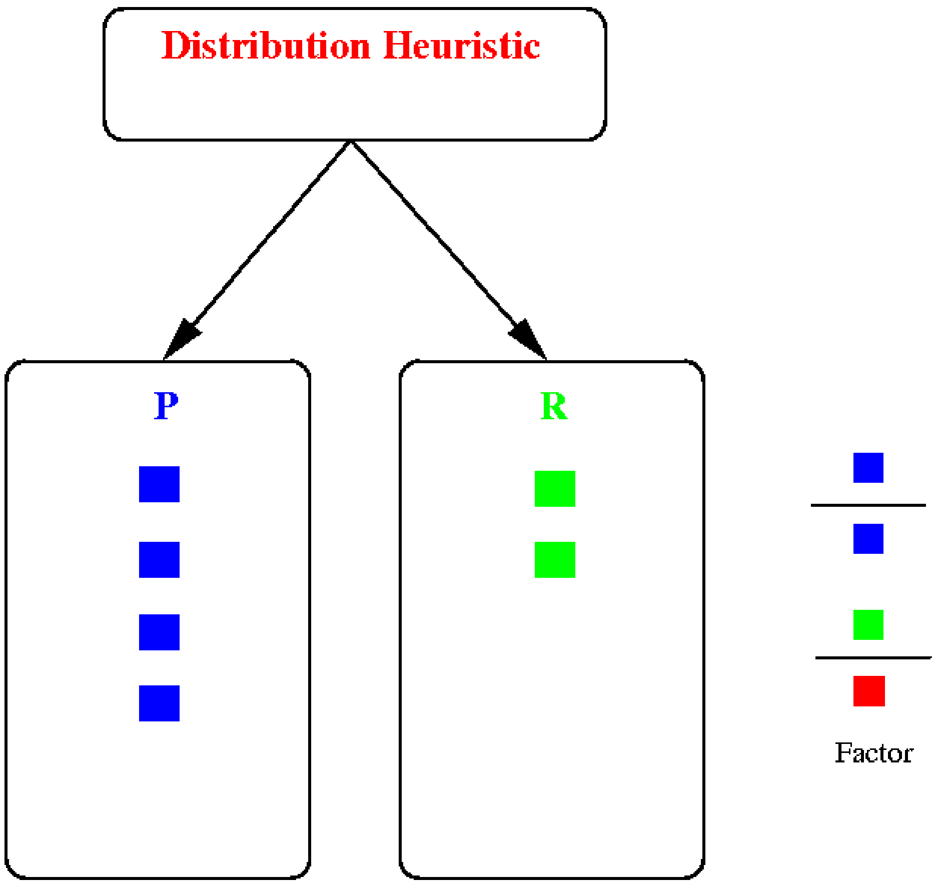
# Factoring

$$\frac{\Gamma \vee P \vee P'}{(\Gamma \vee P)\sigma}$$

where

i) $\sigma$ = mgu(P,P′)

ii) $(\Gamma \vee P)\sigma \in P$ if $\Gamma \vee P \vee P' \notin R$



# Benefits of SIG-Res

- Complete inference system.
- During the initial partition phase, attempts to choose which inference system will be best suited for each clause.
- Allows complete spectrum of solutions from a pure Instance Generation solution to a pure Resolution solution.

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# Spectrum

- Written in $C^{++}$
- TPTP input (CNF form)
- SInst-Gen (Primal P.I. Algorithm)
- Uses Yices SMT for SAT solving
- Ordered Resolution (KBO ordering)
- Safe-Factoring
- SIG-Res with Ground/Single Max partition heuristic
- Redundancy elimination (forward sub, taut elim)

# Results

- Tested 450 easy TPTP problems
  - Spectrum solved 192 in 300s
  - 18 using heuristic alone
    - 16 in LCL class
    - contain transitivity axioms
    - contain growing functions
      - $\neg P(x) \vee P(f(x))$

# Future Work

- Continue developing Spectrum
  - add additional redundancy elimination
  - utilize better data structures and term indexing
  - restrict Sinst-Gen using dismatching constraints
- Investigate extending partioning idea to equalities
  - use SMT to solve ground equalities
  - use rewriting to solve non-ground equalities