Combining Description Logics, Description Graphs, and Rules

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Ontologies and the Semantic Web

Key Idea

Make information on the Web machine-processable

- Develop a vocabulary of a domain—an *ontology*
- Annotate information by ontology terms

Benefits

- Implicit meaning in the data is made explicit
- Formal semantics of the ontology language can be used to explicate implicit information

Description Logics and OWL

DLs are KR formalisms with well-understood formal properties

- Underpin the Ontology Web Language (OWL)
- Basic DL is called $\mathcal{ALC}$
- $\mathcal{ALCF}$ = $\mathcal{ALC}$ + functional roles
- $\mathcal{ALCIF}$ = $\mathcal{ALCF}$ + inverse roles

KBs consist of *concepts* (= unary predicates), *roles* (= binary predicates), and *individuals* (= constants)

$\mathcal{ALC}$: Syntax and Semantics

Interpretation of Roles and Concepts
$(\neg C)^I = x \notin C^I$
$(C \sqcap D)^I = x \in C^I \wedge x \in D^I$
$(C \sqcup D)^I = x \in C^I \vee x \in D^I$
$(\exists R.C)^I = \exists y \in \triangle^I : \langle x, y\rangle \in R^I \wedge y \in C^I$
$(\forall R.C)^I = \forall y \in \triangle^I : \langle x, y\rangle \in R^I \rightarrow y \in C^I$

Interpretation of Axioms and Assertions		
$I \models C \sqsubseteq D$	iff	$\forall x \in \triangle^I : x \in C^I \rightarrow x \in D^I$
$I \models C(a)$	iff	$a^I \in C^I$
$I \models R(a, b)$	iff	$\langle a^I, b^I\rangle \in R^I$

Relevant Reasoning Problems

Concept Satisfiability

Check whether a model of $\mathcal{O}$ exists in which C is not empty

Concept Subsumption

Check whether $\mathcal{O} \models C \sqsubseteq D$

- $\mathit{UKCity} \sqsubseteq \mathit{EUCity}$ is a consequence of
$$\mathcal{O} = \{\ \mathit{UKCity} \sqsubseteq \exists \mathit{cityLocation}.\mathit{UKRegion},\ \mathit{UKRegion} \sqsubseteq \mathit{EURegion},\ \exists \mathit{cityLocation}.\mathit{EURegion} \sqsubseteq \mathit{EUCity}\ \}$$

Query Answering

Check whether $\mathcal{O} \models q$ where q is a *conjunctive query*

Structured Objects

What are structured objects?

Objects composed of other, possibly interrelated, objects

Examples

- the human body
- the benzene molecule
- an airplane
- ...

Why are structured objects important?

At the core of many ontologies (FMA, GALEN, SNOMED, . . .)

Structured Objects

Ontological Representation of Structured Objects

Long-standing open problem

- Modeling not sufficiently precise $\Rightarrow$ more expressive power needed
- Reasoning can be slow $\Rightarrow$ constructed models unnecessarily large

Common solutions based on:

- Modeling patterns $\Rightarrow$ fail to provide the required expressivity
- Language extensions (e.g., rules) $\Rightarrow$ lead to undecidability

No suitable solution known $\Rightarrow$ existing ontologies often inaccurate or wrong

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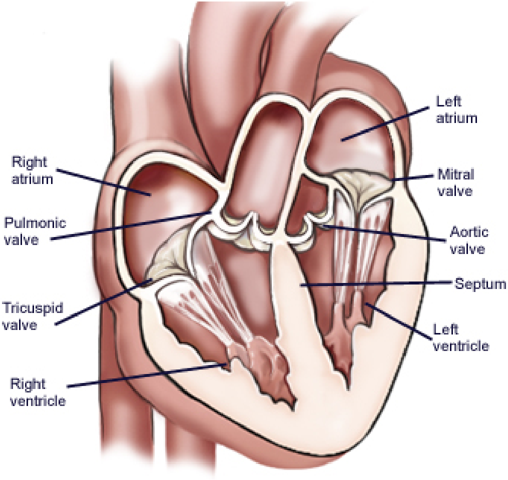
A Novel Solution

Based on an analysis of ontologies in practice:

- Provides the required expressive power
- Improves performance of reasoning

These benefits are not necessarily in conflict!

AN EXAMPLE: THE HUMAN HEART



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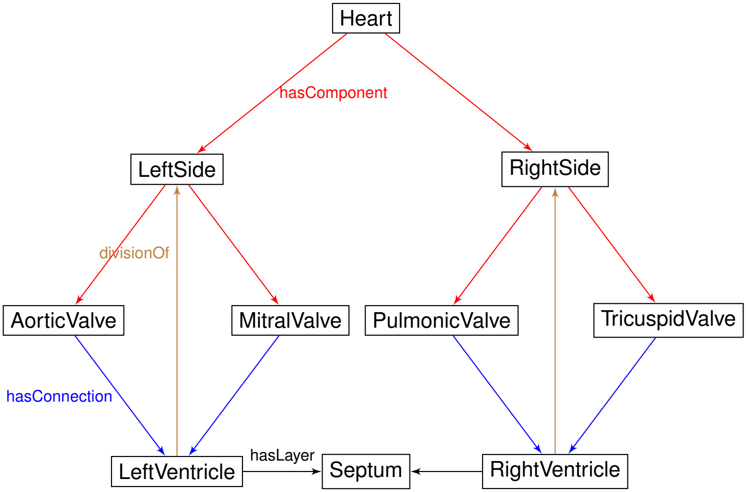
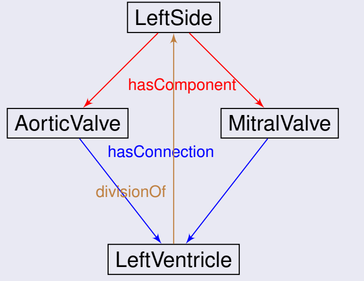


Modeling Structured Objects in DLs

Model the heart in a DL TBox

- We want to represent the structure not of a particular heart, but of all hearts
- The structure should be a “template” that can be instantiated many times

Informal Model



DL Ontology $\mathcal{O}$

$$
\begin{aligned}
\mathit{LeftSide} &\sqsubseteq \exists \mathit{hasComponent}.\mathit{AorticValve} \\
\mathit{LeftSide} &\sqsubseteq \exists \mathit{hasComponent}.\mathit{MitralValve} \\
\mathit{AorticValve} &\sqsubseteq \exists \mathit{hasConnection}.\mathit{LeftVentricle} \\
\mathit{MitralValve} &\sqsubseteq \exists \mathit{hasConnection}.\mathit{LeftVentricle} \\
\mathit{LeftVentricle} &\sqsubseteq \exists \mathit{divisionOf}.\mathit{LeftSide}
\end{aligned}
$$

Is this a Faithful Representation?

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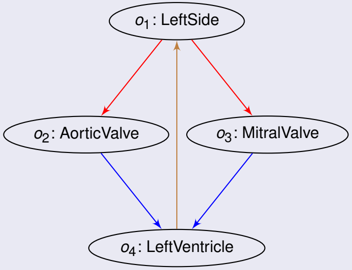
The Intended Model $\mathcal{I}$



$\mathcal{O}$ is satisfied in a model corresponding with our intuitions

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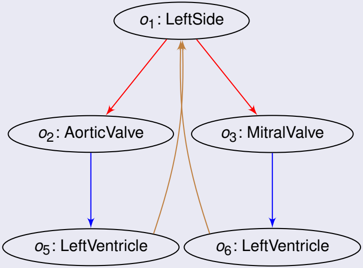
Unintended Model $\mathcal{J}_1$



$\mathcal{O}$ is also satisfied in a model not corresponding with our intuitions

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DL Ontology $\mathcal{O}$

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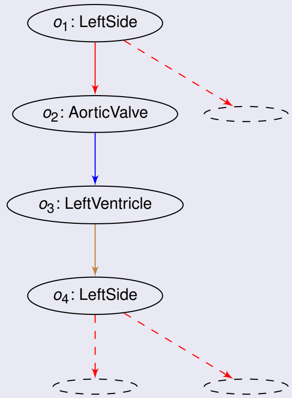
\end{aligned}
$$

Unintended Model $\mathcal{J}_2$



$\mathcal{O}$ is also satisfied in an infinite model not corresponding with our intuitions

Is this a Faithful Representation?

A Tree Model Property

- DL ontology $\mathcal{O}$ has a model $\Rightarrow$ it has a "tree-shaped" one
- Key to ensuring decidability

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- DL ontology $\mathcal{O}$ has a model $\Rightarrow$ it has a “tree-shaped” one
- Key to ensuring decidability

Problems

- Unintended tree models cannot be ruled out in DLs
- Underconstrained representation
- Cannot draw inferences that rely on having only intended models
- Unintended models can be big and expensive to construct $\Rightarrow$ performance problems

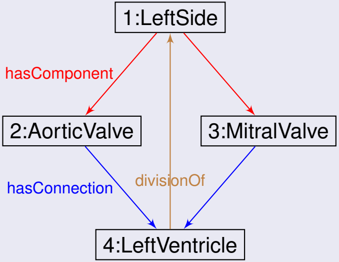
A Novel Solution: Description Graphs

Basic Intuition

"Draw" the intended structure as a graph $G = (V, E, \lambda, M)$

- V: set of nodes
- E: set of edges
- λ: labels nodes with concepts and edges with roles
- M: set of main concepts

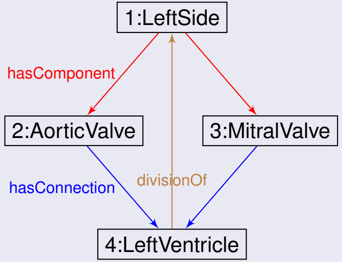
Formal Representation



$$V = \{1, 2, 3, 4\}$$
$$\lambda\langle 1\rangle = \mathit{LeftSide}$$
$$\ldots$$
$$E = \{\langle 1,2\rangle, \langle 1,3\rangle, \langle 2,4\rangle, \langle 3,4\rangle, \langle 4,1\rangle\}$$
$$\lambda\langle 1,2\rangle = \mathit{hasComponent}$$
$$\ldots$$
$$M = \{\mathit{LeftSide}\}$$

Description Graphs: Semantics

Description Graph



Interpretation $\mathcal{I}$

Graph with ℓ vertices $\Rightarrow$ ℓ-ary predicate

Key Property: for each $1 \leq i \leq \ell$,

$$\forall x_1,\ldots,x_\ell,y_1,\ldots,y_\ell \in \triangle^\mathcal{I} : \langle x_1,\ldots,x_\ell\rangle \in G^\mathcal{I} \wedge \langle y_1,\ldots,y_\ell\rangle \in G^\mathcal{I} \wedge x_i = y_i \rightarrow \bigwedge_{1\leq j\leq \ell} x_j = y_j$$

Disjointness Property:

$$\forall x_1,\ldots,x_\ell,y_1,\ldots,y_\ell \in \triangle^\mathcal{I} : \langle x_1,\ldots,x_\ell\rangle \in G^\mathcal{I} \wedge \langle y_1,\ldots,y_\ell\rangle \in G^\mathcal{I} \rightarrow \bigwedge_{1\leq i<j\leq n} x_i \neq y_j$$

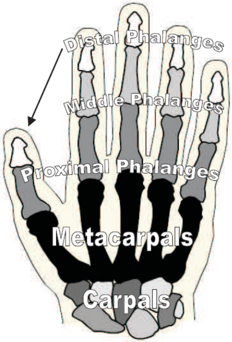
Start Property: for each atomic concept $A \in M$,

$$\forall x \in \triangle^\mathcal{I} : x \in A^\mathcal{I} \rightarrow \exists x_1,\ldots,x_\ell \in \triangle^\mathcal{I} : \langle x_1,\ldots,x_\ell\rangle \in G^\mathcal{I} \wedge \bigvee_{k\in V_A} x = x_k$$

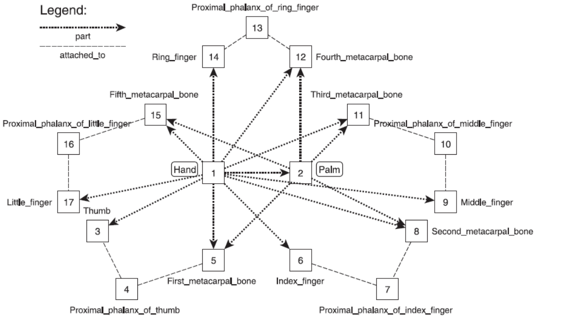
Layout Property:

$$\forall x_1,\ldots,x_\ell \in \triangle^\mathcal{I} : \langle x_1,\ldots,x_\ell\rangle \in G^\mathcal{I} \rightarrow \bigwedge_{i\in V,\ B\in\lambda\langle i\rangle} x_i \in B^\mathcal{I} \wedge \bigwedge_{\langle i,j\rangle\in E,\ R\in\lambda\langle i,j\rangle} \langle x_i,x_j\rangle \in R^\mathcal{I}$$

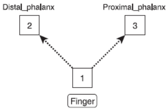
Modularization of the Knowledge Base



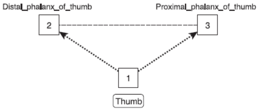
(a) Anatomy of the Hand



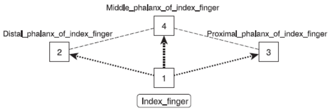
(b) Model of the Hand (G_{hand})



(c) Model of a Finger (G_{finger})



(d) Model of the Thumb (G_{thumb})



(e) Model of the Index Finger (G_{index_finger})

Syntax and Semantics

Syntax

Graph Specialization

$G_1 \lhd G_2$ with $V_1 \subseteq V_2$

G_2 more specific than G_1

$G_{finger} \lhd G_{thumb}$

Graph Alignment

$G_1[u_1, \ldots, u_n] \leftrightarrow G_2[w_1, \ldots, w_n]$

G_1 and G_2 connected at the specified vertices

$G_{hand}[3, 4] \leftrightarrow G_{thumb}[1, 3]$

Semantics

Graph Specialization

$\mathcal{I} \models G_1 \lhd G_2$ if

$$\forall x_1, \ldots, x_{\ell_2} \in \triangle^{\mathcal{I}} : \langle x_1, \ldots, x_{\ell_1}, \ldots, x_{\ell_2} \rangle \in G_2^{\mathcal{I}} \rightarrow \langle x_1, \ldots, x_{\ell_1} \rangle \in G_1^{\mathcal{I}}$$

Graph Alignment

$\mathcal{I} \models G_1[u_1, \ldots, u_n] \leftrightarrow G_2[w_1, \ldots w_n]$ if, for each $1 \leq i \leq n$,

$$\forall x_1, \ldots, x_{\ell_1}, y_1, \ldots, x_{\ell_2} \in \triangle^{\mathcal{I}} : \langle x_1, \ldots, x_{\ell_1} \rangle \in G_1^{\mathcal{I}} \wedge \langle y_1, \ldots, y_{\ell_2} \rangle \in G_2^{\mathcal{I}} \wedge x_{u_i} = y_{w_i} \rightarrow \bigwedge_{1 \leq j \leq n} x_{u_j} = y_{w_j}$$

A COMBINED FORMALISM

A graph-extended knowledge base consists of

- a DL TBox $\mathcal{T}$
- a graph box (GBox) $\mathcal{G}$ composed of:
  - a finite set of description graphs $\mathcal{G}_G$
  - a finite set of graph specializations $\mathcal{G}_S$
  - a finite set of graph alignments $\mathcal{G}_A$
- a set of rules (i.e., function-free implications) $\mathcal{P}$
- an ABox $\mathcal{A}$

Undecidability Results

DLs + Rules

Known from SWRL/CARIN

Description Graphs + Rules

Checking the satisfiability of $\mathcal{K} = (\emptyset, \mathcal{P}, \mathcal{G}, \mathcal{A})$ is undecidable for $\mathcal{P}$ a Horn program and $\mathcal{G} = (\mathcal{G}_G, \emptyset, \emptyset)$.

Description Graphs + DLs

Checking the satisfiability of $\mathcal{K} = (\mathcal{T}, \emptyset, \mathcal{G}, \mathcal{A})$ is undecidable for $\mathcal{T}$ in $\mathcal{ALCF}$ and $\mathcal{G} = (\mathcal{G}_G, \emptyset, \emptyset)$.

Undecidability: DLs + Graphs

Reduction from the DOMINO problem

Knowledge Base $\mathcal{K}$

$$\top \sqsubseteq \leq 1 H \qquad \top \sqsubseteq \leq 1 V$$

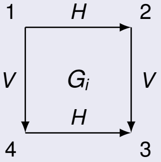
$$\lambda_1 = \left\{ \begin{array}{l} 1 \mapsto \{A_1\}, 2 \mapsto \{A_2\}, \\ 3 \mapsto \{A_3\}, 4 \mapsto \{A_4\} \end{array} \right\}$$

$$\lambda_2 = \left\{ \begin{array}{l} 1 \mapsto \{A_2\}, 2 \mapsto \{A_1\}, \\ 3 \mapsto \{A_4\}, 4 \mapsto \{A_3\} \end{array} \right\}$$

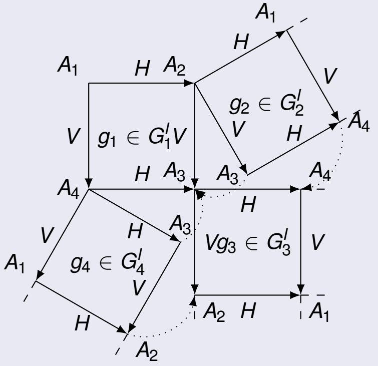
$$\lambda_3 = \left\{ \begin{array}{l} 1 \mapsto \{A_3\}, 2 \mapsto \{A_4\}, \\ 3 \mapsto \{A_1\}, 4 \mapsto \{A_2\} \end{array} \right\}$$

$$\lambda_4 = \left\{ \begin{array}{l} 1 \mapsto \{A_4\}, 2 \mapsto \{A_3\}, \\ 3 \mapsto \{A_2\}, 4 \mapsto \{A_1\} \end{array} \right\}$$

$$M_i = \{A_i\}$$



A Model of $\mathcal{K}$



AN ATTEMPT TO REGAIN DECIDABILITY

ACYCLICITY

Syntactic restriction on $\mathcal{G}$ that prevents cyclic implications of the existence of graph instances

- Limits the number of objects whose existence is implied by the graphs
- Objects in many domains are naturally bounded

DOES NOT DO THE TRICK

Checking the satisfiability of $\mathcal{K} = (\mathcal{T}, \emptyset, \mathcal{G}, \mathcal{A})$ is undecidable for $\mathcal{T}$ in $\mathcal{ALCIF}$ and $\mathcal{G} = (\mathcal{G}_G, \emptyset, \emptyset)$ an acyclic GBox.

Additional Restriction: Role Separation

Weak Separation

The roles occurring in $\mathcal{P}$ are disjoint with the roles occurring in $\mathcal{T}$

- Prevents the application of the rules in $\mathcal{P}$ to the part of the model constructed by $\mathcal{T}$
- "Neutralizes" the undecidability due to rules

Strong Separation

The roles occurring in $\mathcal{P}$ and $\mathcal{G}$ are disjoint with the roles occurring in $\mathcal{T}$

- Separates the parts of the model constructed by $\mathcal{T}$ and $\mathcal{G}$
- "Neutralizes" additionally the problems with inverse roles

Decidability Results

Weak Separation

Checking the satisfiability of a weakly separated acyclic KB $\mathcal{K} = (\mathcal{T}, \mathcal{P}, \mathcal{G}, \mathcal{A})$ is decidable for $\mathcal{T}$ in $\mathcal{SHOQ}^+$.

Strong Separation

Checking the satisfiability of a strongly separated acyclic KB $\mathcal{K} = (\mathcal{T}, \mathcal{P}, \mathcal{G}, \mathcal{A})$ is decidable for $\mathcal{T}$ in $\mathcal{SHOIQ}^+$.

Canonical Models and Reasoning

Decidability

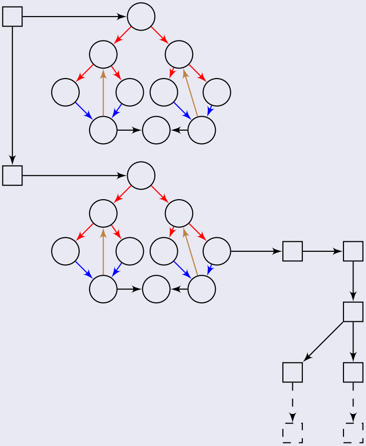
Canonical models composed of:

- A (possibly infinite) tree backbone
  ⇒ Generated by DL axioms
- Arbitrarily connected, yet finite and "isolated" collections of graph instances
  ⇒ Generated by description graphs

A tableau algorithm with the following termination argument:

- Roles in a graph and the backbone interact in a limited way or not at all (role separation)
- As in DLs, an algorithm can eventually "block" the expansion of the tree backbone
- Graphs clusters are bounded (acyclicity, key, and disjointness)

Canonical Model

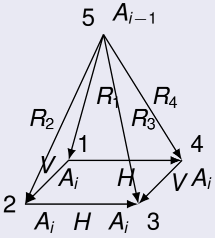


NExpTime-Hardness

Applies to $\mathcal{K} = (\emptyset, \mathcal{P}, \mathcal{G}, \mathcal{A})$ where $\mathcal{G} = (\mathcal{G}_G, \emptyset, \emptyset)$ is an acyclic GBox and each rule in $\mathcal{P}$ contains only atomic concepts and roles and at most four variables

- Reduction from the bounded DOMINO problem

GBox $\mathcal{G}$



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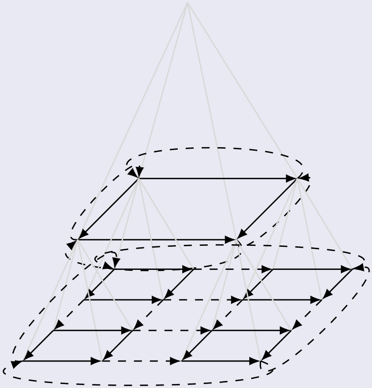

Complexity Results

Weak Separation

Checking the satisfiability of a weakly separated acyclic KB $\mathcal{K} = (\mathcal{T}, \mathcal{P}, \mathcal{G}, \mathcal{A})$ with $\mathcal{T}$ in $\mathcal{SHOQ}^{+}$ is NExpTime-hard.

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Future Work

- Complexity of the strongly separated acyclic case with $\mathcal{T}$ in $\mathcal{SHOIQ}^+$ is open
- Optimizations of the reasoning algorithm
- Tool support in an ontology editor
- New applications (e.g., molecule recognition)