# Course "Automated Reasoning" TEST 

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Consider the following Kripke Model $M$ :


For each of the following facts, say if it is true or false in CTL*.
(a) $M \models \mathbf{A}(\mathbf{G F} p \rightarrow \mathbf{G F} q)$
(b) $M \models \mathbf{A}(\mathbf{G F} p)$
(c) $M \models \mathbf{A}(\mathbf{F G} \neg p)$
(d) $M \models \mathbf{A}(\neg p \mathbf{U} q)$
[SCORING [0...100]:

- +25 pts for each correct answer
- -25pts for each incorrect answer
- 0pts for each unanswered question


## 2

Consider the following Kripke Model $M$ :


For each of the following facts, say if it is true or false in CTL.
(a) $M \models \mathbf{E G} p$
(b) $M \models \mathbf{A F} \neg p$
(c) $M \models \mathbf{A G A F} q$
(d) $M \models \mathbf{E}(\neg p \mathbf{U} q)$
[SCORING [0...100]:

- +25 pts for each correct answer
- -25 pts for each incorrect answer
- 0pts for each unanswered question


## 3

Let $p, q$ be Boolean atoms. For each of the following LTL formulas, say if there exists a CTL formula representing the same property.
(a) $\perp \mathbf{R}(\mathbf{F} q)$
(b) $\top \mathbf{U}(\mathbf{G} q)$
(c) $\mathbf{F G} p \rightarrow q$
(d) $\mathbf{G F} p \rightarrow q$
[SCORING [0...100]:

- +25 pts for each correct answer
- -25pts for each incorrect answer
- 0pts for each unanswered question
]

Consider CDCL SAT solving. For each of the following sentences, say if it is true or false.
(a) Let $\varphi$ be the CNF input Boolean formula, and $C$ denote a generic clause learned during the process. Then $\varphi \vDash C$.
(b) During the CDCL SAT solving process, the formula may contain an exponential number of learned clauses.
(c) Let $C$ be a conflict clause learned using the original backjumping\&learning strategy. Then $C$ contains at least one literal whose negation was unit-propagated in the current branch.
(d) Let $C$ be a conflict clause learned using the state-of-the-art backjumping\&learning strategy. Then $C$ contains at most one literal whose negation was unit-propagated in the current branch.
[SCORING [0...100]:

- +25 pts for each correct answer
- -25 pts for each incorrect answer
- 0pts for each unanswered question


## 5

For each of the following facts regarding theories of interest for SMT, say if it is true or false
(a) The theory of equality and uninterpreted function symbols $(\mathcal{E U F})$ is stably-infinite.
(b) The theory of fixed-width bit-vectors $(\mathcal{B V})$ is stably-infinite.
(c) The theory of linear arithmetic over the rationals $(\mathcal{L R} \mathcal{A})$ is convex.
(d) The theory of linear arithmetic over the integers $(\mathcal{L I A})$ is convex.
[SCORING [0...100]:

- +25 pts for each correct answer
- -25pts for each incorrect answer
- 0pts for each unanswered question


## 6

Consider the following CNF formula in PL:

$$
\begin{aligned}
& \left(\begin{array}{c}
\left.A_{5} \vee \quad A_{1} \vee \quad A_{3}\right) \wedge
\end{array}\right. \\
& \left(A_{2} \vee \neg A_{3} \vee \quad A_{6}\right) \wedge \\
& \left(A_{5} \vee \neg A_{1}\right) \wedge \\
& \left(A_{2} \vee \neg A_{3} \vee \neg A_{6}\right) \wedge \\
& \left(\neg A_{5}\right) \wedge \\
& \left(\neg A_{3} \vee \neg A_{2} \vee \quad A_{7}\right) \wedge \\
& \left(\neg A_{3} \vee \neg A_{2} \vee \neg A_{7}\right)
\end{aligned}
$$

(a) Draw the search tree obtained by applying to the above formula the tableaux algorithm.

Hint: when no better choice is available, clauses should be chosen in order, from top to down.
(b) as a consequence, say if the formula is satisfiable or not.
[SCORING: [0...100], 75pts for a correct answer a), 25pts for correct answer b), no penalties for wrong answers.]

## 7

Consider the following simple $\operatorname{SMT}(\mathcal{E U F} \cup \mathcal{L I} \mathcal{A})$ formula:

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\varphi \stackrel{\text { def }}{=}\left(x_{1}-x_{2} \geq 0\right) \wedge\left(x_{1}-x_{2} \leq 0\right) \wedge\left(f\left(x_{1}\right)<f\left(x_{2}\right)\right)
$$

(a) Purify the formula $\varphi$. Call $\varphi^{\prime}$ the resulting formula.
(b) List the interface variables and interface equalities of $\varphi^{\prime}$. (Order the variables as $x_{1}, x_{2}, x_{3}, x_{4}$.)
(c) Using Nelson-Oppen technique, decide if the formula $\varphi^{\prime}$ is $\mathcal{E U} \mathcal{F} \cup \mathcal{L I} \mathcal{A}$-satisfiable or not.
[SCORING: [0...100], 25pts each for (a) and (b); 50pts for (c). NO penalties for wrong answers..]

Consider the following two Kripke models $M 1$ and $M 2$, which share the variable x:


Compute and draw the graph of the synchronous product of M1 and M2.
Note: unreachable and deadend states should be removed.
[SCORING: [0...100], 100 pts for a correct answer, no penalties for wrong anwers.]

9


Consider the following Kripke model $M$ :
Convert it into an equivalent Buchi automaton.
[SCORING: [0...100], 100 pts for a correct answer, no penalties for wrong anwers.]

## 10

Consider the LTL formula $\varphi \stackrel{\text { def }}{=}(\neg p \mathbf{R} \neg q) \rightarrow \mathbf{G} r$
(a) rewrite $\varphi$ into Negative Normal Form
(b) find the initial states of a corresponding Generalized Büchi Automaton (for each state, define the labels of the incoming arcs and the "next" section.)
(c) How many distinct sets of accepting states will the final Generalized Büchi Automaton have?
[SCORING: [0..100], (a): +25pts for correct answer, (b) +50 points, (c) +25 points. No penalties for wrong answers..]


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