Course "Automated Reasoning"
TEST

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Name (please print):

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1

Let φ be a generic Boolean formula, and let $\varphi_1 \stackrel{\text{def}}{=} CNF(\varphi)$, s.c. $CNF()$ is the "classic" CNF conversion. Let $|\varphi|$ and $|\varphi_1|$ denote the size of φ and φ_1 respectively.

For each of the following sentences, say if it is true or false.

(a) If a DAG representation of formulas is used, then $|\varphi_1|$ is in worst-case polynomial in size wrt. $|\varphi|$.

(b) If φ contains no $\leftrightarrow$'s, then $|\varphi_1|$ is in worst-case polynomial in size wrt. $|\varphi|$.

(c) If φ is valid, then φ_1 is valid.

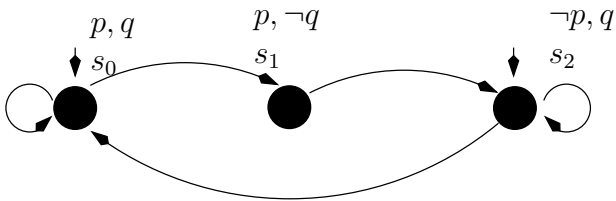(d) If φ_1 is valid, then φ is valid.

[SCORING [0...100]:

- +25pts for each correct answer

- -25pts for each incorrect answer

- 0pts for each unanswered question

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2

Consider the following Kripke Model M:

For each of the following facts, say if it is true or false in LTL.

(a) $M \models \mathbf{F}p$

(b) $M \models \mathbf{G}\neg p$

(c) $M \models \mathbf{G}\mathbf{F}\neg p$

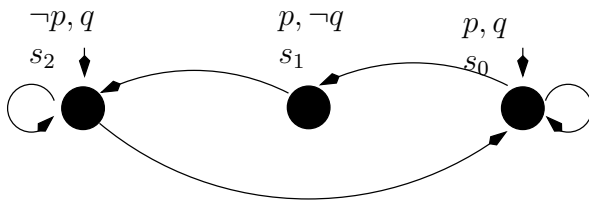(d) $M \models \mathbf{G}(p \vee q)$

[SCORING [0...100]:

- +25pts for each correct answer
- -25pts for each incorrect answer
- 0pts for each unanswered question

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3

Consider the following Kripke Model M:

For each of the following facts, say if it is true or false in CTL.

(a) $M \models \mathbf{EG}q$

(b) $M \models \mathbf{AF}p$

(c) $M \models \mathbf{AF}\neg q$
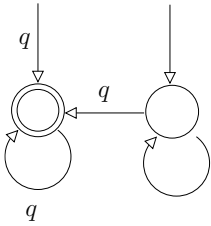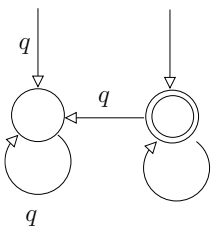
(d) $M \models (\mathbf{AGAF}\neg q)$

4

For each of the following fact regarding Buchi automata, say if it true or false.
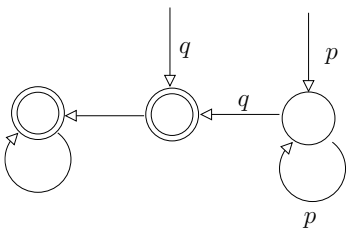
(*a*) The following BA represents **FG**q:

(*b*) The following BA represents **FG**q:

(*c*) The following BA represents pUq:
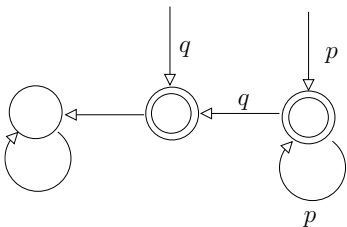
(*d*) The following BA represents pUq:

5

Consider the following two $\mathcal{DL}$ formulas:
$\varphi_1 \stackrel{\text{def}}{=} (x_2 - x_1 \le -6) \wedge (x_3 - x_2 \le 5) \wedge (x_5 - x_4 \le -4) \wedge (x_6 - x_5 \le -7) \wedge (x_8 - x_7 \le 4)$
$\varphi_2 \stackrel{\text{def}}{=} (x_4 - x_3 \le 3) \wedge (x_7 - x_6 \le -1) \wedge (x_1 - x_8 \le 5)$

For each of the following facts, say if it is true or false

(a) The following is a $\mathcal{DL}$ interpolant of $\langle \varphi_1, \varphi_2 \rangle$
$(x_3 - x_1 \le -1) \wedge (x_6 - x_4 \le -11)$

(b) The following is a $\mathcal{LRA}$ interpolant of $\langle \varphi_1, \varphi_2 \rangle$:
$(x_3 - x_1 + x_6 - x_4 + x_8 - x_7 \le -8)$

(c) The following is a $\mathcal{DL}$ interpolant of $\langle \varphi_1, \varphi_2 \rangle$:
$(x_3 - x_1 \le -1) \wedge (x_6 - x_4 \le -11) \wedge (x_8 - x_7 \le 4)$

(d) The following is a $\mathcal{DL}$ interpolant of $\langle \varphi_1, \varphi_2 \rangle$
$(x_2 - x_1 \le -6) \wedge (x_3 - x_2 \le 5) \wedge (x_5 - x_4 \le -4) \wedge (x_6 - x_5 \le -7) \wedge$
$(x_4 - x_3 \le 3) \wedge (x_7 - x_6 \le -1) \wedge (x_1 - x_8 \le 5) \wedge (x_8 - x_7 \le 4)$

[SCORING [0...100]:

- +25pts for each correct answer

- -25pts for each incorrect answer

- 0pts for each unanswered question

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6

Consider the following Boolean formula φ:

$$\neg(((\ A_9 \to \ A_8) \ \wedge \ (\neg A_7 \to \neg A_4)) \ \vee \ ((\neg A_5 \to \neg A_6) \ \wedge \ (\neg A_7 \to \ A_8)))$$

1. Compute the Negative Normal Form of φ, called φ'.

2. For each of the following sentences, only one is true. Say which one.

   (a) φ and φ' are equivalent.

   (b) φ and φ' are not necessarily equivalent. φ' has a model if and only φ has a model.

   (c) There is no relation between the satisfiablity of φ and that of φ'.

[SCORING: [0...100], 75pts for correct answer 1, 25pts for correct answer 2. No penalties for wrong answers..]

7

Let

$$\varphi \stackrel{\text{def}}{=} (\quad A_2 \leftrightarrow \begin{pmatrix} (\quad A_3 \vee \quad A_6 \vee \quad A_8) \quad \wedge \\ (\quad A_5 \vee \quad A_7 \vee \quad A_8) \quad \wedge \\ (\neg A_4 \vee \neg A_6 \vee \neg A_8) \quad \wedge \\ (\neg A_6 \vee \quad A_7 \vee \neg A_8) \quad \wedge \\ (\neg A_3 \vee \quad A_6 \vee \quad A_9) \quad \wedge \\ (\neg A_6 \vee \neg A_8 \vee \neg A_9) \quad \wedge \\ (\quad A_3 \vee \quad A_4 \vee \neg A_5) \quad \wedge \\ (\quad A_5 \vee \quad A_8 \vee \neg A_9) \quad \wedge \\ (\neg A_3 \vee \neg A_8 \vee \neg A_4) \quad \wedge \\ (\quad A_6 \vee \quad A_4 \vee \neg A_7) \quad \wedge \\ (\quad A_5 \vee \quad A_8 \vee \neg A_1) \quad \wedge \\ (\neg A_4 \vee \neg A_7 \vee \neg A_9) \end{pmatrix}).$$

Using the variable ordering:

$$" \quad A_1, \quad A_3, \quad A_4, \quad A_5, \quad A_6, \quad A_7, \quad A_8, \quad A_9",$$
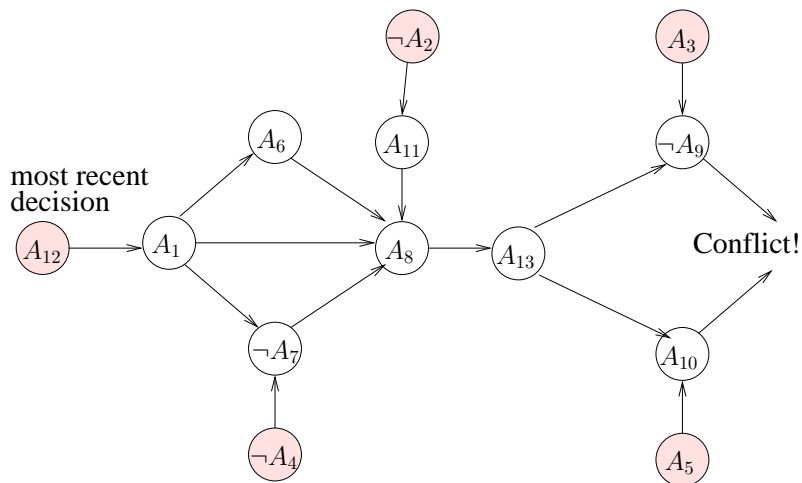
draw the OBDD corresponding to the formula φ' defined as:

$$\varphi' \stackrel{\text{def}}{=} \exists \quad A_2.\varphi.$$

[SCORING: [0...100], 100 pts for a correct answer. No penalties for a wrong answer..]

8

Consider the following implication graph:

A_{12} being the most recent decision literal. Write the conflict clauses generated by

(a) the decision conflict analysis criterion
(b) the last UIP conflict analysis criterion
(c) the 1st UIP conflict analysis criterion

[SCORING: [0...100], 33 points each for each correct answer to questions (a), (b) and (c). No penalties for uncorrect answers..]

9

Consider the following pair of SMT($\mathcal{LRA}$) sets of literals:

$$A \stackrel{\text{def}}{=} \{(0 \leq -3x_1 - 5x_2 + 1), (0 \leq x_1 + x_2)\}$$
$$B \stackrel{\text{def}}{=} \{(0 \leq 3x_3 - 2x_1 - 3), (0 \leq x_1 - 2x_3 + 1)\}.$$

(a) Write a proof P of $\mathcal{LRA}$-unsatisfiablity of $A \wedge B$

(b) From such a proof, compute a $\mathcal{LRA}$-interpolant for $\langle A, B \rangle$ using McMillan's technique.

[SCORING: [0...100], 50 points each for questions a) and b). No penalties for wrong answers..]

10

Consider the LTL formula $\varphi \stackrel{\text{def}}{=} p \vee q$, where p, q are atomic propositions. (Notice: <u>LTL</u> formula!) Compute the corresponding Generalized Büchi Automaton.

[SCORING: [0...100], 100 pts for a correct answer..]