UNIVERSITÀ DI TRENTO

Formal Method Mod. 1 (Automated Reasoning)
Laboratory 6

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Outline

Multi-objective optimization

Exercise 6.1: multiopt

Assuming $x_1, x_2 \in [-10, 10]$ and x_1, x_2 are both integer numbers, use OptiMathSAT to:

2. $\begin{cases} \min (2\,x_1 - x_2, -x_1) \\ x_2 - 2 \leq 0 \\ x_1^2 - x_2 \leq 0 \end{cases}$

- ▶ Check how many Pareto-optimal solutions exists.
- ▶ Is the lexigographic-optimal solution covered by the Pareto optima?

In this case, the encoding is trivial: we know the types of variables, the constraints (aka assertions) and the cost functions.

Multi-objective optimization: optimizations

- ▶ If we expect the solver to return multiple optima (i.e. in the case of Pareto), we can call check-sat and get-objectives multiple times.

- ▶ Once no more solutions are found, the solver will return UNSAT.

- ▶ You can change the type of optimization from the option **:opt.priority**.

Not getting high penalties

Exercise 6.2: penalty

Suppose you are Rose. You choose one character between A and D. After your turn also Collin choose one character. Your goal is to get the lowest penalty, according to the joint choice between Collin and yourself. Assume also that Collin is your worst enemy, and since it is the second it will make his choice to maximize your penalty. Use OptiMathSAT to get the minimum penalty.

Rose\Colin	A	B	C	D
A	4	3	2	5
B	-10	2	0	-1
C	7	5	2	3
D	0	8	-4	-5

▶ Rose searches for the minimum, while Collins search for the maximum among the possible choices... What is this minmax/maxmin in OptiMathSAT?

Not getting high penalties: variables

As always, we first define the variables that efficiently describe the problem:

▶ For each cell in the grid we require a Int constant.
▶ We also add additional variables ri to store, for each choice made by Rose, the possible penalties she can get.

Not getting high penalties: properties

- In each variable ri we store the possible output is Rose chooses character i. We can simulate it using or operators.

- We can minmax the problem: for each row, we minimize the penalty (we want to avoid any risk), and from these values OptiMathSAT will retrieve the maximum (Collin's choice).

Storing files

Exercise 6.2: servers

You are working for UniTN and you are asked to pack sever virtual machines (VMs) into servers. The complete list of available servers (describing the number of RAM and the storage in GBs they provide) is shown below:

```
       RAM  storage
srv0    2       100
srv1    4       800
srv2    4      1000
srv3   16      8000
srv4    8      3000
srv5   16      6000
srv6   16      4000
srv7   32      2000
srv8    8      1000
srv9   16     10000
srv10   8      1000
```

Storing files

Exercise 6.3: servers

On the other hand, this is the list of the virtual machines you are requested to store:

```
      RAM storage
 VM0   1      100
 VM1  16      900
 VM2   4      710
 VM3   2      800
 VM4   4     7000
 VM5   8     4000
 VM6   2      800
 VM7   4     2500
 VM8  16      450
 VM9  16     3700
VM10  12     1300
```

Storing files

Exercise 6.3: servers

On the other hand, this is the list of the virtual machines you are requested to store:

```
       RAM storage
 VM0    1      100
 VM1   16      900
 VM2    4      710
 VM3    2      800
 VM4    4     7000
 VM5    8     4000
 VM6    2      800
 VM7    4     2500
 VM8   16      450
 VM9   16     3700
VM10   12     1300
```

Storing files

Exercise 6.3: servers

Remember that it is **NOT** possible to partially store part of a VM on a server and the remaining part to a different one. The rector of the university is invested in the project and would be pleased to achieve a better efficiency according to the following rule: **We must be sure to retrieve a configuration that minimizes the number of employed servers. The fewer servers you will use, the higher will be your salary**.

Exercise 6.3: servers

Use OptiMathSAT to evaluate the best configuration that could maximize your gain. First try to see if there is at least a way to store the VMs on the servers, independently from the additional rule provided by the rector, and report the result in a comment:

- ▶ If no solution is found, write how you assumed it.
- ▶ If at least a solution is found, write how you assumed it. Next add some constraints to obtain the configuration that will maximize your salary. Be sure to check if the obtained solution is the only one using that minimal number of servers and provide comments about it.

Storing files: hints

▶ If you add multiple assertions to a solver and want to retrieve all of them, the PySMT class Solver contains the attribute **solver.assertions** that generates the list of all assertions (thanks Andrea!)

▶ Can you create additional variables to check how many servers have been used?

▶ If you choose the correct type for variables, the exercise will be drastically easier.

Storing files: variables

As always, we first define the variables that efficiently describe the problem:

- ▶ We create Pseudo-Boolean (PB) variables **srv$i$_vm$j$** such as that we map, for each pair server/vm, if vm j is stored in server i.

- ▶ For the optimization task, we also generate other PB variables, **use_server$i$**, to see if a server i has been used.

Storing files: properties

▶ Set all ranges for the PB variables such that they are actually PB!

▶ A virtual machine can be placed only on one server

▶ Create linear relation that sum, for each server, the amount of RAM and storage used according to the virtual machines associated to it.

▶ If at least one virtual machine has been assigned to a server i, then **use_server**i becomes 1, meaning it has been used and must be counted.

▶ Sum all the **use_server**i PB variables into the cost function that we will minimize.

Outline

Solve the fac-simile of the exam. Remember, 2 hours, you can access only the main documentation shown during classes.