

Formal Methods

Module II: Formal Verification

Ch. 07: **SAT-Based Model Checking**

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- 1 SAT-based Model Checking: Generalities
- 2 Bounded Model Checking
 - Intuitions
 - General Encoding
 - Relevant Subcases
 - An Example
 - Computing Upper Bounds
 - Discussion
- 3 Inductive reasoning on invariants (aka “K-Induction”)
 - K-Induction
 - An Example
- 4 Exercises

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SAT-based Model Checking

- Key problems with BDD's:
 - they can explode in space
- A possible alternative:
 - Propositional Satisfiability Checking (SAT)
 - SAT technology is very advanced
- Advantages:
 - reduced memory requirements
 - limited sensitivity: one good setting, does not require expert users
 - much higher capacity (more variables) than BDD based techniques
- Various techniques:
 - Bounded Model Checking (BMC)
 - K-induction
 - Interpolant-based
 - IC3/PDR
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SAT-based Bounded Model Checking & K-Induction

Key Ideas:

- **BMC**: look for counter-example paths of increasing length k
⇒ oriented to finding bugs
- **K-Induction**: look for an induction proofs of increasing length k
⇒ oriented to prove correctness
- BMC [resp. K-induction]: for each k , build a Boolean formula that is satisfiable [resp. unsatisfiable] iff there is a counter-example [resp. proof] of length k
 - can be expressed using $k \cdot |s|$ variables
 - formula construction is not subject to state explosion
- Satisfiability of the Boolean formulas is checked by a **SAT solver**
 - can manage complex formulae on several 100K variables
 - returns satisfying assignment (i.e., a counter-example)
 - exploit incrementality

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SAT-based Bounded Model Checking & K-Induction

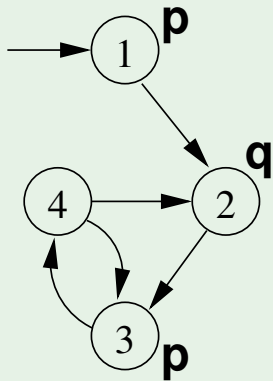
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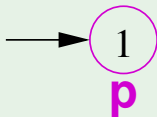
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Bounded Model Checking: Example

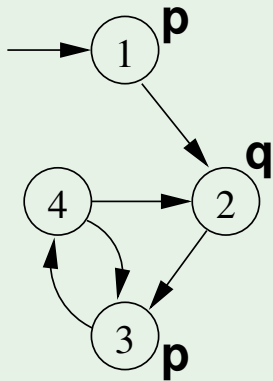


- LTL Formula: $\mathbf{G}(p \rightarrow \mathbf{F}q)$
- Negated Formula (violation): $\mathbf{F}(p \wedge \mathbf{G}\neg q)$
- $k = 0$:

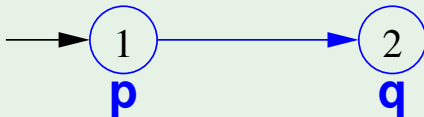


- No counter-example found.

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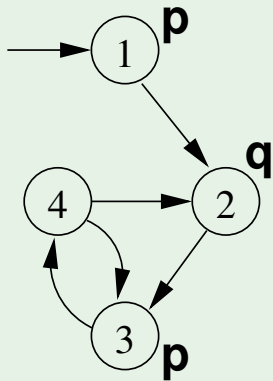


- LTL Formula: $\mathbf{G}(p \rightarrow \mathbf{F}q)$
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- $k = 1$:

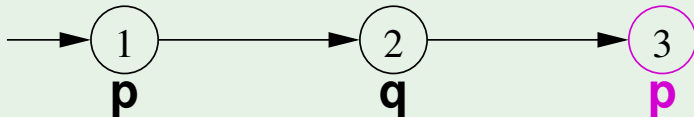


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Bounded Model Checking: Example

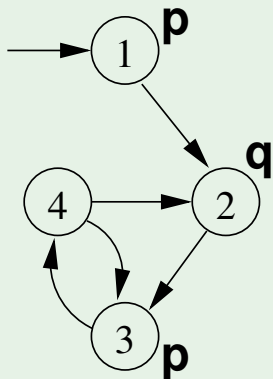


- LTL Formula: $\mathbf{G}(p \rightarrow \mathbf{F}q)$
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- $k = 2$:

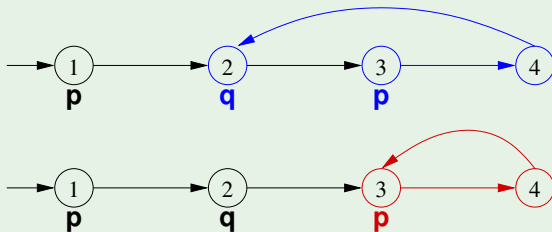


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Bounded Model Checking: Example



- LTL Formula: $\mathbf{G}(p \rightarrow \mathbf{F}q)$
- Negated Formula (violation): $\mathbf{F}(p \wedge \mathbf{G}\neg q)$
- $k = 3$:



- The 2nd trace is a counter-example!

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The problem [Biere et al, 1999]

Ingredients:

Assume states represented by an array s of n Boolean variables

- a **system** written as a Kripke structure $M := \langle I(s), R(s, s') \rangle$
- a **property** f written as a **LTL formula**
- an integer $k \geq 0$ (**bound**)

Problem

Is there an execution path π of M of length k satisfying the temporal property f ?

$$M \models_k \mathbf{E}f$$

Note: f is the negation of the property in the LTL model checking problem $M \models \neg f$, and π is a counter-example of length k (bug).

- The check is repeated for increasing values of $k = 0, 1, 2, 3, \dots$

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The encoding

Equivalent to the satisfiability problem of a Boolean formula $[[M, f]]_k$ defined as follows:

$$[[M, f]]_k := [[M]]_k \wedge [[f]]_k$$

$$[[M]]_k := I(s^0) \wedge \bigwedge_{i=0}^{k-1} R(s^i, s^{i+1}),$$

$$[[f]]_k := (\neg \bigvee_{l=0}^k R(s^k, s^l) \wedge [[f]]_k^0) \vee \bigvee_{l=0}^k (R(s^k, s^l) \wedge I[[f]]_k^0),$$

- The vector s of propositional variables is replicated $k+1$ times
 s^0, s^1, \dots, s^k
- $[[M]]_k$ encodes the fact that the k -path is an execution of M
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The Encoding [cont.]

The encoding for a formula f with k steps, $[[f]]_k$ is the disjunction of:

- The constraints needed to express a model without loopback:

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- $[[f]]_k^i, i \in [0, k]$:
“ f holds in s^i under the assumption that s^0, \dots, s^k is a no-loopback path”
- The constraints needed to express a model with some loopback:

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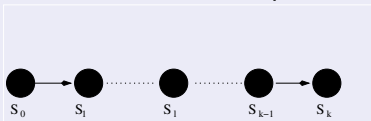
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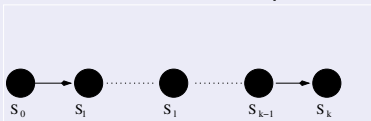
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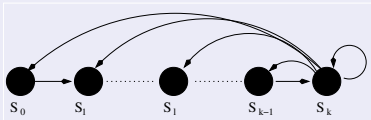


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The Encoding of $[[f]]_k^i$ and ${}_i[[f]]_k^i$

f	$[[f]]_k^i$	${}_i[[f]]_k^i$
p	p_i	p_i
$\neg p$	$\neg p_i$	$\neg p_i$
$h \wedge g$	$[[h]]_k^i \wedge [[g]]_k^i$	${}_i[[h]]_k^i \wedge {}_i[[g]]_k^i$
$h \vee g$	$[[h]]_k^i \vee [[g]]_k^i$	${}_i[[h]]_k^i \vee {}_i[[g]]_k^i$
Xg	$[[g]]_k^{i+1}$ if $i < k$ \perp otherwise.	${}_i[[g]]_k^{i+1}$ if $i < k$ ${}_i[[g]]_k^i$ otherwise.
Gg	\perp	$\bigwedge_{j=\min(i,l)}^k {}_i[[g]]_k^j$
Fg	$\bigvee_{j=i}^k [[g]]_k^j$	$\bigvee_{j=\min(i,l)}^k {}_i[[g]]_k^j$
hUg	$\bigvee_{j=i}^k \left([[g]]_k^j \wedge \bigwedge_{n=i}^{j-1} [[h]]_k^n \right)$	$\bigvee_{j=i}^k \left({}_i[[g]]_k^j \wedge \bigwedge_{n=i}^{j-1} {}_i[[h]]_k^n \right) \vee$ $\bigvee_{j=l}^{i-1} \left({}_i[[g]]_k^j \wedge \bigwedge_{n=i}^k {}_i[[h]]_k^n \wedge \bigwedge_{n=l}^{j-1} {}_i[[h]]_k^n \right)$
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Relevant Subcase: $\mathbf{F}p$ (reachability)

- $f := \mathbf{F}p$, s.t. p Boolean:
is there a reachable state in which p holds?
- a finite path can show that the property holds
- $[[M, f]]_k$ is:

$$I(s^0) \wedge \bigwedge_{i=0}^{k-1} R(s^i, s^{i+1}) \wedge \bigvee_{j=0}^k p^j$$



Important: incremental encoding

if done for increasing value of k , then it suffices that $[[M, f]]_k$ is:

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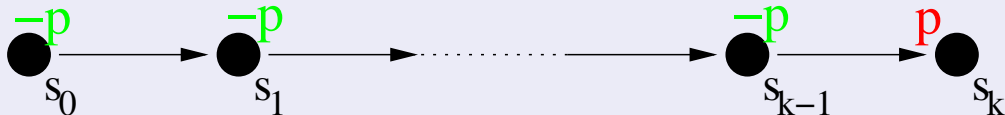
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- We need to produce an infinite behaviour, with a finite number of transitions
- We can do it by imposing that the path loops back

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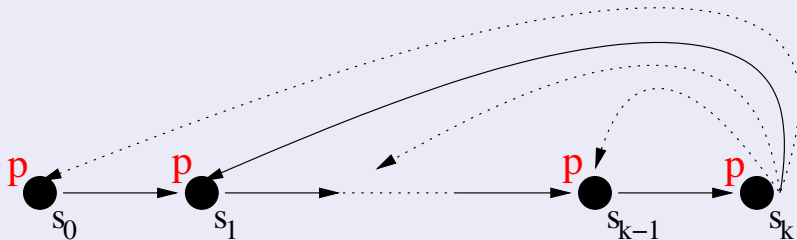
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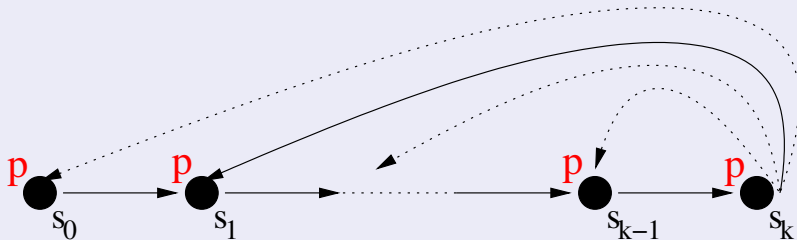


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$$I(s^0) \wedge \bigwedge_{i=0}^{k-1} R(s^i, s^{i+1}) \wedge \bigvee_{l=0}^k R(s^k, s^l) \wedge \bigwedge_{j=0}^k p^j$$

Relevant Subcase: Gp

- $f := Gp$, s.t. p Boolean: is there a path where p holds forever?
- We need to produce an infinite behaviour, with a finite number of transitions
- We can do it by imposing that the path loops back



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Relevant Subcase: $\mathbf{GF}q$ (fair states)

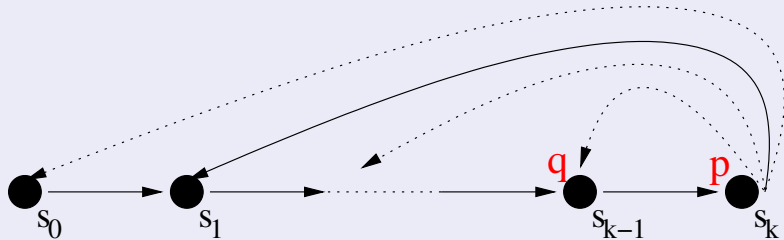
- $f := \mathbf{GF}q$, s.t. q Boolean: **does q hold infinitely often?**
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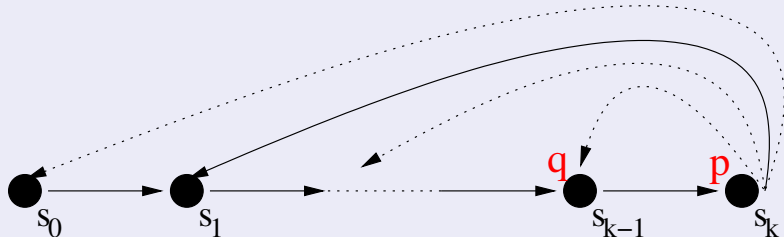


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Subcase Combination: $\mathbf{GF}q \wedge \mathbf{F}p$ (fair reachability)

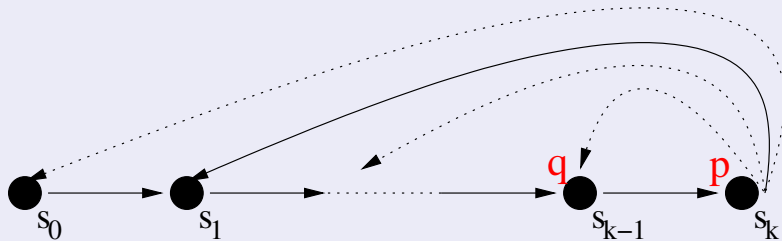
- $f := \mathbf{GF}q \wedge \mathbf{F}p$, s.t. p, q Boolean: provided that q holds infinitely often, is there a reachable state in which p holds?
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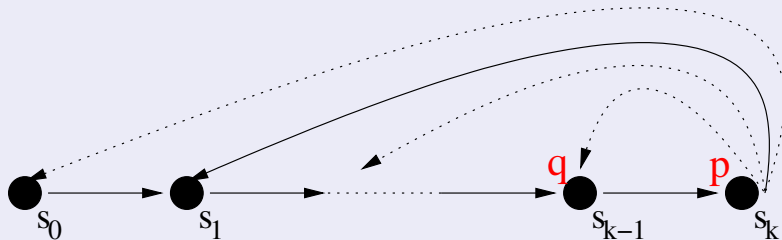


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Example: a bugged 3-bit shift register

- System M :

- $I(x) := \neg x[0] \wedge \neg x[1] \wedge x[2]$

- Correct R : $R(x, x') := (x'[0] \leftrightarrow x[1]) \wedge (x'[1] \leftrightarrow x[2]) \wedge (x'[2] \leftrightarrow 0)$

- Bugged R : $R(x, x') := (x'[0] \leftrightarrow x[1]) \wedge (x'[1] \leftrightarrow x[2]) \wedge (x'[2] \leftrightarrow 1)$

- Property: $\mathbf{F}(\neg x[0] \wedge \neg x[1] \wedge \neg x[2])$

- BMC Problem: is there an execution π of \mathcal{M} of length k s.t. $\pi \models \mathbf{G}((x[0] \vee x[1] \vee x[2]))?$

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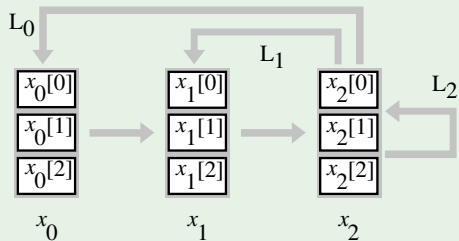
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Example: a bugged 3-bit shift register [cont.]

$k = 0$:



$$I: \quad (\neg x_0[0] \wedge \neg x_0[1] \wedge x_0[2]) \wedge$$
$$\bigvee_{l=0}^0 L_l: \quad (((x_0[0] \leftrightarrow x_0[1]) \wedge (x_0[1] \leftrightarrow x_0[2]) \wedge (x_0[2] \leftrightarrow 1))) \wedge$$
$$\bigwedge_{i=0}^0 (x \neq 0): \quad ((x_0[0] \vee x_0[1] \vee x_0[2]))$$

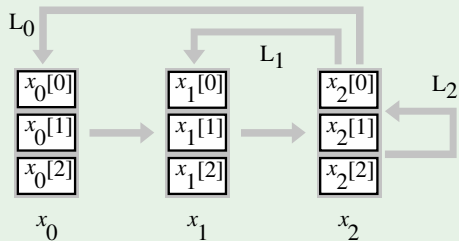
\Rightarrow UNSAT: unit propagation:

$\neg x_0[0], \neg x_0[1], x_0[2]$

\Rightarrow loop violated

Example: a bugged 3-bit shift register [cont.]

$k = 0$:



$$\begin{aligned} I : & \quad (\neg x_0[0] \wedge \neg x_0[1] \wedge x_0[2]) \wedge \\ \bigvee_{l=0}^0 L_l : & \quad (((x_0[0] \leftrightarrow x_0[1]) \wedge (x_0[1] \leftrightarrow x_0[2]) \wedge (x_0[2] \leftrightarrow 1))) \wedge \\ \bigwedge_{i=0}^0 (x \neq 0) : & \quad ((x_0[0] \vee x_0[1] \vee x_0[2])) \end{aligned}$$

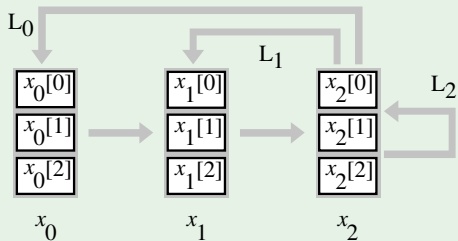
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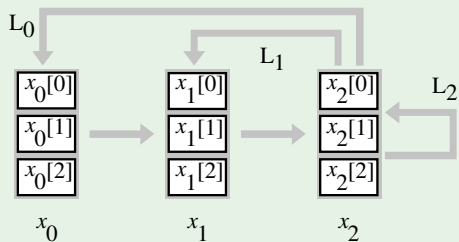
\Rightarrow UNSAT: unit propagation:

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\Rightarrow loop violated

Example: a bugged 3-bit shift register [cont.]

$k = 1$:



$$\begin{aligned}
 I : & \quad (\neg x_0[0] \wedge \neg x_0[1] \wedge x_0[2]) \wedge \\
 [[M]]_1 : & \quad \left((x_1[0] \leftrightarrow x_0[1]) \wedge (x_1[1] \leftrightarrow x_0[2]) \wedge (x_1[2] \leftrightarrow 1) \right) \wedge \\
 \bigvee_{l=0}^1 L_l : & \quad \left(\left((x_0[0] \leftrightarrow x_1[1]) \wedge (x_0[1] \leftrightarrow x_1[2]) \wedge (x_0[2] \leftrightarrow 1) \right) \vee \right. \\
 & \quad \left. \left((x_1[0] \leftrightarrow x_1[1]) \wedge (x_1[1] \leftrightarrow x_1[2]) \wedge (x_1[2] \leftrightarrow 1) \right) \right) \wedge \\
 \bigwedge_{i=0}^1 (x \neq 0) : & \quad \left(\begin{array}{l} (x_0[0] \vee x_0[1] \vee x_0[2]) \wedge \\ (x_1[0] \vee x_1[1] \vee x_1[2]) \end{array} \right)
 \end{aligned}$$

\Rightarrow UNSAT: unit propagation:

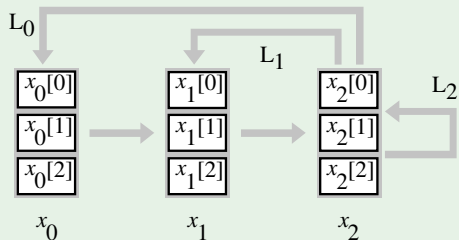
$\neg x_0[0], \neg x_0[1], x_0[2]$

$\neg x_1[0], x_1[1], x_1[2]$

\Rightarrow both loop disjuncts violated

Example: a bugged 3-bit shift register [cont.]

$k = 1$:



$$\begin{aligned}
 I : & \quad (\neg x_0[0] \wedge \neg x_0[1] \wedge x_0[2]) \wedge \\
 [[M]]_1 : & \quad \left((x_1[0] \leftrightarrow x_0[1]) \wedge (x_1[1] \leftrightarrow x_0[2]) \wedge (x_1[2] \leftrightarrow 1) \right) \wedge \\
 \bigvee_{l=0}^1 L_l : & \quad \left(\left((x_0[0] \leftrightarrow x_1[1]) \wedge (x_0[1] \leftrightarrow x_1[2]) \wedge (x_0[2] \leftrightarrow 1) \right) \vee \right. \\
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 \bigwedge_{i=0}^1 (x \neq 0) : & \quad \left(\begin{array}{l} (x_0[0] \vee x_0[1] \vee x_0[2]) \wedge \\ (x_1[0] \vee x_1[1] \vee x_1[2]) \end{array} \right)
 \end{aligned}$$

\Rightarrow UNSAT: unit propagation:

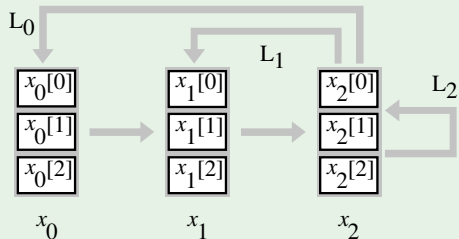
$\neg x_0[0], \neg x_0[1], x_0[2]$

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Example: a bugged 3-bit shift register [cont.]

$k = 1$:



$$\begin{aligned}
 I: & \quad (\neg x_0[0] \wedge \neg x_0[1] \wedge x_0[2]) \wedge \\
 [[M]]_1: & \quad \left((x_1[0] \leftrightarrow x_0[1]) \wedge (x_1[1] \leftrightarrow x_0[2]) \wedge (x_1[2] \leftrightarrow 1) \right) \wedge \\
 \bigvee_{l=0}^1 L_l: & \quad \left(\begin{aligned} & ((x_0[0] \leftrightarrow x_1[1]) \wedge (x_0[1] \leftrightarrow x_1[2]) \wedge (x_0[2] \leftrightarrow 1)) \vee \\ & ((x_1[0] \leftrightarrow x_1[1]) \wedge (x_1[1] \leftrightarrow x_1[2]) \wedge (x_1[2] \leftrightarrow 1)) \end{aligned} \right) \wedge \\
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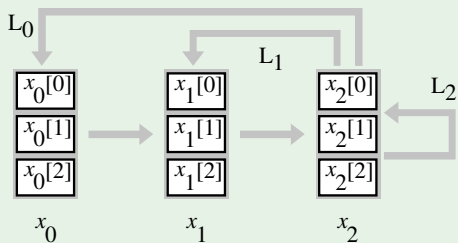
$\neg x_0[0], \neg x_0[1], x_0[2]$

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Example: a bugged 3-bit shift register [cont.]

$k = 2$:

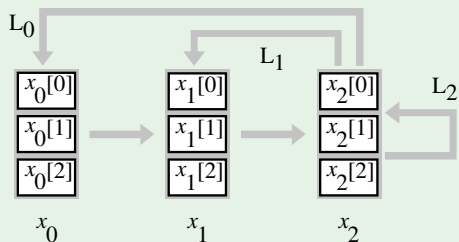


$$\begin{aligned}
 I: & (\neg x_0[0] \wedge \neg x_0[1] \wedge x_0[2]) \wedge \\
 [M]_2: & \left(\begin{array}{l} (x_1[0] \leftrightarrow x_0[1]) \wedge (x_1[1] \leftrightarrow x_0[2]) \wedge (x_1[2] \leftrightarrow 1) \wedge \\ (x_2[0] \leftrightarrow x_1[1]) \wedge (x_2[1] \leftrightarrow x_1[2]) \wedge (x_2[2] \leftrightarrow 1) \end{array} \right) \wedge \\
 \bigvee_{i=0}^2 L_i: & \left(\begin{array}{l} ((x_0[0] \leftrightarrow x_1[1]) \wedge (x_0[1] \leftrightarrow x_2[2]) \wedge (x_0[2] \leftrightarrow 1)) \vee \\ ((x_1[0] \leftrightarrow x_2[1]) \wedge (x_1[1] \leftrightarrow x_2[2]) \wedge (x_1[2] \leftrightarrow 1)) \vee \\ ((x_2[0] \leftrightarrow x_2[1]) \wedge (x_2[1] \leftrightarrow x_2[2]) \wedge (x_2[2] \leftrightarrow 1)) \end{array} \right) \wedge \\
 \bigwedge_{i=0}^2 (x \neq 0): & \left(\begin{array}{l} (x_0[0] \vee x_0[1] \vee x_0[2]) \wedge \\ (x_1[0] \vee x_1[1] \vee x_1[2]) \wedge \\ (x_2[0] \vee x_2[1] \vee x_2[2]) \end{array} \right)
 \end{aligned}$$

\implies SAT: $x_0[0] = x_0[1] = x_1[0] = 0$; $x_i[j] := 1 \forall i, j$

Example: a bugged 3-bit shift register [cont.]

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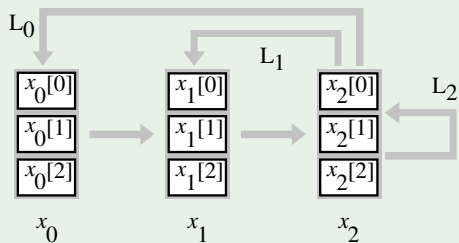


$$\begin{aligned}
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Basic bounds for k

Theorem [Biere et al. TACAS 1999]

Let f be a LTL formula.

Then $M \models \mathbf{E}f \iff M \models_k \mathbf{E}f$ for some $k \leq |M| \cdot 2^{|f|}$.

- $|M| \cdot 2^{|f|}$ is always a bound of k .
 - $|M|$ huge!
 \implies not so easy to compute in a symbolic setting.
- \implies need to find better bounds!

Note: [Biere et al. TACAS 1999] use " $M \models \mathbf{E}f$ " as "there exists a path of M verifying f ", so that $M \not\models \neg f \iff M \models \mathbf{E}f$

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Other bounds for k

ACTL & ECTL

- **ACTL** is a subset of CTL in which “**A...**” (resp. “**E...**”) sub-formulas occur only positively (resp. negatively) in each formula. (e.g. **AG**($p \rightarrow$ **AGAF** q))
- Many frequently-used LTL properties $\neg f$ have equivalent ACTL representations **A** $\neg f'$
 - e.g. **X** $q \iff$ **AX** q , **G** $q \iff$ **AG** q , **F** $q \iff$ **AF** q , **pUq** \iff **A**(**pUq**),
GF $q \iff$ **AGAF** q , **G**($p \rightarrow$ **GF** q) \iff **AG**($p \rightarrow$ **AGAF** q)
 - ... but not all of them (e.g., **FG** $\not\iff$ **AFAG** p)
- **ECTL** is a subset of CTL in which “**E...**” (resp. “**A...**”) sub-formulas occur only positively (resp. negatively) in each formula. (e.g. **EF**($p \wedge$ **EFEG** $\neg q$))
- ECTL is the dual subset of ACTL: $\phi \in$ **ECTL** \iff $\neg\phi \in$ **ACTL**.

Theorem [Biere et al. TACAS 1999]

Let f be an ECTL formula.

Then $M \models$ **E** $f \iff M \models_k$ **E** f for some $k \leq |M|$.

Other bounds for k

ACTL & ECTL

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Other bounds for k (cont)

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The diameter

Definition: Diameter

Given M , the **diameter** of M is the smallest integer d s.t. for every path s_0, \dots, s_{d+1} there exist a path t_0, \dots, t_l s.t. $l \leq d$, $t_0 = s_0$ and $t_l = s_{d+1}$.

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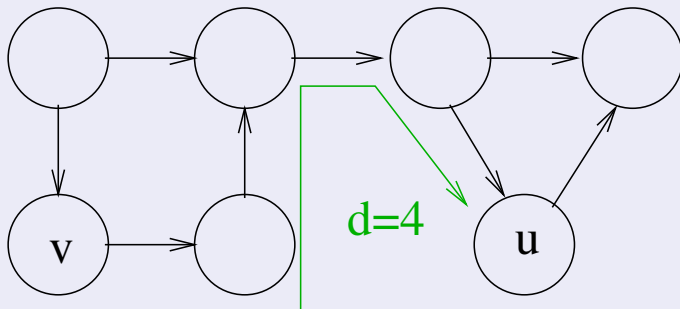
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Definition: recurrence diameter

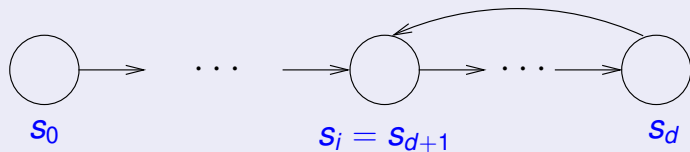
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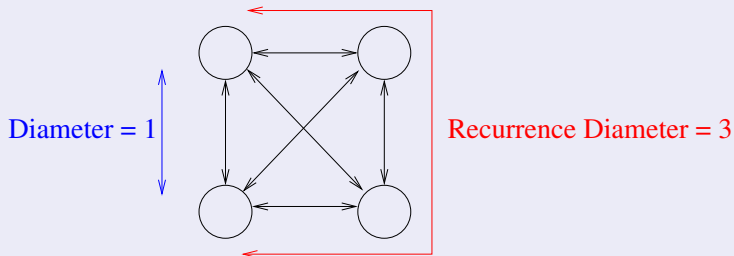
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 - Relevant Subcases
 - An Example
 - Computing Upper Bounds
 - **Discussion**
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Efficiency Issues in Bounded Model Checking

- **Incrementality:**
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Other Successful SAT-based MC Techniques

- Inductive reasoning on invariants (aka “K-Induction”)
- Counter-example guided abstraction refinement (CEGAR)
[Clarke et al. CAV 2002]
- Interpolant-based MC
[Mc Millan, TACAS 2005]
- IC3/PDR
[Bradley, VMCAI 2011]
- ...

For a survey see e.g.
[Amla et al., CHARME 2005, Prasad et al. STTT 2005].

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Inductive Reasoning on Invariants

Invariant: “**G***Good*”, *Good* being a Boolean formula

- (i) If all the initial states are good,
 - (ii) and if from good states we only go to good states
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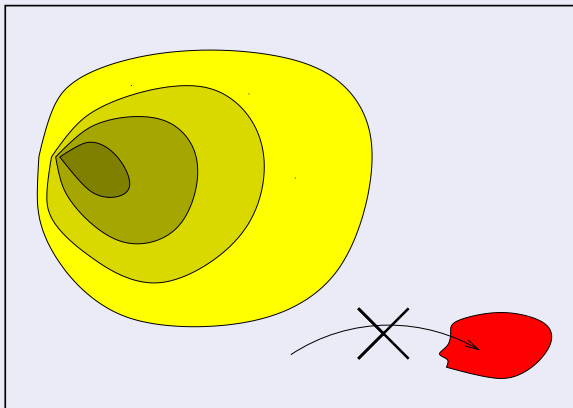
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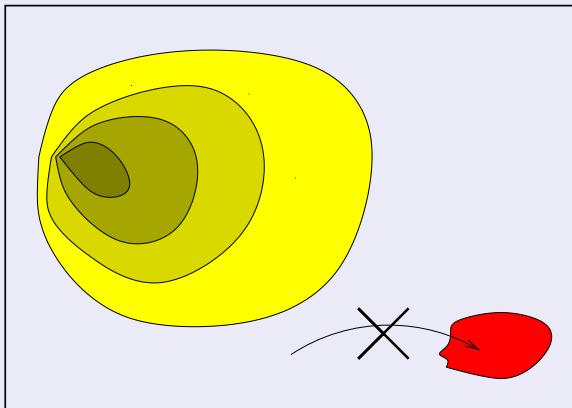
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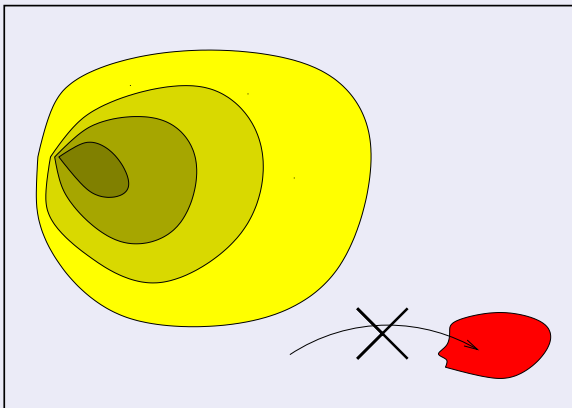
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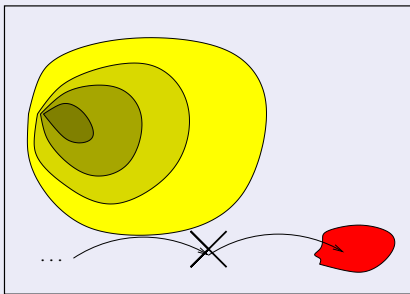


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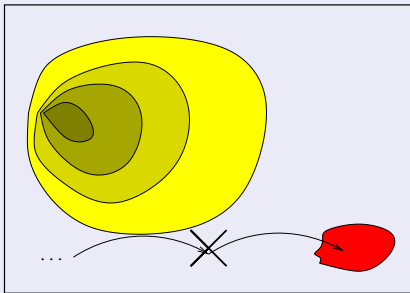
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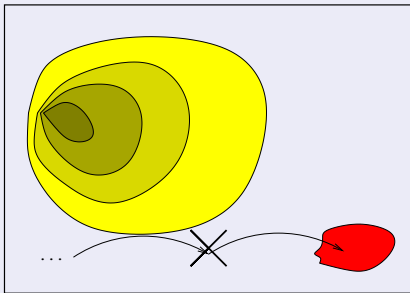
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...

- Repeat for increasing values of the gap 1, 2, 3, 4,
- **Intuition:** increasingly tighten the constraint for “spurious” counterexamples: a spurious counterexample must be a chain s_{k-n}, \dots, s_k of **unreachable** and **different** states s.t. $\neg \text{Good}(s_k)$ and $R(s_i, s_{i+1}), \forall i$.

- Dual to –and interleaved with– bounded model checking steps
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Strengthening of Invariants [cont.]

⇒ Check for the [un]satisfiability of the Boolean formulas:

$$I(s^0) \wedge \neg \text{Good}(s^0); \text{ [BMC}_0\text{]}$$

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K-Induction Algorithm [Sheeran et al. 2000]

Algorithm

Given:

$$\begin{aligned} \text{Base}_n &:= I(\mathbf{s}_0) \wedge \bigwedge_{i=0}^{n-1} (R(\mathbf{s}_i, \mathbf{s}_{i+1}) \wedge \varphi(\mathbf{s}_i)) \wedge \neg\varphi(\mathbf{s}_n) \\ \text{Step}_n &:= \bigwedge_{i=0}^n (R(\mathbf{s}_i, \mathbf{s}_{i+1}) \wedge \varphi(\mathbf{s}_i)) \wedge \neg\varphi(\mathbf{s}_{n+1}) \\ \text{Unique}_n &:= \bigwedge_{0 \leq i < j \leq n} \neg(\mathbf{s}_i = \mathbf{s}_{j+1}) \end{aligned}$$

1. **function** CHECK_PROPERTY (I, R, φ)
2. **for** $n := 0, 1, 2, 3, \dots$ **do**
3. **if** (DPLL(Base_n) == SAT)
4. **then return** PROPERTY_VIOLATED;
5. **else if** (DPLL($\text{Step}_n \wedge \text{Unique}_n$) == UNSAT)
6. **then return** PROPERTY_VERIFIED;
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⇒ Reuses previous search if DPLL is incremental!!

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Example: a correct 3-bit shift register

- System M :

- $I(x) := (\neg x[0] \wedge \neg x[1] \wedge \neg x[2])$

- $R(x, x') := ((x'[0] \leftrightarrow x[1]) \wedge (x'[1] \leftrightarrow x[2]) \wedge (x'[2] \leftrightarrow 0))$

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$$\left(\begin{array}{l} (\neg x^0[0] \wedge ((x^1[0] \leftrightarrow x^0[1]) \wedge (x^1[1] \leftrightarrow x^0[2]) \wedge (x^1[2] \leftrightarrow 0))) \\ \wedge x^1[0] \end{array} \right)$$

\implies (partly by unit-propagation)

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\implies not proved

Remark

Both $\{\neg x^0[0], x^0[1], x^0[2]\}$ and $\{x^1[0], x^1[1], \neg x^1[2]\}$ are non-reachable.

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Example: a correct 3-bit shift register [cont.]

- BMC Step 1: (...) \implies unsat
- K-Induction Step 2:

$$\left(\begin{array}{l} (\neg x^0[0] \wedge ((x^1[0] \leftrightarrow x^0[1]) \wedge (x^1[1] \leftrightarrow x^0[2]) \wedge (x^1[2] \leftrightarrow 0)) \wedge \\ \neg x^1[0] \wedge ((x^2[0] \leftrightarrow x^1[1]) \wedge (x^2[1] \leftrightarrow x^1[2]) \wedge (x^2[2] \leftrightarrow 0)) \\) \wedge x^2[0] \end{array} \right) \wedge \neg((x^1[0] \leftrightarrow x^0[0]) \wedge (x^1[1] \leftrightarrow x^0[1]) \wedge (x^1[2] \leftrightarrow x^0[2]))$$

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$\{\neg x^0[0], \neg x^0[1], x^0[2]\}$, $\{\neg x^1[0], x^1[1], \neg x^1[2]\}$, and $\{x^2[0], \neg x^2[1], \neg x^2[2]\}$ are non-reachable.

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Example: a correct 3-bit shift register [cont.]

- BMC Step 2: (...) \implies unsat
- K-Induction Step 3:

$$\left(\begin{array}{l} (\neg x^0[0] \wedge ((x^1[0] \leftrightarrow x^0[1]) \wedge (x^1[1] \leftrightarrow x^0[2]) \wedge (x^1[2] \leftrightarrow 0)) \wedge \\ \neg x^1[0] \wedge ((x^2[0] \leftrightarrow x^1[1]) \wedge (x^2[1] \leftrightarrow x^1[2]) \wedge (x^2[2] \leftrightarrow 0)) \wedge \\ \neg x^2[0] \wedge ((x^3[0] \leftrightarrow x^2[1]) \wedge (x^3[1] \leftrightarrow x^2[2]) \wedge (x^3[2] \leftrightarrow 0)) \\) \wedge x^3[0] \end{array} \right)$$
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\implies (unit-propagation) $\{x^3[0], x^2[1], x^1[2]\}$

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\implies proved!

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Ex: Bounded Model Checking

Given the symbolic representation of a FSM M , expressed in terms of the two Boolean formulas: $I(x, y) \stackrel{\text{def}}{=} \neg x \wedge y$,
 $T(x, y, x', y') \stackrel{\text{def}}{=} (x' \leftrightarrow (x \leftrightarrow \neg y)) \wedge (y' \leftrightarrow \neg y)$, and the LTL property: $\varphi \stackrel{\text{def}}{=} \neg \mathbf{F}(x \wedge y)$,

Ex: Bounded Model Checking

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[Solution: The question corresponds to the Bounded Model Checking problem $M \models_2 \mathbf{E F}f$, s.t. $f(x, y) \stackrel{\text{def}}{=} (x \wedge y)$. Thus we have:

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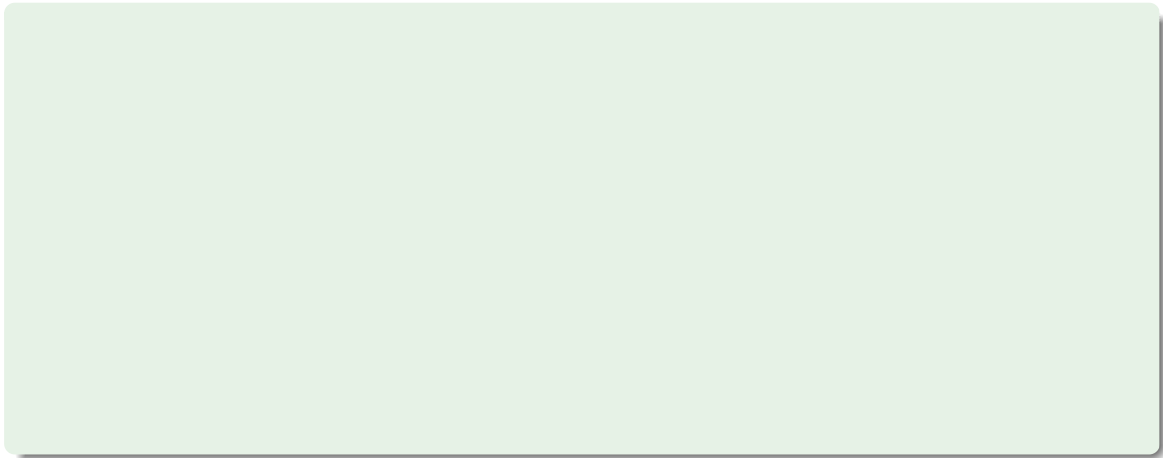
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[Solution: Yes: $\{\neg x_0, y_0, x_1, \neg y_1, x_2, y_2\}$, corresponding to the execution: $(0, 1) \rightarrow (1, 0) \rightarrow (1, 1)$]

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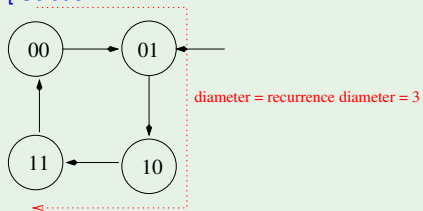
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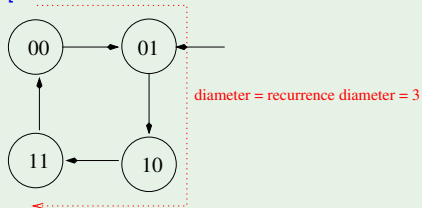


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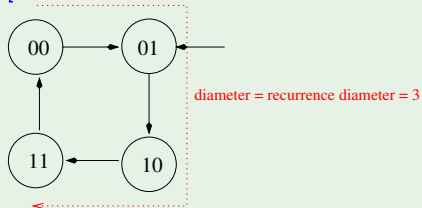
4. From the solutions to question #1 and #2 we can conclude that:

- (a) $M \models \varphi$
- (b) $M \not\models \varphi$
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[Solution: No: it is easy to see that the formula above is inconsistent]

Ex: Bounded Model Checking [cont.]

1 ...

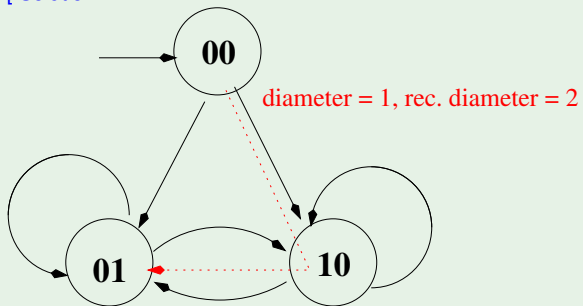
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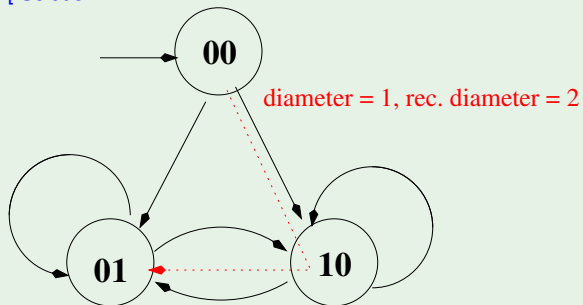
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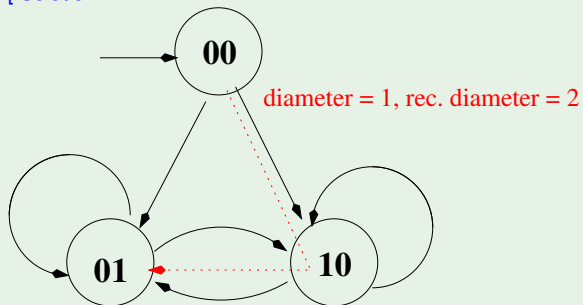
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[Solution:



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- 4 Can we conclude anything about the model-checking problem $M \models \varphi$? Explain why.
[Solution: yes, we can conclude that $M \models \varphi$, since $M \not\models_2 \mathbf{E F} \neg \varphi$ and rec. diameter=2.]

Ex: K-Induction

Given the following LTL Model Checking problem $M \models \varphi$ expressed in NuSMV input language:

```
MODULE main
VAR x : boolean; y : boolean; z : boolean;
INIT (!x & !y & z)
TRANS ((next(x) <-> (y)) & (next(y) <-> z) & (next(z) <-> x) )
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[Solution: The LTL property is in the form “**G**Good(x, y, z)”, hence, applying k-induction:

$$\begin{aligned} \varphi_{Base} &\stackrel{\text{def}}{=} (\neg x_0 \wedge \neg y_0 \wedge z_0) && \wedge && // I(x_0, y_0, z_0) \wedge \\ &\neg(x_0 \vee y_0 \vee z_0) && && // \neg \text{Good}(x_0, y_0, z_0) \\ \varphi_{Ind1} &\stackrel{\text{def}}{=} (x_i \vee y_i \vee z_i) && \wedge && // \text{Good}(x_i, y_i, z_i) \wedge \\ &((x_{i+1} \leftrightarrow y_i) \wedge (y_{i+1} \leftrightarrow z_i) \wedge (z_{i+1} \leftrightarrow x_i)) && \wedge && // T(x_i, y_i, z_i, x_{i+1}, y_{i+1}, z_{i+1}) \wedge \\ &\neg(x_{i+1} \vee y_{i+1} \vee z_{i+1}) && && // \neg \text{Good}(x_{i+1}, y_{i+1}, z_{i+1}) \end{aligned}$$

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 - φ_{Base} is not satisfiable. In fact, the second row forces the assignments $\neg x_0, \neg y_0, \neg z_0$, which makes the first row false.
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[Solution: a) $M \models \varphi$. In fact, we have proved it in one induction step.

]