Foundations of Artificial Intelligence
Chapter 04: **Beyond Classical Search**

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Generalities

- So far we addresses a single category of problems:
  1. observable,
  2. deterministic,
  3. with known environment,
  4. s.t. the solution is a sequence of actions.
- What happens when these assumptions are relaxed?
- In order we will:
  - release condition 4 $\Longrightarrow$ local search
  - release condition 2 $\Longrightarrow$ search with non-deterministic actions
  - release condition 1 $\Longrightarrow$ search with no observability or with partial observability
  - release condition 3 $\Longrightarrow$ online search

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Outline

1. Local Search and Optimization
   - General Ideas
   - Hill-Climbing
   - Simulated Annealing
   - Local Beam Search & Genetic Algorithms
2. Search with Nondeterministic Actions
3. Search with Partial or No Observations (Deterministic/Nondeterministic Actions)
   - Search with No Observations
   - Search with Partial Observations
4. Online Search (aka Exploration)

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General Ideas

- Search techniques: systematic exploration of search space
  - solution to problem: the path to the goal state
  - ex: 8-puzzle
- With many problems, the path to goal is irrelevant
  - goals expressed as conditions, not as explicit list of goal states
  - solution to problem: only the goal state itself
  - ex: N-queens
  - many important applications:
    integrated-circuit design, factory-floor layout, job-shop scheduling, automatic programming, telecommunications network optimization, vehicle routing, portfolio management...
- The state space is a set of "complete" configurations
  - decision problems: find goal configuration satisfying constraints/rules (ex: N-queens)
  - optimization problems: find optimal configurations
    (ex: Travelling Salesperson Problem TSP)
- If so, we can use iterative-improvement algorithms (in particular local search algorithms):
  - keep a single "current" state, try to improve it

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Local Search

- Idea: use single current state and move to “neighbouring” states
  - operate using a single current node
  - the paths followed by the search are not retained
- Two key advantages:
  - use very little memory (usually constant)
  - can often find reasonable solutions in large or infinite (continuous) state spaces, for which systematic algorithms are unsuitable
- Also useful for pure optimization problems
  - find the best state according to an objective function
  - often do not fit the “standard” search model of previous chapter
  - ex: Darwinian survival of the fittest: metaphor for optimization, but no “goal test” and no “path cost”
- A complete local search algorithm: guaranteed to always find a solution (if exists)
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  - maximization and minimization dual (switch sign)

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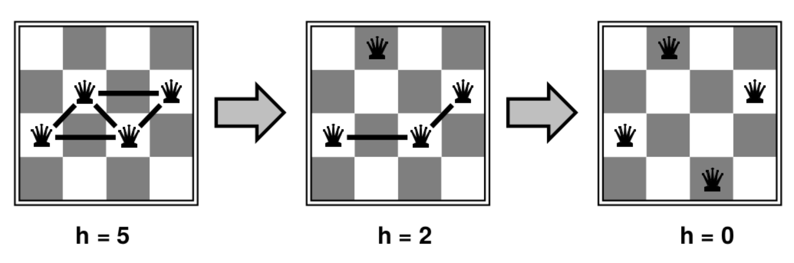
Local Search Example: N-Queens

- One queen per column (incremental representation)
- Cost (h): # of queen pairs on the same row, column, or diagonal
- Goal: h=0
- Step: move a queen vertically to reduce number of conflicts



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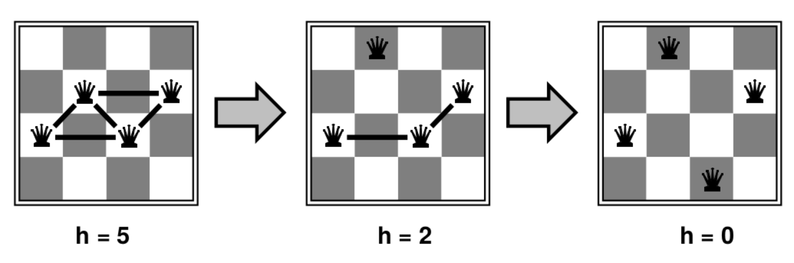

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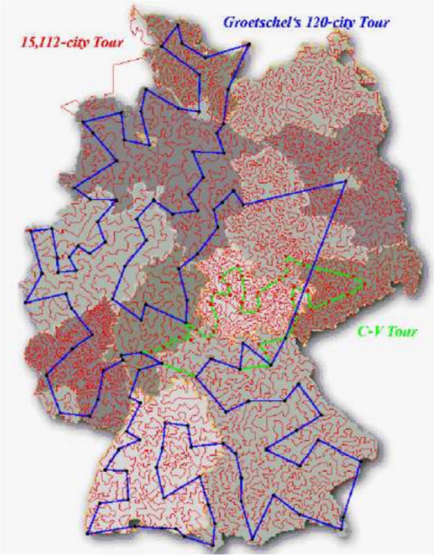
Optimization Local Search Example: TSP

Travelling Salesperson Problem (TSP)

Given an undirected graph, with n nodes and each arc associated with a positive value, find the Hamiltonian tour with the minimum total cost.

Very hard for classic search!



(Courtesy of Michela Milano, UniBO)

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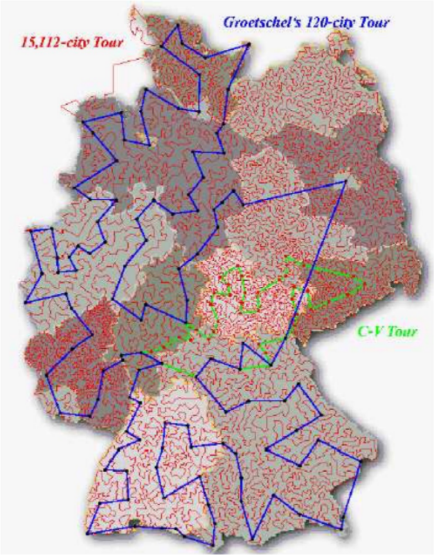

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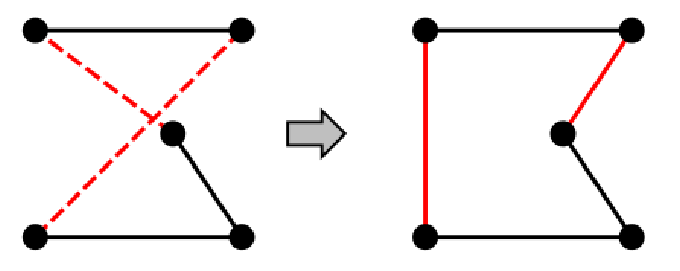
Optimization Local Search Example: TSP

- State represented as a permutation of numbers $(1, 2, ..., n)$
- Cost (h): total cycle length
- Start with any complete tour
- Step: (2-swap) perform pairwise exchange



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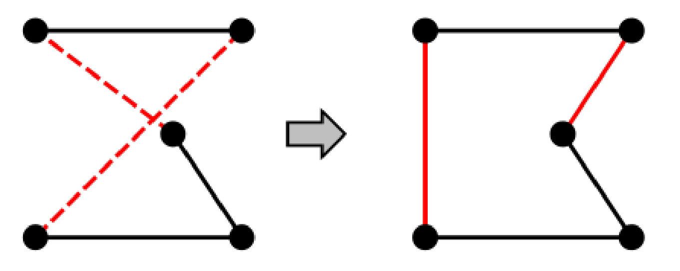

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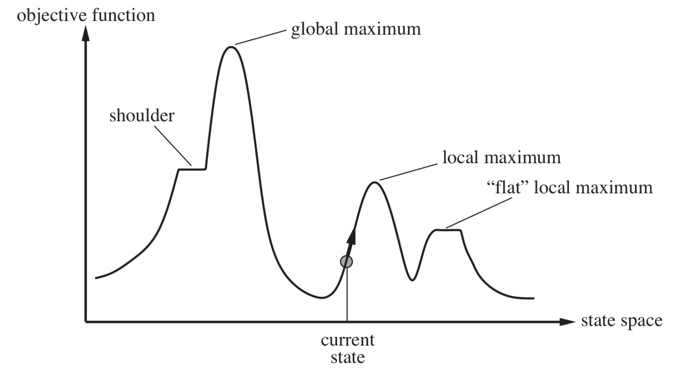
Local Search: State-Space Landscape

State-space landscape (Maximization)

- Local search algorithms explore state-space landscape
  - state space n-dimensional (and typically discrete)
  - move to “nearby” states (neighbours)
- NP-Hard problems may have exponentially-many local optima



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Outline

Hill-Climbing Search (aka Greedy Local Search)

Hill-Climbing

- Very-basic local search algorithm
- Idea: a move is performed only if the solution it produces is better than the current solution
  - (steepest-ascent version): selects the neighbour with best score improvement (select randomly among best neighbours if ≥ 1)
  - does not look ahead of immediate neighbors of the current state
  - stops as soon as it finds a (possibly local) minimum
- Several variants (Stochastic H.C., Random-Restart H.C., ...)
- Often used as part of more complex local-search algorithms

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum

  current ← MAKE-NODE(*problem*.INITIAL-STATE)
  loop do
    neighbor ← a highest-valued successor of *current*
    if neighbor.VALUE ≤ current.VALUE **then return** *current*.STATE
    current ← *neighbor*

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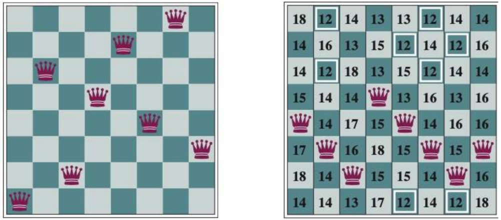
Hill-Climbing Search: Example

8-queen puzzle (minimization)

- Neighbour states: generated by moving one queen vertically
  - Cost (h): # of queen pairs on the same row, column, or diagonal
  - Goal: h=0
- Two example scenarios ((b) $\Longrightarrow$ (a) in 5 steps) :
  - (a) h=1, but all neighbours have higher costs $\Longrightarrow$ local minimum
  - (b) 8-queens state with heuristic cost estimate h = 17 (12d, 5h)
    - 8 moves with minimum cost 12: hill-climbing will pick one



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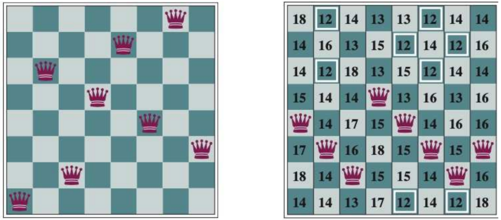

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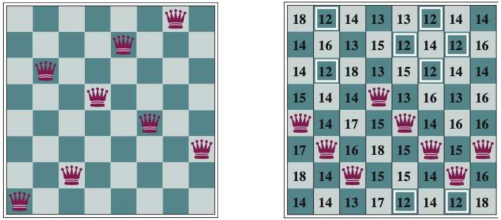

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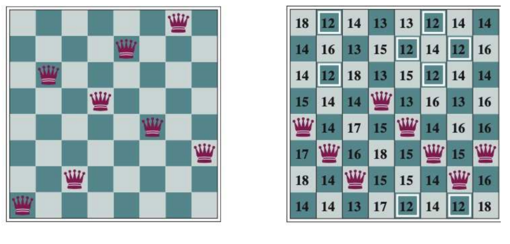

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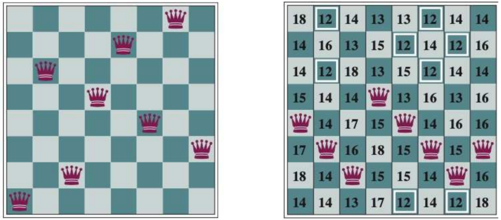

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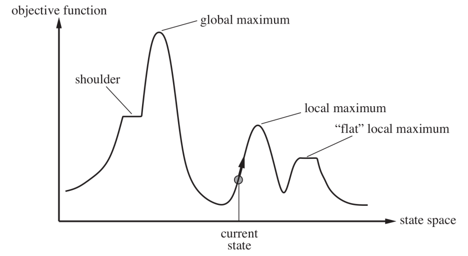
Hill-Climbing Search: Drawbacks

- Incomplete: gets stuck in local optima, flat local optima & shoulders (aka plateaux), ridges (sequences of local optima)
  - Ex: with 8-queens, gets stuck 86% of the time, fast when succeed

  note: converges very fast till (local) minima or plateaux
- Possible idea: allow 0-progress moves (aka sideways moves)
  - pros: may allow getting out of shoulders
  - cons: may cause infinite loops with flat local optima

  $\Longrightarrow$ set a limit to consecutive sideways moves (e.g. 100)
  - Ex: with 8-queens, pass from 14% to 94% success, slower



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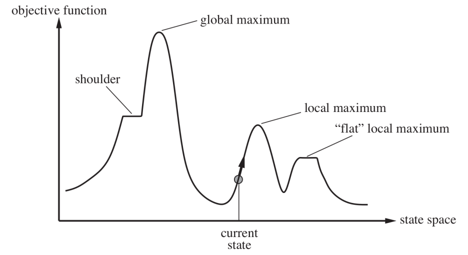

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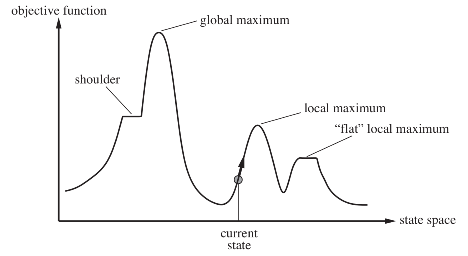

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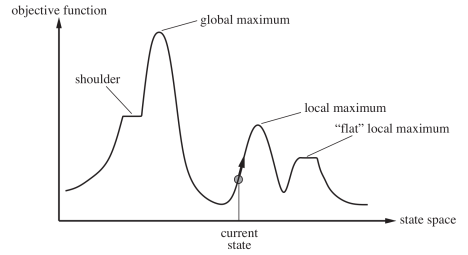

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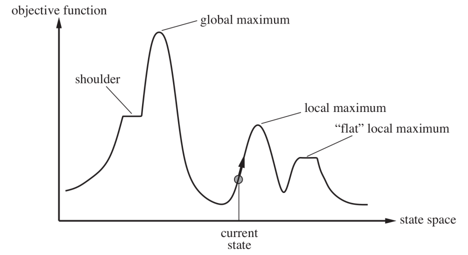

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Hill-climbing: Variations

- Stochastic hill-climbing
  - random selection among the uphill moves
  - selection probability can vary with the steepness of uphill move
  - sometimes slower, but often finds better solutions
- First-choice hill-climbing
  - generates successors randomly until a better one is found
  - good when there are large amounts of successors
- Random-restart hill-climbing
  - conducts a series of hill-climbing searches from randomly generated initial states
  - tries to avoid getting stuck in local maxima

Outline

Simulated Annealing

- Inspired to statistical-mechanics analysis of metallurgical annealing (Boltzmann's state distributions)
- Idea: Escape local maxima by allowing "bad" moves...
  - "bad move": move toward states with worse value
  - typically pick a move taken at random ("random walk")
- ... but gradually decrease their size and frequency.
  - sideways moves progressively less likely
- Analogy: get a ball into the deepest crevice in a bumpy surface
  - initially shaking hard ("high temperature")
  - progressively shaking less hard ("decrease the temperature")

Widely used in large-scale optimization tasks (e.g. VSLI layout problems, factory scheduling,...)

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Simulated Annealing [cont.]

Simulated Annealing (minimization)

- A "temperature" parameter T slowly decreases with steps ("schedule")
- The probability of picking a "bad move":
  - decreases exponentially with the "badness" of the move $|\Delta E|$ (bad move if $\Delta E < 0$)
  - decreases as the "temperature" T goes down
- If schedule lowers T slowly enough,
  then the algorithm will find a global optimum with probability approaching 1

function SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state
  current ← *problem*.INITIAL
  for t = 1 **to** ∞ **do**
    $T \leftarrow schedule(t)$
    if $T = 0$ **then return** *current*
    next ← a randomly selected successor of *current*
    $\Delta E \leftarrow$ VALUE(*current*) – VALUE(*next*)
    if $\Delta E > 0$ **then** *current* ← *next* // if improved, then it is accepted
    else *current* ← *next* only with probability $e^{\Delta E/T}$ //if worse, accepted with probability // which decreases with time

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Outline

Local Beam Search

Local Beam Search

Idea: keep track of k states instead of one

- Initially: k random states
- Step:
  1. determine all successors of k states
  2. if any of successors is goal $\Longrightarrow$ finished
  3. else select k best from successors
- Different from k searches run in parallel:
  - searches that find good states recruit other searches to join them
    $\Longrightarrow$ information is shared among k search threads
- Lack of diversity: quite often, all k states end up in the same local hill

$\Longrightarrow$ Stochastic Local Beam: choose k successors randomly, with probability proportional to state success.

Resembles natural selection with asexual reproduction:
the successors (offspring) of a state (organism) populate the next generation according to its value (fitness), with a random component.

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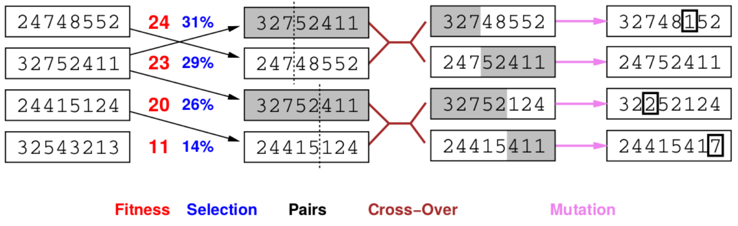
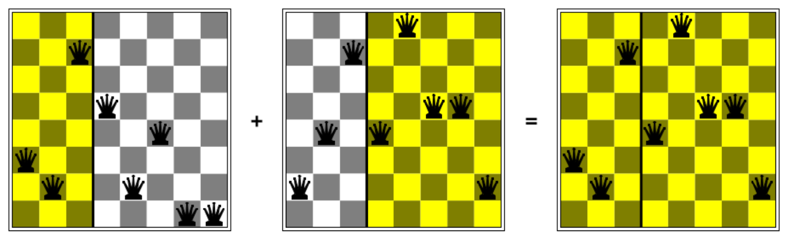
```
function GENETIC-ALGORITHM(population, fitness) returns an individual
  repeat
      weights ← WEIGHTED-BY(population, fitness) // list of fitness values for each individual
      population2 ← empty list
      for i = 1 to SIZE(population) do
          parent1, parent2 ← WEIGHTED-RANDOM-CHOICES(population, weights, 2)
          child ← REPRODUCE(parent1, parent2)
          if (small random probability) then child ← MUTATE(child)
          add child to population2
      population ← population2
  until some individual is fit enough, or enough time has elapsed
  return the best individual in population, according to fitness

function REPRODUCE(parent1, parent2) returns an individual
  n ← LENGTH(parent1)
  c ← random number from 1 to n
  return APPEND(SUBSTRING(parent1, 1, c), SUBSTRING(parent2, c + 1, n))
```

Genetic Algorithms: Example

Example: 8-Queens

state[*i*]: (upward) position of the queen in *i*th column





327|52411 + 247|48552 = 327|48552

Genetic Algorithms: Intuitions, Pros & Cons

Intuitions

- Selection drives the population toward high fitness
- Crossover combines good parts from good solutions
  (but it might achieve the opposite effect)
- Mutation introduces diversity

Pros & Cons

- Pros:
  - extremely simple
  - general purpose
  - tractable theoretical models
- Cons:
  - not completely understood
  - good coding is crucial (e.g., Gray codes for numbers)
  - too simple genetic operators

Widespread impact on optimization problems, i.e. circuit layout and job-shop scheduling

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Outline

Recall: Generalities

- So far we addresses a single category of problems:
  1. observable,
  2. deterministic,
  3. with known environment,
  4. s.t. the solution is a sequence of actions.
- What happens when these assumptions are relaxed?
- In order we will:
  - release condition 4 $\Longrightarrow$ local search
  - release condition 2 $\Longrightarrow$search with non-deterministic actions
  - release condition 1 $\Longrightarrow$ search with no observability or with partial observability
  - release condition 3 $\Longrightarrow$ online search

Generalities (cont.)

- Assumptions so far (see ch. 2 and 3):
  - the environment is deterministic
  - the environment is fully observable
  - the agent knows the effects of each action

$\Longrightarrow$ The agent does not need perception:
  - can calculate which state results from any sequence of actions
  - always knows which state it is in
- If one of the above does not hold, then percepts are useful
  - the future percepts cannot be determined in advance
  - the agent's future actions will depend on future percepts
- Solution: not a sequence but a contingency plan (aka conditional plan, strategy)
  - specifies the actions depending on what percepts are received
- We analyze first the case of nondeterministic environments

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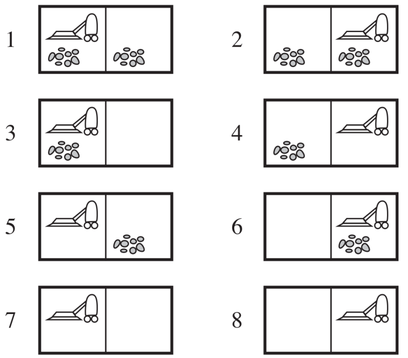
Example: The Erratic Vacuum Cleaner

Erratic Vacuum-Cleaner Example

- actions: Left, Right, Suck
- goal: A and B cleaned (states 7, 8)
- if environment is observable, deterministic, and completely known $\Longrightarrow$ solvable by search algos
- ex: if initially in 1, then [suck,right,suck] leads to 8: [1,5,6,8]



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- Nondeterministic version (erratic vacuum cleaner):
  - if dirty square: cleans the square, sometimes cleans also the other square. Ex: $1 \overset{suck}{\Longrightarrow} \{5, 7\}$
  - if clean square: sometimes deposits dirt on the carpet

    Ex: $5 \overset{suck}{\Longrightarrow} \{1, 5\}$

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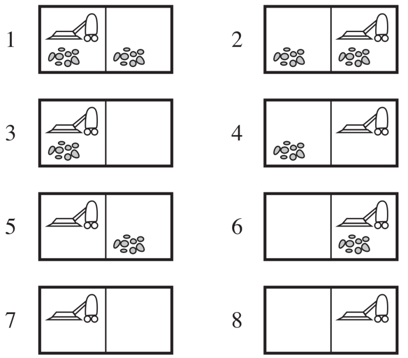

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Searching with Nondeterministic Actions

Generalized notion of transition model

- RESULTS(S,A) returns a set of possible outcomes states
  - Ex: RESULTS(1,SUCK)=$\{5, 7\}$, RESULTS(5,SUCK)=$\{1, 5\}$, ...
- A solution is a contingency plan (aka conditional plan, strategy)
  - contains nested conditions on future percepts (if-then-else, case-switch, ...)
  - Ex: from state 1 we can act the following contingency plan:
    [SUCK, IF STATE = 5 THEN [RIGHT, SUCK] ELSE [ ]]
- Can cause loops (see later)

Remark

In practice, we don't reason on states, rather on state variable values:
[Suck; if B.Dirty then [Right, Suck] else [ ]]

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Searching with Nondeterministic Actions [cont.]

And-Or Search Trees

- In a deterministic environment, we branch on agent's choices

  $\Longrightarrow$ OR nodes, hence OR search trees
  - OR nodes correspond to states
- In a nondeterministic environment, we branch also on outcomes for each action
  - the agent has to handle all such outcomes

  $\Longrightarrow$ AND nodes, hence AND-OR search trees
  - AND nodes correspond to actions
  - leaf nodes are goal, dead-end or loop OR nodes
- A solution for an AND-OR search problem is a subtree s.t.:
  - has a goal node at every leaf
  - specifies one action at each of its OR nodes
  - includes all outcome branches at each of its AND nodes

OR tree: AND-OR tree with 1 outcome each AND node (determinism)

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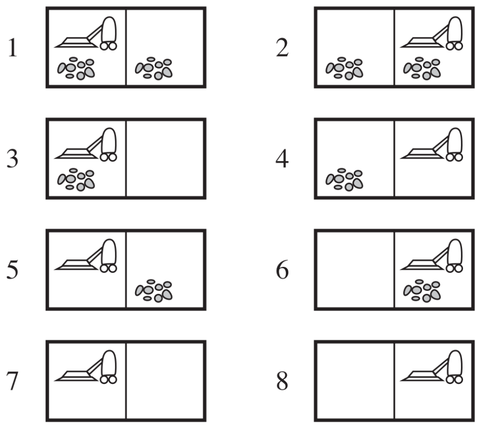
And-Or Search Trees: Example

(Part of) And-Or Search Tree for Erratic Vacuum Cleaner Example.

Problem: Init: 1, Goal: 7,8.

Solution: [SUCK, IF STATE = 5 THEN [RIGHT, SUCK] ELSE [ ]] (solid arcs)



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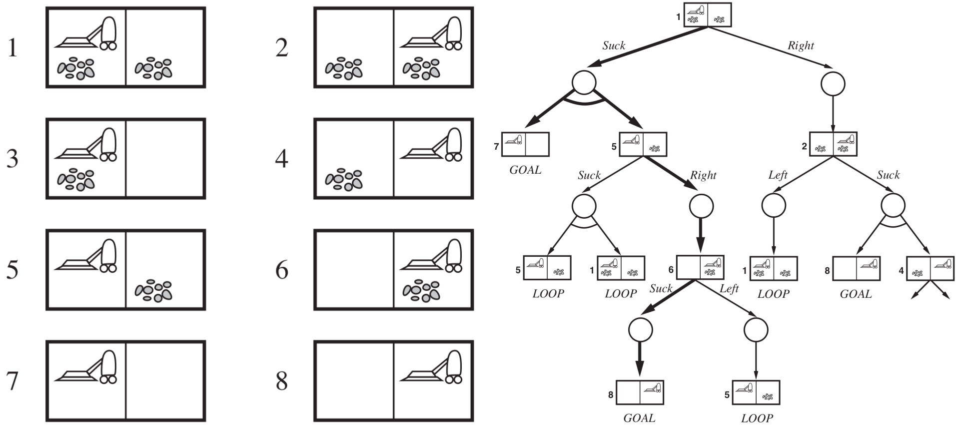

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AND-OR Search

Recursive Depth-First (Tree-based) AND-OR Search

function AND-OR-SEARCH(*problem*) **returns** a conditional plan, or *failure*
  return OR-SEARCH(*problem*, *problem*.INITIAL, [ ])

function OR-SEARCH(*problem*, *state*, *path*) **returns** *a conditional plan, or failure*
  if *problem*.IS-GOAL(*state*) **then return** the empty plan
  if IS-CYCLE(*state*, *path*) **then return** *failure*
  for each *action* **in** *problem*.ACTIONS(*state*) **do**
    $plan \leftarrow$ AND-SEARCH(*problem*, RESULTS(*state*, *action*), [*state*] + [*path*])
    if $plan \neq failure$ **then return** [*action*] + [*plan*]
  return *failure*

function AND-SEARCH(*problem*, *states*, *path*) **returns** *a conditional plan, or failure*
  for each s_i **in** *states* **do**
    $plan_i \leftarrow$ OR-SEARCH(*problem*, s_i, *path*)
    if $plan_i = failure$ **then return** *failure*
  return [**if** s_1 **then** $plan_1$ **else if** s_2 **then** $plan_2$ **else** ... **if** s_{n-1} **then** $plan_{n-1}$ **else** $plan_n$]

Note: nested if-then-else can be rewritten as case-switch

AND-OR Search [cont.]

Recursive Depth-First (Tree-based) AND-OR Search

- Cycles: if the current state already occurs in the path $\Longrightarrow$ failure
  - cycle detection like with ordinary DFS
  - does not mean “no solution”
  - means “if there is a non-cyclic solution,
    then it must be reachable from the earlier incarnation of the current state”
    $\Longrightarrow$ the new incarnation can be discharged

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- Implictly Depth-First, but can also be explored by breadth-first or best-first method
  - e.g. A^* variant for AND-OR search available (see AIMA book)

AND-OR Search [cont.]

Recursive Depth-First (Tree-based) AND-OR Search

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- Some problems have no acyclic solutions
- A cyclic plan may be considered a cyclic solution provided that:
  - every leaf is a goal state (loop states not considered leaves), and
  - a leaf is reachable from every point in the plan
- Can be expressed by means of introducing
  - labels, and backward goto's to labels
  - loop syntax (e.g., while-do)

$\Longrightarrow$ Executing a cyclic solution eventually reaches a goal, provided that each outcome of a nondeterministic action eventually occurs

- Is this assumption reasonable?
- Yes, provided we distinguish: $\langle$*nondeterministic, observable*$\rangle \neq \langle$*deterministic, partially-observable*$\rangle$
- Ex: device may not always work $\neq$ device is broken (but we don't know it)

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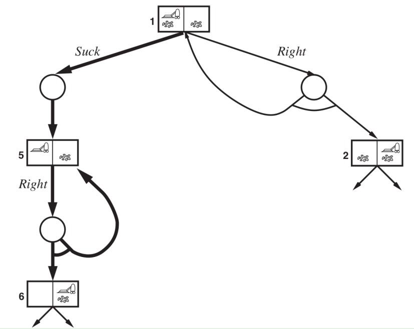
Cyclic Solution: Example

Example: Slippery Vacuum Cleaner

- Movement actions may fail: e.g., $\mathit{Results}(1, \mathit{Right}) = \{1, 2\}$

$\Longrightarrow$ A cyclic solution

- Use labels: [Suck, L1 : Right, if State = 5 then L1 else Suck]
- Use cycles: [Suck, While State = 5 do Right, Suck]



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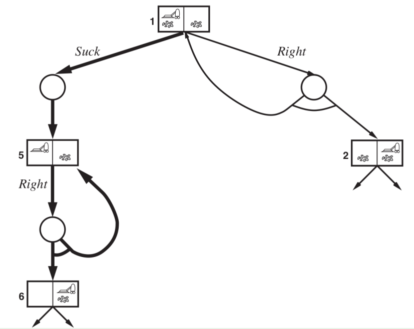

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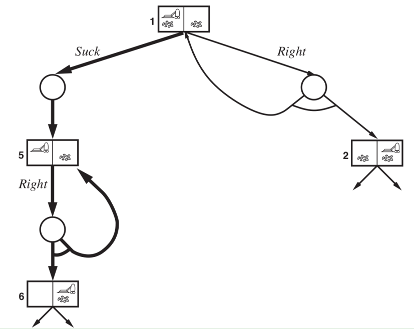

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Exercises

1. Consider the slippery vacuum cleaner problem s.t. initial is $\langle B, \langle \mathit{dirty}, \mathit{clean} \rangle\rangle$ and goal is $\langle A, \langle \mathit{clean}, \mathit{clean} \rangle\rangle$:
   - Using the AND-OR-tree search method, find a plan to achieve the goal.
   - Draw the AND-OR (sub)tree which produces the plan
   - Track explicitly all the the recursive calls of AND-OR-SEARCH, OR-SEARCH, and AND-SEARCH, and their returned (sub)plans
2. Do the same with a defective variant of the basic vacuum cleaner, in which “suck” may either succeed or not in making the current location clean.
3. Do the same with other initial/goal state pairs

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Outline

Recall: Generalities

- So far we addresses a single category of problems:
  1. observable,
  2. deterministic,
  3. with known environment,
  4. s.t. the solution is a sequence of actions.
- What happens when these assumptions are relaxed?
- In order we will:
  - release condition 4 $\Longrightarrow$ local search
  - release condition 2 $\Longrightarrow$ search with non-deterministic actions
  - release condition 1 $\Longrightarrow$search with no observability or with partial observability
  - release condition 3 $\Longrightarrow$ online search

Generalities

Partial Observability

- Partial observability: percepts do not capture the whole state
  - not all state variable values known $\Longrightarrow$ corresponds to a set of possible physical states
- If the agent is in one of several possible physical states, then an action may lead to one of several possible outcomes, even if the environment is deterministic

Belief States

- Belief state: the agent's current belief about the possible physical states it might be in, given the previous sequence of actions and percepts
  - is a set of physical states: (but does not know in which one)
  - contains the actual physical state the agent is in
  - ex: $\{1, 2\}$: the agent is either in state 1 or in state 2 (but it does not know in which one)
  - if the belief state contains only one state, then the agent knows it is in that state
- 2^n possible belief states out of n possible physical states!

In practice, we reason in terms of (disjunctions of) partial states rather than of sets of states

- a partial state assigns values to a subset of state variables (e.g. $\langle ?, \langle ?, \mathit{Clean} \rangle \rangle = \{3, 4, 7, 8\}$

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Outline

Search with No Observation

Search with No Observation (aka Sensorless Search or Conformant Search)

- Idea: To solve sensorless problems, the agent searches in the space of belief states rather than in that of physical states
  - fully observable, because the agent knows its own belief space
  - solutions are always sequences of actions (no contingency plan), because percepts are always empty and thus predictable
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Search with No Observation: Example

Example: Sensorless Vacuum Cleaner

- the vacuum cleaner knows the geography of its world, but it doesn't know its location or the distribution of dirt
  - initial state: $\{1, 2, 3, 4, 5, 6, 7, 8\}$ (i.e. $\langle ?, \langle ?, ? \rangle\rangle$)
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- In practice, the information on the state is made progressively less partial by the actions:

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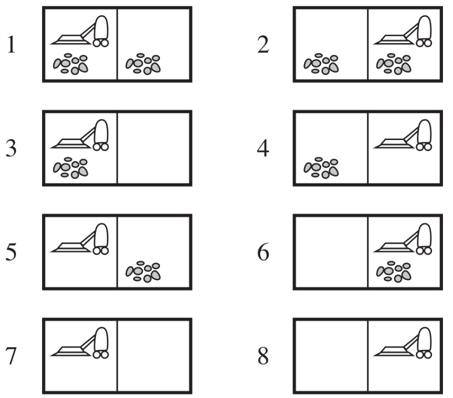

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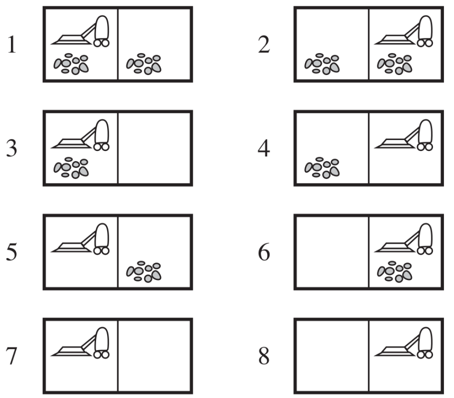

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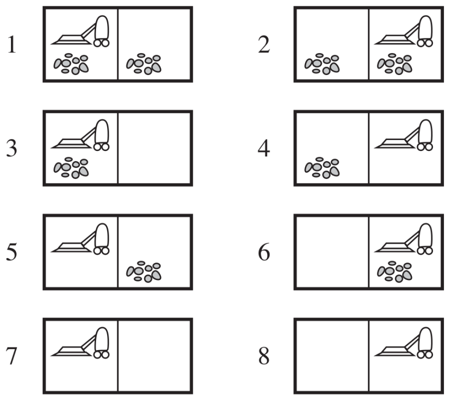

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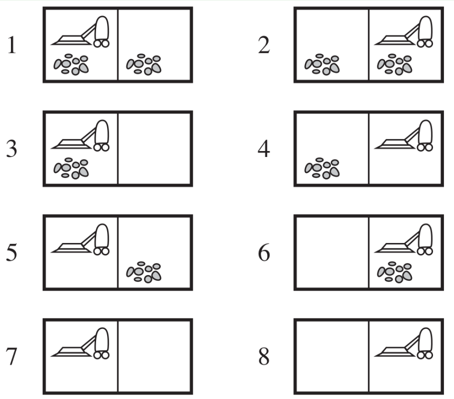

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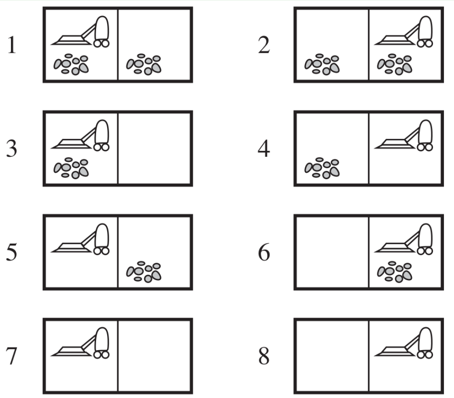

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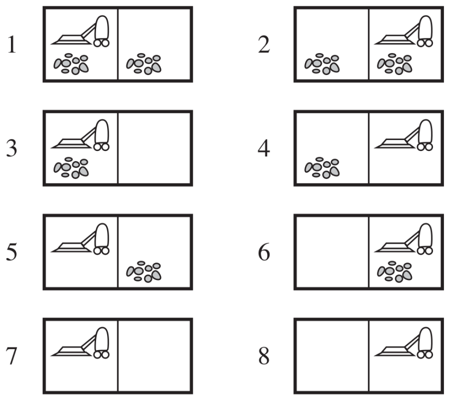

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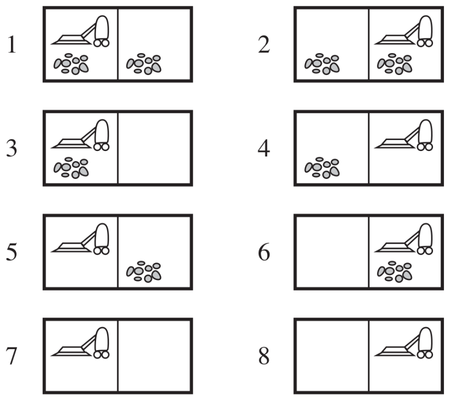

Search with No Observation: Example

Example: Sensorless Vacuum Cleaner

- the vacuum cleaner knows the geography of its world, but it doesn't know its location or the distribution of dirt
  - initial state: $\{1, 2, 3, 4, 5, 6, 7, 8\}$ (i.e. $\langle ?, \langle ?, ? \rangle\rangle$)
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  - after action sequence [RIGHT,SUCK,LEFT,SUCK], state is $\{7\}$ (i.e. $\langle A, \langle Clean, Clean \rangle\rangle$)
- In practice, the information on the state is made progressively less partial by the actions:

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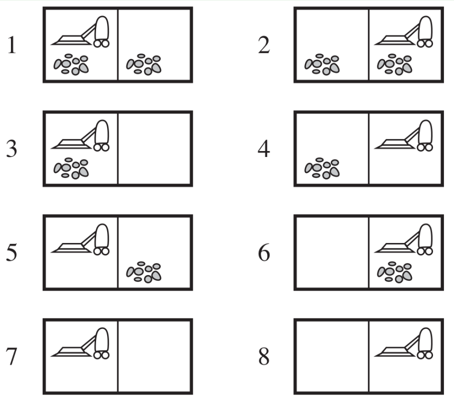

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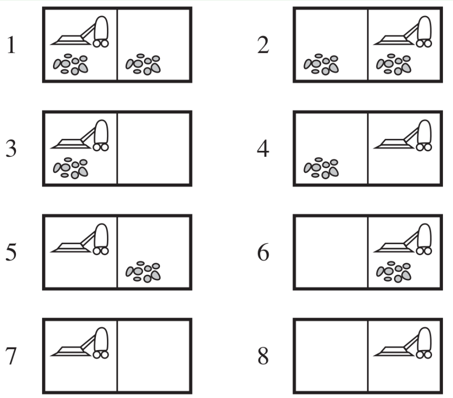

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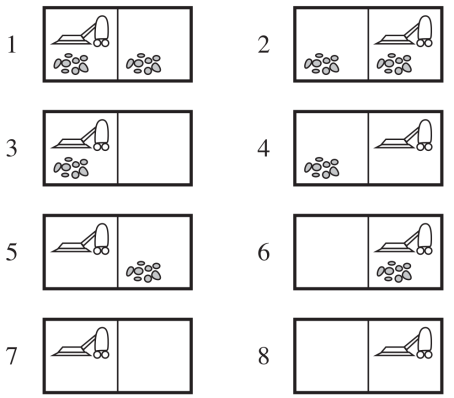

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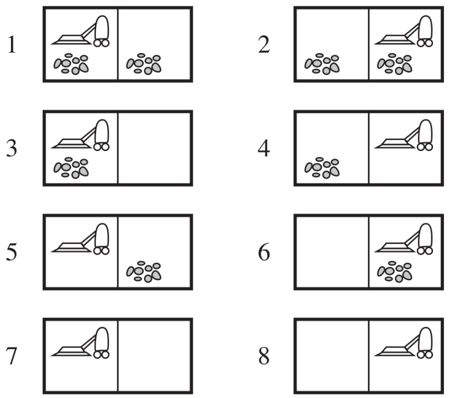

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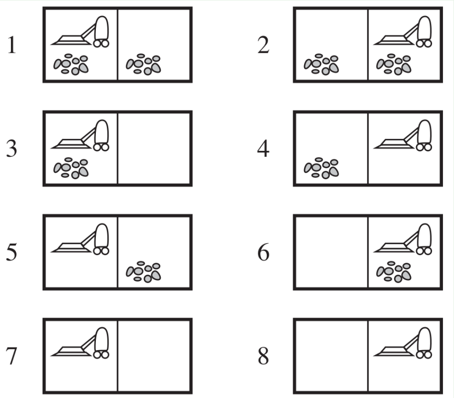

Remark

Single partial state vs. disjunction of partial states

- Often belief state can be represented as a single partial state:

  Ex: $\overbrace{\langle ?, \langle ?, ? \rangle\rangle}^{\{1,2,3,4,5,6,7,8\}} \overset{RIGHT}{\Longrightarrow} \overbrace{\langle B, \langle ?, ? \rangle\rangle}^{\{2,4,6,8\}}$

- Sometimes a belief state requires beeing represented as a disjunction of partial states.

  Ex: $\overbrace{\langle ?, \langle ?, ? \rangle\rangle}^{\{1,2,3,4,5,6,7,8\}} \overset{SUCK}{\Longrightarrow} \overbrace{\overbrace{\langle A, \langle Clean, ? \rangle\rangle}^{\{5,7\}} \text{ or } \overbrace{\langle B, \langle ?, Clean \rangle\rangle}^{\{4,8\}}}^{\{4,5,7,8\}}$

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Belief-State Problem Formulation

Let $Actions_P()$, $Result_P()$, $GoalTest_P()$, $StepCost_P()$ refer to physical System P:

- Belief states: subsets of physical states (typically partial states)
  - If P has N states, then the sensorless problem has up to 2^N states
- Initial state: typically the set of all physical states in P
- Actions: (assumption: illegal actions have no effects)
  - $Actions(b) \stackrel{\text{def}}{=} \bigcup_{s \in b} Actions_P(s)$ (i.e., must consider all possible actions in all possible states)
- Transition model:
  - for deterministic actions: $b' = Result(b, a) \stackrel{\text{def}}{=} \{s' \mid s' = Result_P(s, a) \text{ and } s \in b\}$;
  - for nondeterministic actions: [a]
    $b' = Result(b, a) \stackrel{\text{def}}{=} \{s' \mid s' \in Result_P(s, a) \text{ and } s \in b\} = \bigcup_{s \in b} Result_P(s, a)$;
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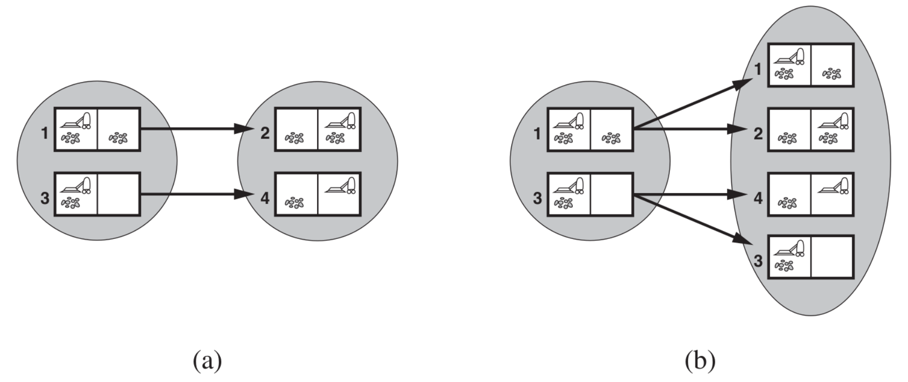
Example: Sensorless Vacuum Cleaner, plain and slippery versions

Prediction: $\mathit{Result}(\{1,3\}, \mathit{Right})$, deterministic (a) and nondeterministic action (b)



(© S. Russell & P. Norwig, AIMA)

(a): $\overbrace{\langle A, \langle \mathit{Dirty}, ?\rangle\rangle}^{\{1,3\}} \overset{\mathit{Right}}{\Longrightarrow} \overbrace{\langle B, \langle \mathit{Dirty}, ?\rangle\rangle}^{\{2,4\}}$

(b): $\overbrace{\langle A, \langle \mathit{Dirty}, ?\rangle\rangle}^{\{1,3\}} \overset{\mathit{Right}}{\Longrightarrow} \overbrace{\langle ?, \langle \mathit{Dirty}, ?\rangle\rangle}^{\{1,2,3,4\}}$

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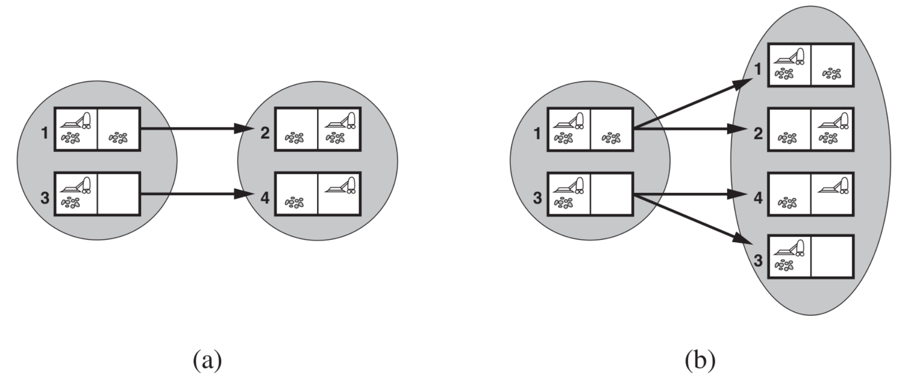

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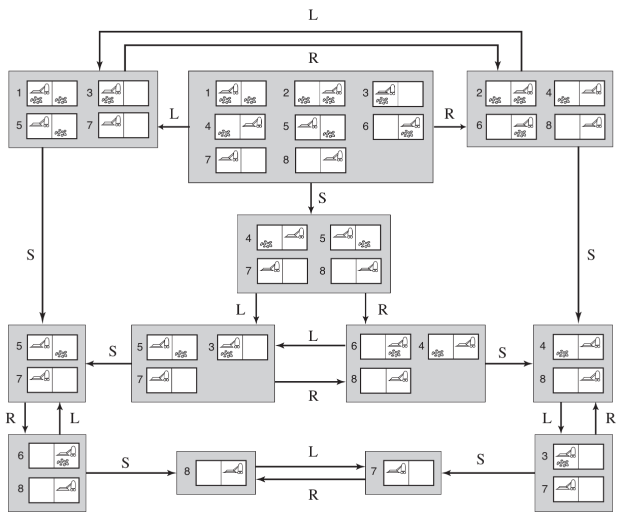
Belief-State Problem Formulation [cont.]

Example: Sensorless Vacuum Cleaner: Belief State Space

(self-loops are omitted)



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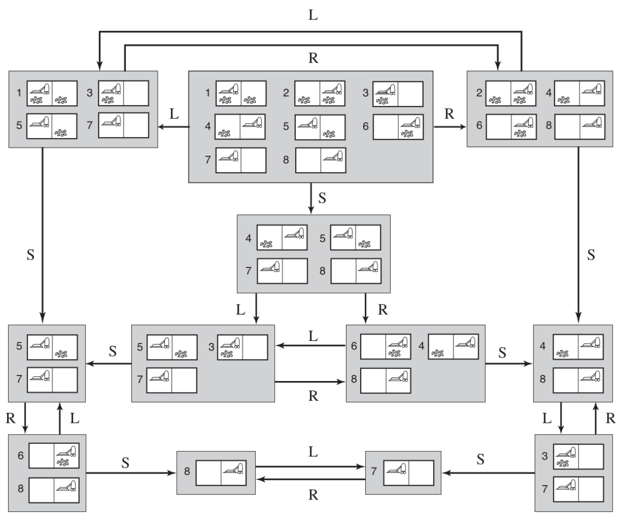

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Belief-State Problem Formulation [cont.]

Remarks

1. if $b \subseteq b'$, then $Result(b, a) \subseteq Result(b', a)$ (b more informative than b')
2. If a is deterministic, then $|Result(b, a)| \leq |b|$
3. The agent might achieve the goal earlier than $GoalTest(b)$ holds, but it does not know it (because he knows it only when all states in the belief state are goal states)

Properties

- An action sequence is a solution for b iff it leads b to a goal
- If an action sequence is a solution for a belief state b, then it is also a solution for any belief state b' s.t. $b' \subseteq b$
  - if $b \xrightarrow{a_1} \dots \xrightarrow{a_k} g$, then $b' \xrightarrow{a_1} \dots \xrightarrow{a_k} g$

We can apply to the Belief-State space any search algorithm.

- if a solution for b has been found, then any $b' \subseteq b$ is solvable
- if $b' \subseteq b$ has already been generated and discarded, then we can discard a path reaching a belief state b

$\Longrightarrow$ Dramatically improves efficiency

Belief-State Problem Formulation [cont.]

Remarks

1. if $b \subseteq b'$, then $Result(b, a) \subseteq Result(b', a)$ (b more informative than b')
2. If a is deterministic, then $|Result(b, a)| \leq |b|$
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Properties

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We can apply to the Belief-State space any search algorithm.

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Exercises

1. Consider the Sensorless Vacuum Cleaner (goal: both A,B clean)
   - applying graph BFS or DFS, expansion order $\{L, R, S\}$, find a valid plan or show there is none
   - applying tree BFS or DFS with loop detection, expansion order $\{L, R, S\}$, find a valid plan or show there is none
2. Consider (the sensorless version of) the Slippery vacuum cleaner (goal: both A,B clean)
   - draw the Belief State Space
   - applying graph BFS or DFS, expansion order $\{L, R, S\}$, find a valid plan or show there is none
   - applying tree BFS or DFS with loop detection, expansion order $\{L, R, S\}$, find a valid plan or show there is none
3. Consider (the sensorless version of) the Erratic vacuum cleaner (goal: both A,B clean)
   - do the same as with exercise 2
4. Consider a defective variant of the basic vacuum cleaner, in which “suck” may either succeed or not in making the current location clean
   - do the same as with exercise 2

Outline

Search with Observations

Perception and Belief-State Problem Formulation

- *Percept*(s) returns the percept received in state s
  - ex: local-sensing vacuum cleaner, can perceive dirty/clean only on the current position: $Percept(1) = [A, Dirty]$
  - with fully observable problems: $Percept(s) = s, \forall s$
  - with sensorless problems: $Percept(s) = null, \forall s$
- Partial observations: many states can produce the same percept
  - ex: $Percept(1) = Percept(3) = [A, Dirty]$

  $\Longrightarrow$ $Percept(s)$ may correspond to many different candidate states s
- *Actions*(), *StepCost*(), *GoalTest*(): as with sensorless case

Note

- If sensing were nondeterministic, then a function *Percepts*(s) would return a set of possible percepts
- Hereafter, we assume that sensing is deterministic

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Transition Model with (Partial) Perceptions

The Prediction-Observation-Update process

- Three steps:
  1. Prediction (same as for sensorless): predict the belief state after action a
     $\hat{b} = Predict(b, a) \stackrel{\text{def}}{=} Result_{(sensorless)}(b, a) = \{s' \mid s' = Result_P(s, a) \text{ and } s \in b\}$
  2. Observation prediction: determines the set of percepts that could be observed in the predicted belief state: $PossiblePercepts(\hat{b}) \stackrel{\text{def}}{=} \{o \mid o = Percept(s) \text{ and } s \in \hat{b}\}$
  3. Update: for each percept o, determine the belief state b_o, i.e., the subset of states in $\hat{b}$ that could have produced the percept o:
     - $b_o = Update(\hat{b}, o) \stackrel{\text{def}}{=} \{s \mid s \in \hat{b} \text{ and } o = Percept(s)\}$

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- set (not union!) of belief states, one for each possible percepts o
- for each o, $b_o \subseteq \hat{b} \Longrightarrow$ sensing reduces uncertainty!
- (assume sensing deterministic) the b_o's are all disjoint (each s belongs to b_o s.t. $o = Percept(s)$)
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  $\Longrightarrow$ each next possible percepts o is used to partition $\hat{b}$ into a smaller belief state b_o

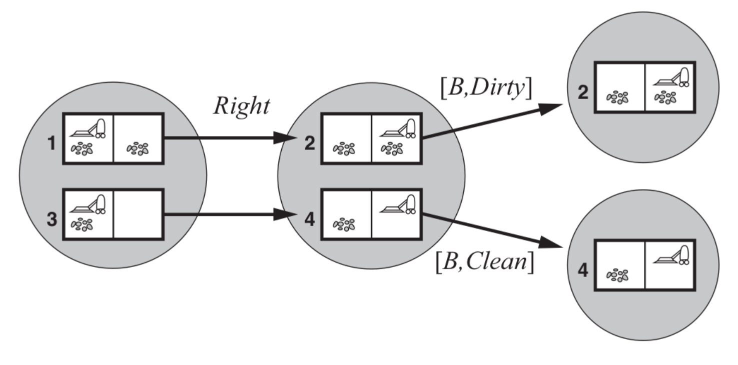
$\Longrightarrow$ Non-deterministic belief-state problem

- due to the inability to predict exactly the next percept

Transition Model with Perceptions: Example

Deterministic actions: Local-sensing vacuum cleaner

- $\hat{b} = \mathit{Predict}(\{1,3\}, \mathit{Right}) = \{2,4\}$
- $\mathit{PossiblePercepts}(\hat{b}) = \{[B, \mathit{Dirty}], [B, \mathit{Clean}]\}$
- $\mathit{Result}(\{1,3\}, \mathit{Right}) = \{\{2\},\{4\}\}$



Transition Model with Perceptions: Example

Nondeterministic actions: Slippery local-sensing vacuum cleaner

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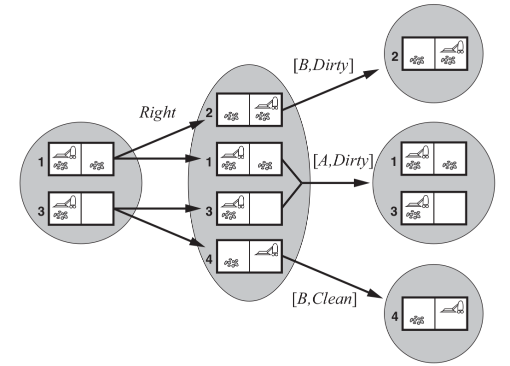

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Solving Partially-Observable Problems

- Formulation as a nondeterministic belief-state search problem
  - non-determinism due to different possible percepts

$\Longrightarrow$ The AND-OR search algorithms can be applied

$\Longrightarrow$ The solution is a conditional plan

Solution for initial percept [A, Dirty] (deterministic): [Suck, Right, if Bstate = {6} then Suck else [ ]]

First level:
(draw second level
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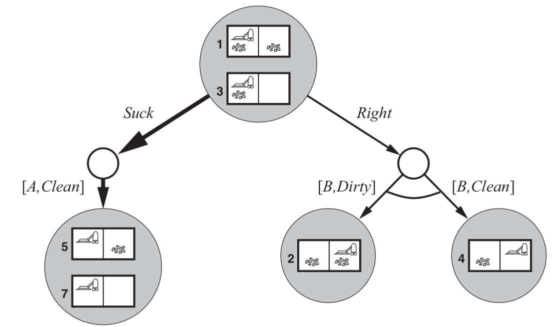

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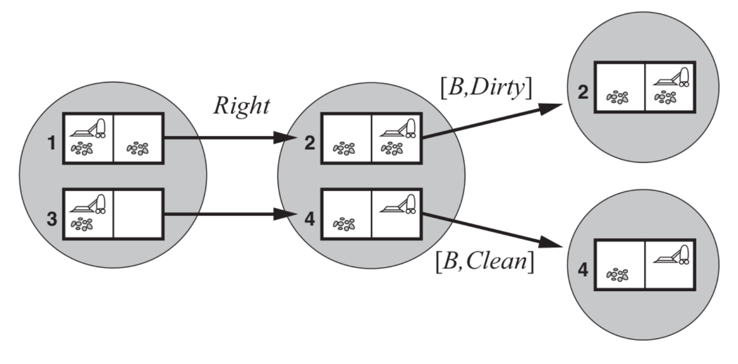

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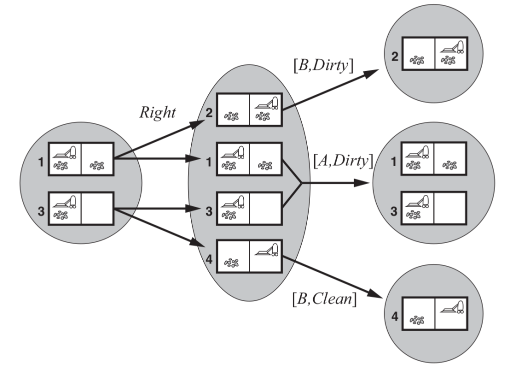

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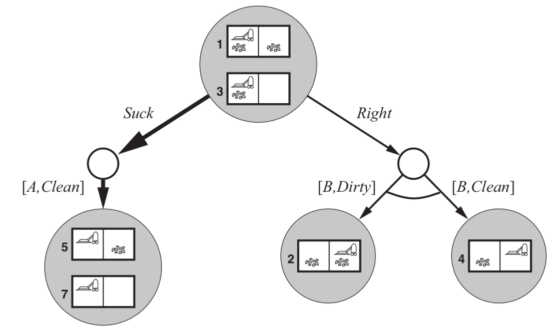

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Exercises

1. Consider the deterministic Local-sensing Vacuum Cleaner (goal: both A,B clean)
   - applying graph AND-OR search expansion order $\{L, R, S\}$, find a valid plan or show there is none
   - do it for different initial belief states (e.g., $\langle ?, \langle ?, ? \rangle\rangle$, $\langle A, \langle ?, ? \rangle\rangle$, $\langle A, \langle \mathit{Dirty}, ? \rangle\rangle$, $\langle A, \langle \mathit{Clean}, ? \rangle\rangle$, ...)
2. Same as #1, with the slippery Local-sensing Vacuum Cleaner
3. Same as #1, with the defective variant ("Suck" may or may not be effective) of Local-sensing Vacuum Cleaner

An Agent for Partially-Observable Environments

- Agent quite similar to the simple problem-solving agent [Ch.3]:
  - formulates a problem (as a belief-state search)
  - calls a search algorithm (an AND-OR-GRAPH one)
  - executes the solution
- Two main differences:
  - the solution is a conditional plan, not an action sequence
  - in step (3) the agent needs to maintain its belief state as it performs actions and receives percepts (aka monitoring, filtering, state estimation)
- State estimation resembles the prediction-observation-update process:
  - simpler, because the percept o is given by the environment $\Longrightarrow$ no need to calculate it
  - given b, a and o: $b' = Update(Predict(b, a), o)$

Remark

The computation has to happen as fast as percepts are coming in
$\Longrightarrow$ in some complex applications, compute approximate belief states

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Example: Belief-State Maintenance

Example: Kindergarden Vacuum-Cleaner

- local sensing $\Longrightarrow$ partially observable
- any square may become dirty at any time unless the agent is actively cleaning it at that moment $\Longrightarrow$ nondeterministic
- Ex: $\mathit{Update}(\overbrace{\mathit{Predict}(\{1,3\}, \mathit{Suck})}^{\{5,7\}}, [A, \mathit{Clean}]) = \{5,7\}$
- Ex: $\mathit{Update}(\overbrace{\mathit{Predict}(\{5,7\}, \mathit{Right})}^{\{2,4,6,8\}}, [B, \mathit{Dirty}]) = \{2,6\}$

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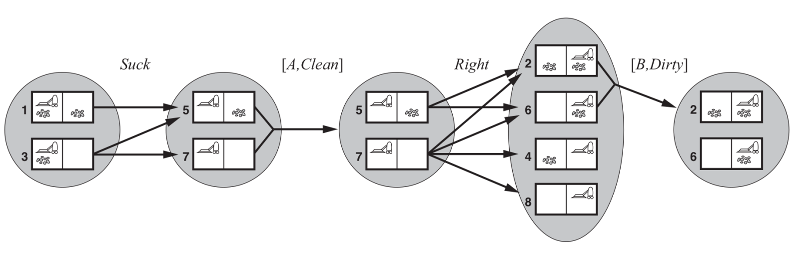

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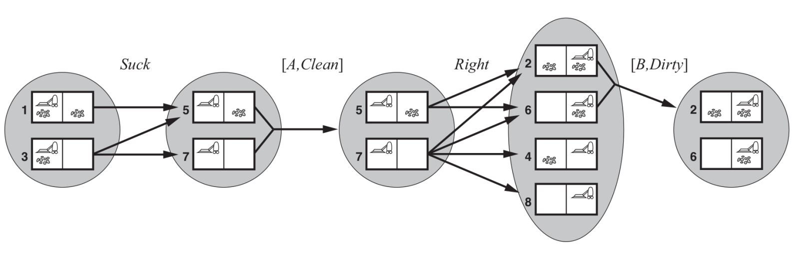

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Example:

- Knows the map, senses walls in the four directions (NESW)
  - localization broken: does not know where it is
  - navigation broken: does not know the direction is moving to $\Longrightarrow$ move is nondeterministic
  - goal: localization (know where it is)
- $b = \{all\ locations\}$, $o = NSW$
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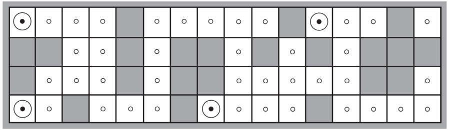
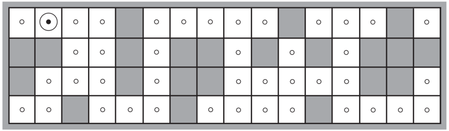


(a) Possible locations of robot after $E_1 = NSW$



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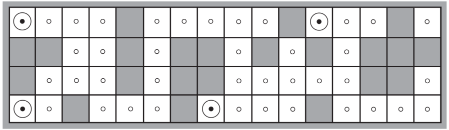

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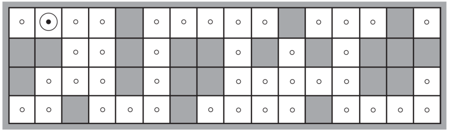

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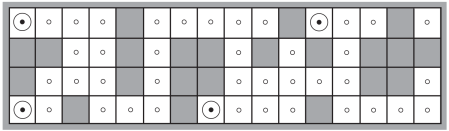

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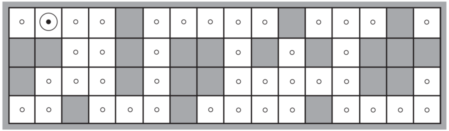

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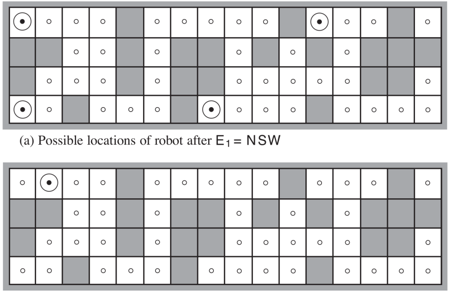

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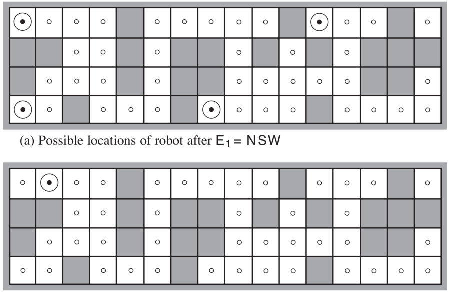

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Outline

Recall: Generalities

- So far we addresses a single category of problems:
  1. observable,
  2. deterministic,
  3. with known environment,
  4. s.t. the solution is a sequence of actions.
- What happens when these assumptions are relaxed?
- In order we will:
  - release condition 4 $\Longrightarrow$ local search
  - release condition 2 $\Longrightarrow$ search with non-deterministic actions
  - release condition 1 $\Longrightarrow$ search with no observability or with partial observability
  - release condition 3 $\Longrightarrow$online search

Generalities

Online vs. offline search

- So far: Offline search
  - it computes a complete solutions before executing it
- Online search: agent interleaves computation and action
  - it takes an action,
  - then it observes the environment and computes the next action
  - (repeat)
- Necessary in dynamic domains or unknown domains
  - cannot know the states and consequences of actions
  - faces an exploration problem: must use actions as experiments in order to learn enough
  - ex: a robot placed in a new building $\Longrightarrow$ must explore it to build a map for getting from A to B
  - ex: newborn baby $\Longrightarrow$ acts to learn the outcome of his/her actions
- Useful in nondeterministic domains
  - prevents search blowup

Must be solved by executing actions, rather than by pure computation

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Online Search

Working Hypotheses

- Assumption: a deterministic and fully observable environment
- The agent knows only
  - $Actions(s)$, which returns the list of actions allowed in s
  - the step-cost function $c(s, a, s')$ (cannot be used until s' is known)
  - $GoalTest(s)$
- Remark: The agent cannot determine $Result(s, a)$
  - except by actually being in s and doing a
- The agent knows an admissible heuristic function h(s), that estimates the distance from the current state to a goal state
- Objective: reach goal with minimal cost
  - Cost: total cost of traveled path
  - Competitive ratio: ratio of cost over cost of the solution path if search space is known ($+\infty$ if agent in a deadend)

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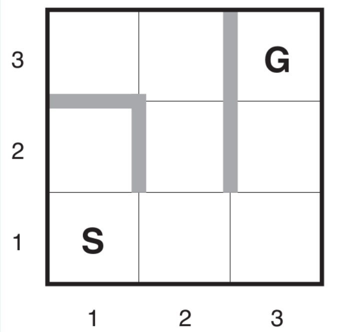
Online Search: Example

Example: a simple maze problem

- the agent does not know that going Up from (1,1) leads to (1,2)
- having done that, it does not know that going Down leads to (1,1)



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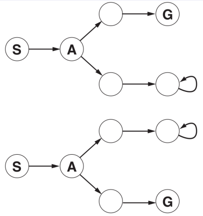
Online Search: Deadends

Inevitability of Deadends

- Online search may face deadends (e.g., with irreversible actions)
- No algorithm can avoid dead ends in all state spaces
- Adversary argument: for each algo, an adversary can construct the state space while the agent explores it
  - If states S and A are visited. What next?

  $\Longrightarrow$ if algo goes right, adversary builds (top), otherwise builds (bot)

  $\Longrightarrow$ adversary builds a deadend
- Assumption the state space is safely explorable: some goal state is reachable from every reachable state (ex: reversible actions)



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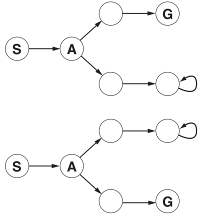

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Online Search Agents

Online Search Agents: Basic Ideas

- Idea: The agent creates & maintains a map of the environment (*result*[s, a])
  - map is updated based on percept input after every action
  - map is used to decide next action
- Difference wrt. offline algorithms (ex A^*, BFS)
  - Can only expand the node it is physically in
    - $\Longrightarrow$ expand nodes in local order
    - $\Longrightarrow$ DFS natural candidate for an online version
  - Needs to backtrack physically
    - DFS: go back to the state from which the agent most recently entered the current state
    - must keep a table with the predecessor states of each state to which the agent has not yet backtracked (*unbacktracked*[s])
    - $\Longrightarrow$ backtrack physically (find an action reversing the generation of s)

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Online DFS Search Agents

```
function ONLINE-DFS-AGENT(problem, s′) returns an action
            s, a, the previous state and action, initially null
            result, a table mapping (s, a) to s′, initially empty
            untried, a table mapping s to a list of untried actions
            unbacktracked, a table mapping s to a list of states never backtracked to

  if problem.IS-GOAL(s′) then return stop
  if s′ is a new state (not in untried) then untried[s′] ← problem.ACTIONS(s′)
  if s is not null then   // if neither initial nor result of backtracking
      result[s, a] ← s′
      add s to the front of unbacktracked[s′]
  if untried[s′] is empty then   //backtrack   // results[s',b] exists because untried[s'] is empty
      if unbacktracked[s′] is empty then return stop   // added in 4th ed. AIMA
      a ← an action b such that result[s′, b] = POP(unbacktracked[s′]); s′ ← null
  else a ← POP(untried[s′])   // all actions in actions(s') have been tried
  s ← s′
  return a
```

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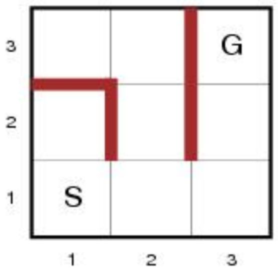
Online DFS: Example

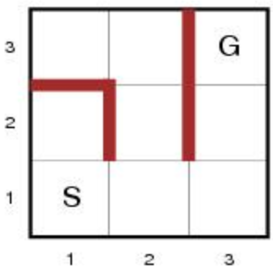


- Assume maze problem on 3x3 grid.
- s’ = (1,1) is initial state
- Result, untried, unbacktracked, … are empty
- S,a are also empty

(Courtesy of Tom Lenaerts, IRIDIA)

Online DFS: Example

S'=(1,1)

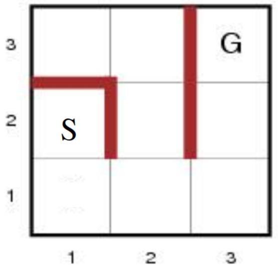


- GOAL-TEST((1,1))?
  - **S not = G thus false**
- (1,1) a new state?
  - **True**
  - **ACTIONS((1,1)) -> untried[(1,1)]**
    - {RIGHT,UP}
- s is null?
  - **True (initially)**
- untried[(1,1)] empty?
  - **False**
- POP(untried[(1,1)])->a
  - **A=UP**
- s = (1,1)
- Return a

(Courtesy of Tom Lenaerts, IRIDIA)

Online DFS: Example

S'=(2,1)

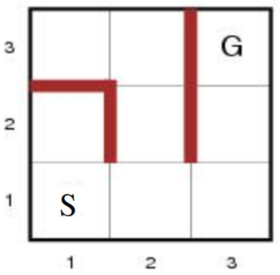


- GOAL-TEST((2,1))?
  - **S not = G thus false**
- (2,1) a new state?
  - **True**
  - **ACTION((2,1)) -> untried[(2,1)]**
    - {DOWN}
- s is null?
  - **false (s=(1,1))**
  - **result[UP,(1,1)] <- (2,1)**
  - **unbacktracked[(2,1)]={(1,1)}**
- untried[(2,1)] empty?
  - **False**
- A=DOWN, s=(2,1) return A

(Courtesy of Tom Lenaerts, IRIDIA)

Online DFS: Example

S'=(1,1)



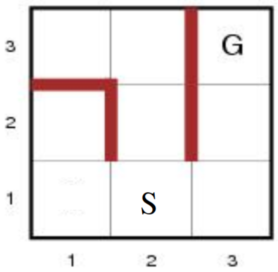
- GOAL-TEST((1,1))?
  - **S not = G thus false**
- (1,1) a new state?
  - **false**
- s is null?
  - **false (s=(2,1))**
  - **result[DOWN,(2,1)] <- (1,1)**
  - **unbacktracked[(1,1)]={(2,1)}**
- untried[(1,1)] empty?
  - **False**
- A=RIGHT, s=(1,1) return A

// Notice: here (2.1)->(1.1) is a move, not a form of backtracking

(Courtesy of Tom Lenaerts, IRIDIA)

Online DFS: Example

S'=(1,2)

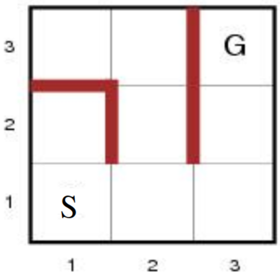


- GOAL-TEST((1,2))?
  - **S not = G thus false**
- (1,2) a new state?
  - **True, untried[(1,2)]={RIGHT,UP,LEFT}**
- s is null?
  - **false (s=(1,1))**
  - **result[RIGHT,(1,1)] <- (1,2)**
  - **unbacktracked[(1,2)]={(1,1)}**
- untried[(1,2)] empty?
  - **False**
- A=LEFT, s=(1,2) return A

(Courtesy of Tom Lenaerts, IRIDIA)

Online DFS: Example

S'=(1,1)



- GOAL-TEST((1,1))?
  - **S not = G thus false**
- (1,1) a new state?
  - **false**
- s is null?
  - **false (s=(1,2))**
  - **result[LEFT,(1,2)] <- (1,1)**
  - **unbacktracked[(1,1)]={(1,2),(2,1)}**
- untried[(1,1)] empty?
  - **True**
  - **unbacktracked[(1,1)] empty? False**
- A= b for b in result[b,(1,1)]=(1,2)
  - **B=RIGHT**
- A=RIGHT, s=(1,1) …

// Note: here (1,2)->(1,1) ia a move, not a form of backtracking

(Courtesy of Tom Lenaerts, IRIDIA)

Online Search Agents

Online Search Agents: Facts

- Works only if actions are always reversible
- Worst case: each link $\langle s, a, s' \rangle$ is visited at most twice
  - one as exploration ($a \in \mathit{untried}[s]$)
  - one as backtracking ($a \in \mathit{unbacktracked}[s]$)
- An agent can go on a long walk even if it is close to the solution
  - an online iterative deepening approach solves this problem

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Online Local Search

- Hill Climbing natural candidate for online search
  - locality of search
  - only one state is stored
  - unfortunately, stuck in local minima
  - random restarts not possible
- Possible solution: Random Walk
  - selects randomly one available actions from the current state
  - preference can be given to actions that have not yet been tried
  - eventually finds a goal or complete its exploration if space is finite
  - unfortunately, very slow

Random Walk: example

- random walk takes exponentially many steps to find a goal (backward progress is twice as likely as forward progress)

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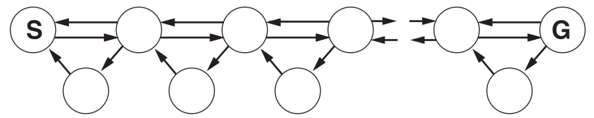

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Online A^*: $LRTA^*$

$LRTA^*$: General ideas

- Better possible solution: add memory to hill climbing
- Idea: store a "current best estimate" H(s) of the cost to reach the goal from each state that has been visited
  - initially $h(s)$
  - updated as the agent gains experience in the state space

  (recall that $h(s)$ is in general "too optimistic")

$\Longrightarrow$ Learning Real-Time A^* ($LRTA^*$)

- builds a map of the environment in the result[s,a] table
- chooses the "apparently best" move a according to current $H()$
- updates the cost estimate $H(s)$ for the state s it has just left, using the cost estimate of the target state s'
  - $H(s) := c(s, a, s') + H(s')$
- "optimism under uncertainty": untried actions in s are assumed to lead immediately to the goal with the least possible cost $h(s)$

$\Longrightarrow$ encourages the agent to explore new, possibly promising paths

An $LRTA^*$ agent is guaranteed to find a goal in any finite, safely explorable environment.

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function LRTA*-AGENT(s') **returns** an action
  inputs: s', a percept that identifies the current state
  persistent: $result$, a table, indexed by state and action, initially empty
    H, a table of cost estimates indexed by state, initially empty
    s, a, the previous state and action, initially null

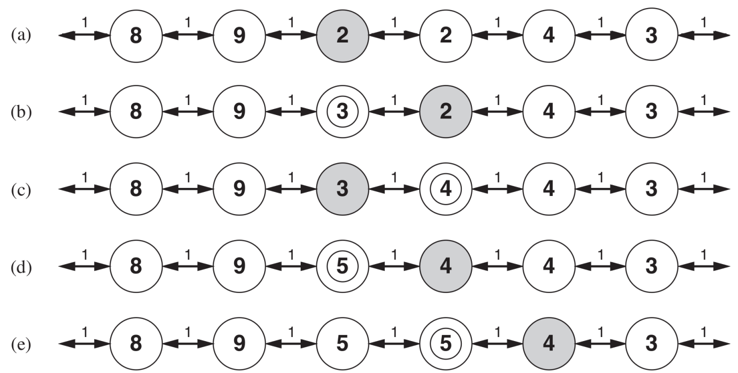
  if GOAL-TEST(s') **then return** $stop$
  if s' is a new state (not in H) **then** $H[s'] \leftarrow h(s')$
  if s is not null
    $result[s, a] \leftarrow s'$
    $H[s] \leftarrow \min\limits_{b \in \text{ACTIONS}(s)} \text{LRTA*-COST}(s, b, result[s, b], H)$
  $a \leftarrow$ an action b in ACTIONS(s') that minimizes LRTA*-COST($s', b, result[s', b], H$)
  $s \leftarrow s'$
  return a

function LRTA*-COST(s, a, s', H) **returns** a cost estimate
  if s' is undefined **then return** $h(s)$
  else return $c(s, a, s') + H[s']$

Example: *LRTA**

Five iterations of *LRTA** on a one-dimensional state space

- states labeled with current H(s), arcs labeled with step cost
- shaded state marks the location of the agent,
- updated cost estimates a each iteration are circled



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