

Fundamentals of Artificial Intelligence

Chapter 10: **Classical Planning**

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`https://disi.unitn.it/rseba/DIDATTICA/fai\_2025/`

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- 1 Basics on Planning
 - The Problem
 - The PDDL Language
- 2 Search Strategies and Heuristics
 - Forward and Backward Search
 - Heuristics
- 3 Planning Graphs, Heuristics and Graphplan
 - Planning Graphs
 - Heuristics Driven by Planning Graphs
 - The Graphplan Algorithm
- 4 Other Approaches (hints)
 - Planning as SAT Solving

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Automated Planning (aka “Planning”)

Automated Planning

Synthesize a sequence of actions (plan) to be performed by an agent leading from an initial state of the world to a set of target states (goal)

- Planning is both:
 - an application per se
 - a common activity in many applications
(e.g. design & manufacturing, scheduling, robotics,...)
- Similar to problem-solving agents (Ch.03), with factored/structured representation of states
- “Classical” Planning (this chapter):
fully observable, deterministic, static environments with single agents

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Automated Planning [cont.]

Automated Planning

- Given:
 - an initial state
 - a set of actions you can perform
 - a (set of) state(s) to achieve (goal)
- Find:
 - a **plan**: a partially- or totally-ordered set of actions needed to achieve the goal from the initial state

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Decidability and Complexity

- **PlanSAT**: the question of whether there exists any plan that solves a planning problem
 - decidable for classical planning
 - with function symbols, the number of states becomes infinite
 - ⇒ undecidable
 - in PSPACE
 - harder than NP, no polynomial-size witness (e.g., Tower of Hanoi)
- **Bounded PlanSAT**: the question of whether there exists any plan of a given length k or less
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A Language for Planning: PDDL

Planning Domain Definition Language (PDDL)

- A **state** is a conjunction of **fluents**: **ground, function-less atoms**
 - ex: $Poor \wedge Unknown, At(Truck_1, Melbourne) \wedge At(Truck_2, Sydney)$
 - examples of non-fluents:
 - $At(x, y)$ (non ground), $\neg Poor$ (negated), $At(Father(Fred), Sydney)$ (not function-less)
 - **closed-world assumption**: all non-mentioned fluents are false
 - **unique-name assumption**: distinct names refer to distinct objects
- **Actions** are described by a set of **action schemata**
 - concise description: **describe which fluent change**
 $\implies$ the other fluents implicitly maintain their values
- **Action Schema**: consists in **action name**, a **list of variables** in the schema, the **precondition**, the **effect** (aka **postcondition**)
 - precondition and effect are **conjunctions of literals** (positive or negated atomic sentences)
 - **lifted representation**: variables implicitly **universally quantified**
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PDDL: Example

- Action schema:

Action(*Fly*(*p*, *from*, *to*),

PRECOND : *Plane*(*p*) $\wedge$ *Airport*(*from*) $\wedge$ *Airport*(*to*) $\wedge$ *At*(*p*, *from*)

EFFECT : $\neg$ *At*(*p*, *from*) $\wedge$ *At*(*p*, *to*)

- Action instantiation:

Action(*Fly*(*P*₁, *SFO*, *JFK*),

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Action(*Fly*(*P*₁, *SFO*, *JFK*),

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A Language for Planning: PDDL [cont.]

- **Precondition**: must hold to ensure the action can be executed
 - defines the states in which the action can be executed
 - action is **applicable** in state s if the preconditions are satisfied by s
- **Effect**: represent the effects of the action on the world
 - defines the result of executing the action
- **Add list (ADD(a))**: (the fluents in) the positive literals in the action's effects
 - ex: $\{At(p, to)\}$
- **Delete list (DEL(a))**: (the fluents in) the negative literals in the action's effects
 - ex: $\{At(p, from)\}$
- **Result of action a in state s** : $RESULT(s, a) \stackrel{\text{def}}{=} (s \setminus DEL(a) \cup ADD(a))$
 - start from s
 - remove the fluents that appear as negative literals in effect
 - add the fluents that appear as positive literals in effect
 - ex: $Fly(P_1, SFO, JFK) \implies$ remove $At(P_1, SFO)$, add $At(P_1, JFK)$

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EFFECT : $\neg$ *At*(*P*₁, *SFO*) $\wedge$ *At*(*P*₁, *JFK*)

- $s : \textit{At}(P_1, \textit{SFO}) \wedge \textit{Plane}(P_1) \wedge \textit{Airport}(\textit{SFO}) \wedge \textit{Airport}(\textit{JFK}) \wedge \dots$

$\Rightarrow s' : \textit{At}(P_1, \textit{JFK}) \wedge \textit{Plane}(P_1) \wedge \textit{Airport}(\textit{SFO}) \wedge \textit{Airport}(\textit{JFK}) \wedge \dots$

Sometimes we want to **propositionalize** a PDDL problem:
replace each action schema with a set of ground actions.

- Ex: $\dots \textit{At_P}_1_ \textit{SFO} \wedge \textit{Plane_P}_1 \wedge \textit{Airport_SFO} \wedge \textit{Airport_JFK} \dots$

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A Language for Planning: PDDL [cont.]

Time in PDDL

- Fluents do not explicitly refer to time
- Times and states are **implicit** in the action schemata:
 - the precondition always refers to time t
 - the effect to time $t+1$.

PDDL Problem

- A set of action schemata defines a planning domain
- PDDL problem: a planning domain, an initial state and a goal
 - the initial state is a **conjunction of ground atoms** (positive literals)
 - closed-world assumption: any not-mentioned atoms are false
 - the goal is a **conjunction of literals** (positive or negative)
 - may contain variables, which are implicitly existentially quantified
 - a goal g may represent a **set of states** (the set of states entailing g)
- Ex: goal: $At(p, SFO) \wedge Plane(p)$:
 - variable "p" implicitly means "for some plane p"
 - the state $Plane(Plane_1) \wedge At(Plane_1, SFO) \wedge \dots$ entails g

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Time in PDDL

- Fluents do not explicitly refer to time
- Times and states are **implicit** in the action schemata:
 - the precondition always refers to time t
 - the effect to time $t+1$.

PDDL Problem

- A set of action schemata defines a **planning domain**
- **PDDL problem**: a **planning domain**, an **initial state** and a **goal**
 - the **initial state** is a **conjunction of ground atoms** (positive literals)
 - closed-world assumption: any not-mentioned atoms are false
 - the **goal** is a **conjunction of literals (positive or negative)**
 - **may contain variables**, which are implicitly **existentially quantified**
 - a goal g may represent a **set of states** (the set of states entailing g)
- Ex: **goal**: $At(p, SFO) \wedge Plane(p)$:
 - variable “ p ” implicitly means “for some plane p ”
 - the state $Plane(Plane_1) \wedge At(Plane_1, SFO) \wedge \dots$ entails g

A Language for Planning: PDDL [cont.]

Planning as a search problem

All components of a search problem

- an initial state
- an ACTIONS function
- a RESULT function
- and a goal test

Example: Air Cargo Transport

$Init(At(C_1, SFO) \wedge At(C_2, JFK) \wedge At(P_1, SFO) \wedge At(P_2, JFK)$
 $\wedge Cargo(C_1) \wedge Cargo(C_2) \wedge Plane(P_1) \wedge Plane(P_2)$
 $\wedge Airport(JFK) \wedge Airport(SFO))$

$Goal(At(C_1, JFK) \wedge At(C_2, SFO))$

$Action(Load(c, p, a),$

PRECOND: $At(c, a) \wedge At(p, a) \wedge Cargo(c) \wedge Plane(p) \wedge Airport(a)$

EFFECT: $\neg At(c, a) \wedge In(c, p)$

$Action(Unload(c, p, a),$

PRECOND: $In(c, p) \wedge At(p, a) \wedge Cargo(c) \wedge Plane(p) \wedge Airport(a)$

EFFECT: $At(c, a) \wedge \neg In(c, p)$

$Action(Fly(p, from, to),$

PRECOND: $At(p, from) \wedge Plane(p) \wedge Airport(from) \wedge Airport(to)$

EFFECT: $\neg At(p, from) \wedge At(p, to)$

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One solution: $[Load(C_1, P_1, SFO), Fly(P_1, SFO, JFK), Unload(C_1, P_1, JFK),$
 $Load(C_2, P_2, JFK), Fly(P_2, JFK, SFO), Unload(C_2, P_2, SFO)]$

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Example: Spare Tire Problem

$Init(Tire(Flat) \wedge Tire(Spare) \wedge At(Flat, Axle) \wedge At(Spare, Trunk))$
 $Goal(At(Spare, Axle))$
 $Action(Remove(obj, loc),$
 PRECOND: $At(obj, loc)$
 EFFECT: $\neg At(obj, loc) \wedge At(obj, Ground)$)
 $Action(PutOn(t, Axle),$
 PRECOND: $Tire(t) \wedge At(t, Ground) \wedge \neg At(Flat, Axle)$
 EFFECT: $\neg At(t, Ground) \wedge At(t, Axle)$)
 $Action(LeaveOvernight,$
 PRECOND:
 EFFECT: $\neg At(Spare, Ground) \wedge \neg At(Spare, Axle) \wedge \neg At(Spare, Trunk)$
 $\wedge \neg At(Flat, Ground) \wedge \neg At(Flat, Axle) \wedge \neg At(Flat, Trunk)$)

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(We assume that the car is parked in a particularly bad neighborhood, so that the effect of leaving it overnight is that the tires disappear.)

One solution: $[Remove(Flat, Axle), Remove(Spare, Trunk), PutOn(Spare, Axle)]$

Example: Spare Tire Problem

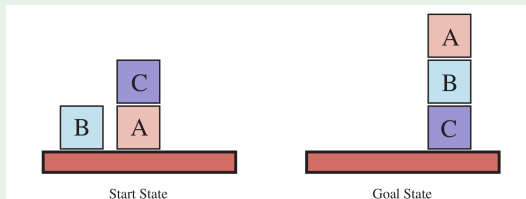
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Example: Blocks World

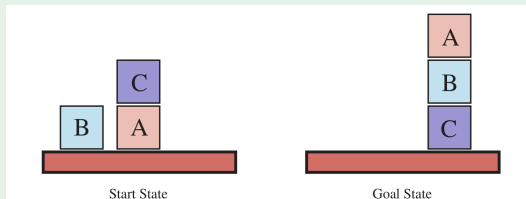


$Init(On(A, Table) \wedge On(B, Table) \wedge On(C, A)$
 $\wedge Block(A) \wedge Block(B) \wedge Block(C) \wedge Clear(B) \wedge Clear(C))$
 $Goal(On(A, B) \wedge On(B, C))$
 $Action(Move(b, x, y),$
 PRECOND: $On(b, x) \wedge Clear(b) \wedge Clear(y) \wedge Block(b) \wedge Block(y) \wedge$
 $(b \neq x) \wedge (b \neq y) \wedge (x \neq y),$
 EFFECT: $On(b, y) \wedge Clear(x) \wedge \neg On(b, x) \wedge \neg Clear(y))$
 $Action(MoveToTable(b, x),$
 PRECOND: $On(b, x) \wedge Clear(b) \wedge Block(b) \wedge (b \neq x),$
 EFFECT: $On(b, Table) \wedge Clear(x) \wedge \neg On(b, x))$

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One solution: $[MoveToTable(C, A), Move(B, Table, C), Move(A, Table, B)]$

Example: Blocks World



$Init(On(A, Table) \wedge On(B, Table) \wedge On(C, A)$
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 - The Problem
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- 2 **Search Strategies and Heuristics**
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 - Heuristics
- 3 Planning Graphs, Heuristics and Graphplan
 - Planning Graphs
 - Heuristics Driven by Planning Graphs
 - The Graphplan Algorithm
- 4 Other Approaches (hints)
 - Planning as SAT Solving

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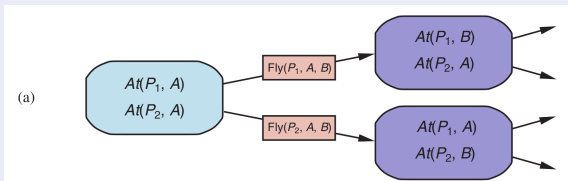
Two Main Approaches

(a) Forward search (aka progression search)

- start in the initial state
- use actions to search forward for a goal state

(b) Backward search (aka regression search)

- start from goals
- use reverse actions to search forward for the initial state



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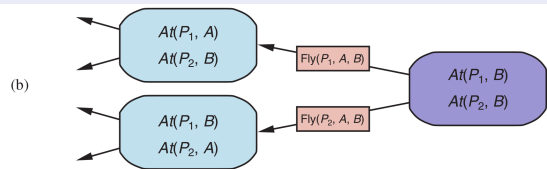
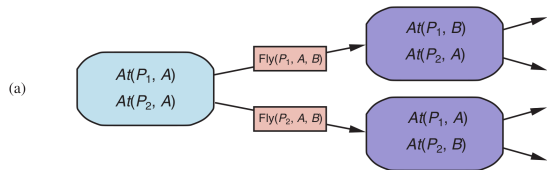
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Forward Search

- Forward search (aka progression search)

- choose actions whose preconditions are satisfied
- add positive effects, delete negative

- Goal test: does the state satisfy the goal?

- Step cost: each action costs 1

⇒ We can use any of the search algorithms from Ch. 03, 04

- need keeping track of the actions used to reach the goal

- Breadth-first and best-first

- Sound: if they return a plan, then the plan is a solution
- Complete: if a problem has a solution, then they will return one
- Require exponential memory wrt. solution length! ⇒ unpractical

- Depth-first search and greedy search

- Sound
- Not complete
 - may enter in infinite loops
 - (classical planning only): made complete by loop-checking
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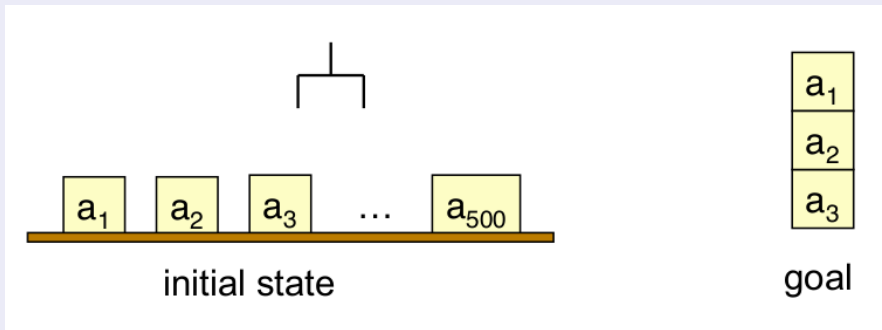
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Branching Factor of Forward Search

- Planning problems can have huge state spaces
- Forward search can have a very large branching factor
 - ex: *pickup*(a_1), *pickup*(a_2), ..., *pickup*(a_{500})

⇒ Forward-search can waste time trying lots of irrelevant actions

⇒ Need a good heuristic to guide the search

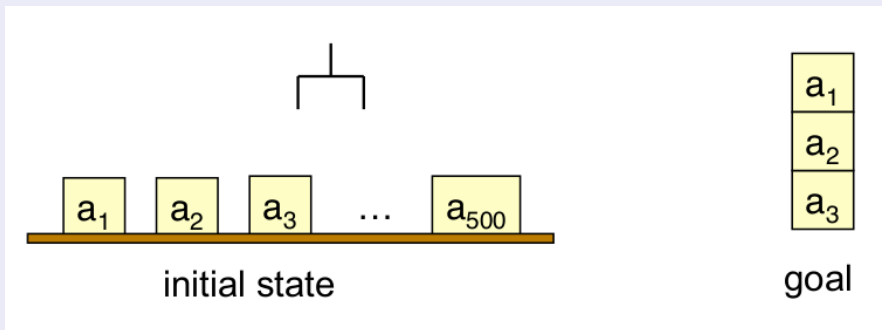


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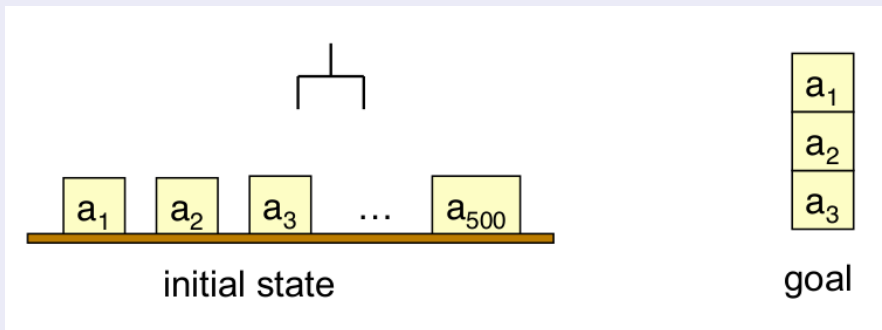


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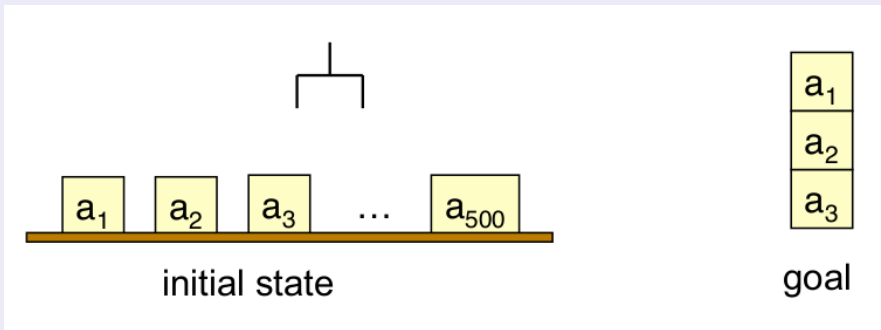


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Backward Search (aka Regression or Relevant-States)

- Predecessor (sub)goal g' of ground goal g via ground action a :

$$Pos(g') \stackrel{\text{def}}{=} (Pos(g) \setminus Add(a)) \cup Pos(Precond(a))$$

$$Neg(g') \stackrel{\text{def}}{=} (Neg(g) \setminus Del(a)) \cup Neg(Precond(a))$$

- Note: Both g and g' represent many states
 - irrelevant ground atoms unassigned

- Consider the goal $At(C_1, SFO) \wedge At(C_2, JFK)$
- Consider the ground action:
 $Action(Unload(C_1, P_1, SFO),$
 $PRECOND : In(C_1, P_1) \wedge At(P_1, SFO) \wedge Cargo(C_1) \wedge Plane(P_1) \wedge Airport(SFO)$
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Backward Search [cont.]

- Idea: deal with partially un-instantiated actions and states
 - avoid unnecessary instantiations
 - ⇒ no need to produce a goal for every possible instantiation
 - use the most general unifier ⇒ compute weakest precondition
 - standardize action schemata first (rename vars into fresh ones)

- Consider the goal $At(C_1, SFO) \wedge At(C_2, JFK)$
- Consider the partially-instantiated action:
 $Action(Unload(C_1, p', SFO),$
 $PRECOND : In(C_1, p') \wedge At(p', SFO) \wedge Cargo(C_1) \wedge Plane(p') \wedge Airport(SFO)$
 $EFFECT : At(C_1, SFO) \wedge \neg In(C_1, p'))$
- This produces the sub-goal g' :
 $In(C_1, p') \wedge At(p', SFO) \wedge Cargo(C_1) \wedge Plane(p') \wedge Airport(SFO) \wedge At(C_2, JFK)$
- Represents states with all possible planes
 - ⇒ no need to produce a subgoal for every plane $P_1, P_2, P_3, \dots$

Backward Search [cont.]

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- Represents states with all possible planes
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- Idea: deal with partially un-instantiated actions and states
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 - The Problem
 - The PDDL Language
- 2 **Search Strategies and Heuristics**
 - Forward and Backward Search
 - **Heuristics**
- 3 Planning Graphs, Heuristics and Graphplan
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 - The Graphplan Algorithm
- 4 Other Approaches (hints)
 - Planning as SAT Solving

Heuristics for (Forward-Search) Planning

A* for Planning

- Recall: A* is a best-first algorithm which
 - uses an **evaluation function** $f(s) = g(s) + h(s)$,
 - $g(s)$: (exact) cost to reach s
 - $h(s)$: **admissible (optimistic) heuristics**
(never overestimates the distance to the goal)
- A technique for admissible heuristics: **problem relaxation**
 $\implies h(s)$: the exact cost of a solution to the relaxed problem
- Forms of problem relaxation exploiting problem structure
 - Add arcs to the search graph $\implies$ make it easier to search
 - ignore-preconditions heuristics
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Ignore (some) Preconditions Heuristics

- **Ignore all preconditions** drops all preconditions from actions
 - every action is applicable in any state
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Ignore-Preconditions Heuristics: Example

Sliding tiles

Action(*Slide*(*t*, *s*₁, *s*₂),

PRECOND : *Tile*(*t*) $\wedge$ *Blank*(*s*₂) $\wedge$ *On*(*t*, *s*₁) $\wedge$ *Adjacent*(*s*₁, *s*₂)

EFFECT : *On*(*t*, *s*₂) $\wedge$ *Blank*(*s*₁) $\wedge$ $\neg$ *On*(*t*, *s*₁) $\wedge$ $\neg$ *Blank*(*s*₂)

- Remove the preconditions *Blank*(*s*₂) $\wedge$ *Adjacent*(*s*₁, *s*₂)

$\Rightarrow$ we get the **number-of-misplaced-tiles** heuristics

- Remove the precondition *Blank*(*s*₂)

$\Rightarrow$ we get the **Manhattan-distance** heuristics

7	2	4
5		6
8	3	1

Start State

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Goal State

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Ignore Delete-list Heuristics

- Assumption: goals & preconditions contain only positive literals
 - reasonable in many domains
- Idea: Remove the delete lists from all actions
 - No action will ever undo the effect of actions,
⇒ there is a monotonic progress towards the goal
- Still NP-hard to find the optimal solution of the relaxed problem
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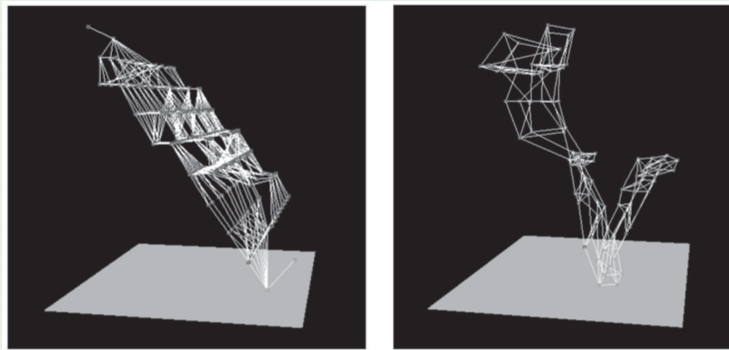
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Ignore Delete-list Heuristics: Example (Hoffmann'05)

- Planning state spaces with ignore-delete-lists heuristic
 - height above the bottom plane is the heuristic score of a state
 - states on the bottom plane are goals

⇒ No local minima, non dead-ends, non backtracking

⇒ Search for the goal is straightforward for hill-climbing

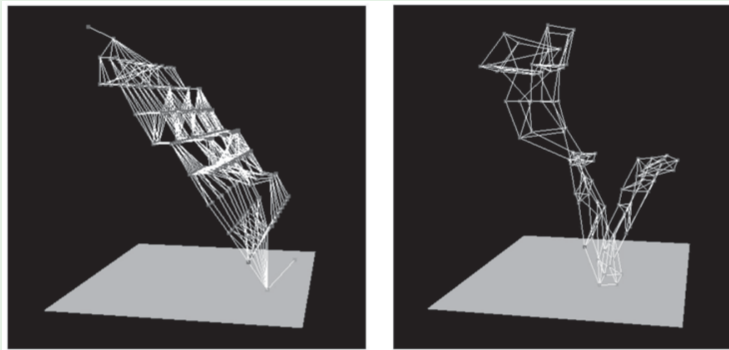


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State Abstractions

- Many-to-one mapping from states in the ground/original representation of the problem to a more abstract representation
 - drastically reduces the number of states
- Common strategy: ignore some (less-relevant) fluents
 - drop k fluents $\Rightarrow$ reduce search space by 2^k factors
 - relevance based on (heuristic) evaluation or domain knowledge

- Air cargo problem: 10 airports, 50 planes, 200 pieces of cargo
 $\Rightarrow 10^{50} \cdot (50 + 10)^{200} \approx 10^{405}$ states (*)
- Consider particular problem in that domain
 - all packages are at 5 airports
 - all packages at a given airport have the same destination
- Abstraction: drop all "At" fluents except for these involving one plane and one package at each of the the 5 airports
 $\Rightarrow 10^5 \cdot (5 + 10)^5 \approx 10^{11}$ states (*)
 - abstract solution shorter than ground solutions $\Rightarrow$ admissible
 - abstract solution easy to extend: add Load and Unload actions

(*) wrong in AIMA III Ed, corrected in later editions

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• Abstraction: drop all "At" fluents except for these involving one plane and one package at each of the the 5 airports

$\implies 10^5 \cdot (5 + 10)^5 \approx 10^{11}$ states (*)

- abstract solution shorter than ground solutions $\implies$ admissible
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State Abstractions

- Many-to-one mapping from states in the ground/original representation of the problem to a more abstract representation
 - drastically reduces the number of states
- Common strategy: **ignore some (less-relevant) fluents**
 - drop k fluents $\implies$ reduce search space by 2^k factors
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Other Strategies for Planning

Other strategies to define heuristics

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 - “divide & conquer” problem into subproblem
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Planning Graph

- A data structure which is a rich source of information:
 - can be used to give **better heuristic estimates** $h(s)$
 - can drive an algorithm called **Graphplan**
- A **polynomial-size over-approximation to the (exponential) search tree**
 - can be constructed very quickly
- cannot answer definitively if goal g is reachable from initial state
- + may discover that the goal is not reachable
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Planning Graph: Definition

- A directed graph, built **forward** and organized into **levels**
 - **level S_0** : contain each ground fluent that holds in the initial state
 - **level A_0** : contains each ground action with preconditions in S_0 (i.e. applicable in S_0)
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 - **level A_i** : contains all ground actions with preconditions in S_i
 - **level S_{i+1}** : all the effects of all the actions in A_i
 - each S_i may contain both P_j and $\neg P_j$

until $S_N = S_{N-1}$ ("leveled off").

- Contains **persistence actions** (aka **maintenance actions**, **no-ops**)
 - say that a literal l persists if no action negates it
- **Mutual exclusion links (mutex)** connect
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Planning Graph: Example

Init(Have(Cake))

Goal(Have(Cake) $\wedge$ Eaten(Cake))

Action(Eat(Cake))

PRECOND: *Have(Cake)*

EFFECT: $\neg$ *Have(Cake)* $\wedge$ *Eaten(Cake)*

Action(Bake(Cake))

PRECOND: $\neg$ *Have(Cake)*

EFFECT: *Have(Cake)*

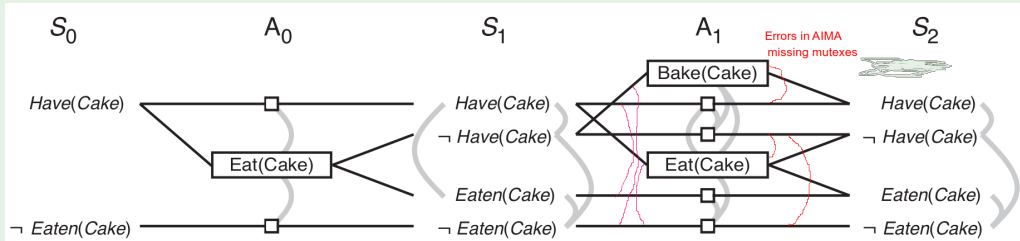
*You would like to eat your cake and still have a cake.
Fortunately, you can bake a new one.*

Rectangles indicate actions

Small squares persistence actions (**no-ops**)

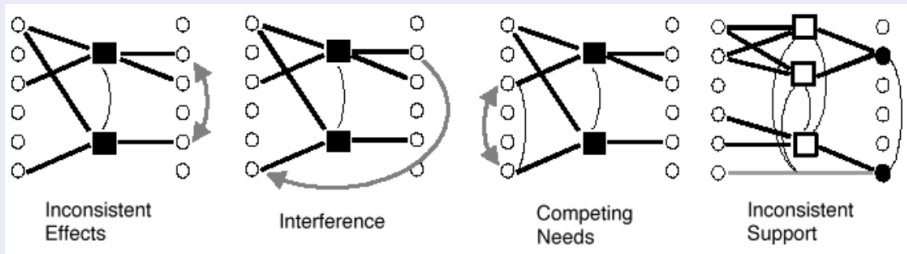
Straight lines indicate preconditions
and effects

Mutex links are shown as curved gray lines



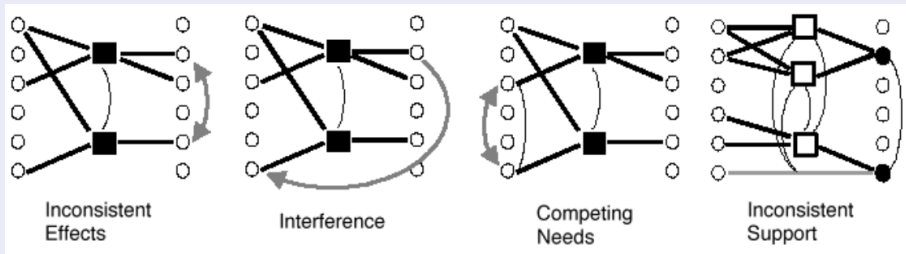
Mutex Computation

- Two **actions** at the same action-level have a mutex relation if
 - **Inconsistent effects**: an effect of one negates an effect of the other
 - **Interference**: one deletes a precondition of the other
 - **Inconsistent preconditions** (aka **competing needs**): they have mutually exclusive preconditions
- Otherwise they don't interfere with each other
⇒ both may appear in a solution plan
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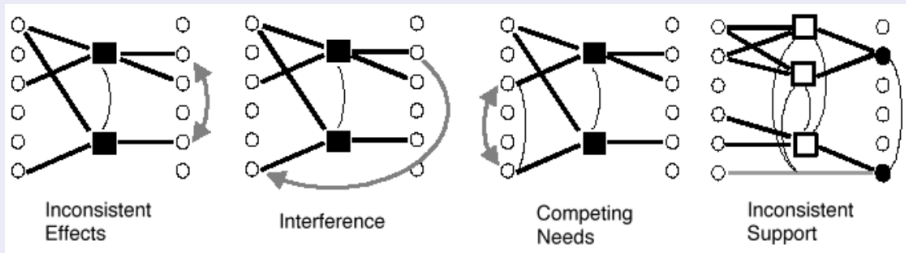
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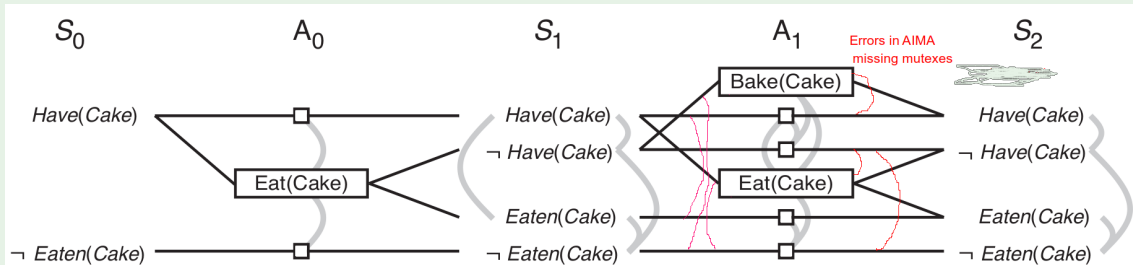
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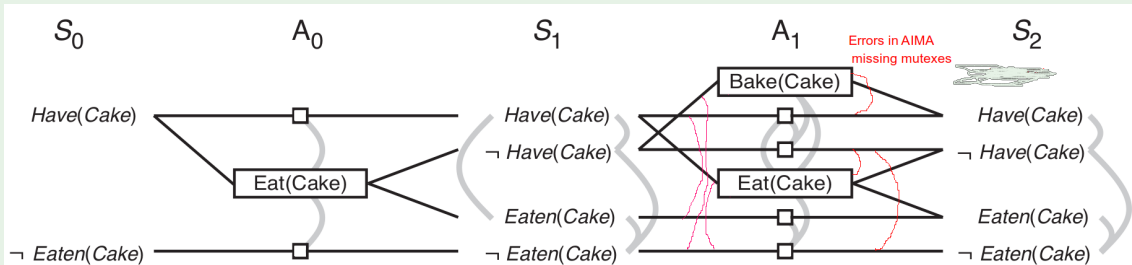
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- Two **actions** at the same action-level have a mutex relation if
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ex: *Bake(Cake)*, *Eat(Cake)* have competing effects
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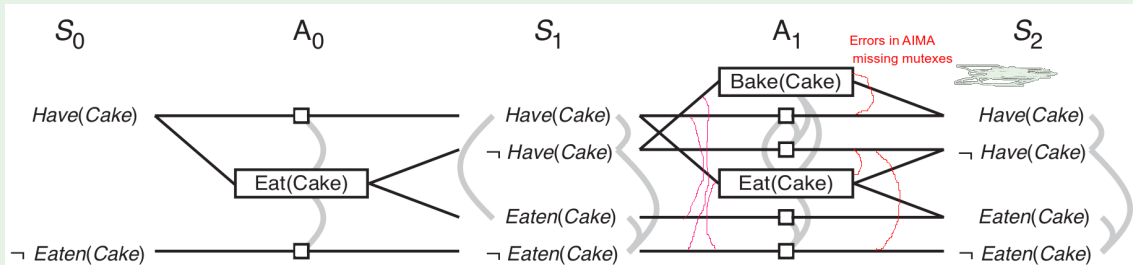
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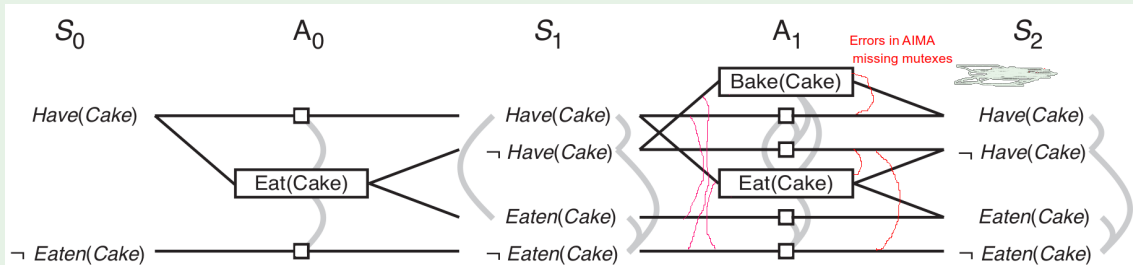
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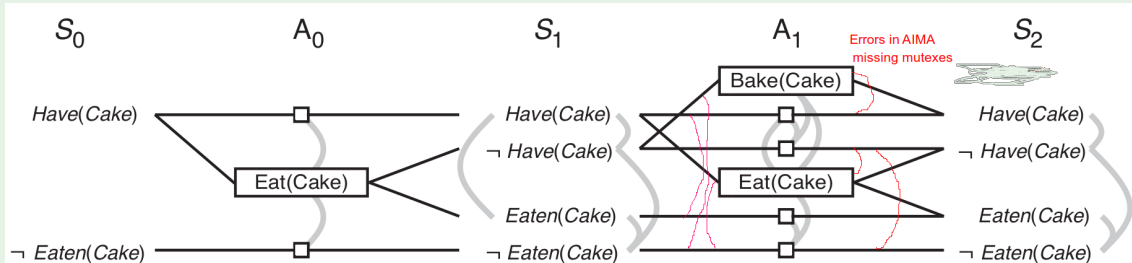
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Mutex Computation: Example [cont.]

- Two **literals** at the same state-level have a mutex relation if
 - inconsistent support**: one is the negation of the other
ex.: $Have(Cake)$, $\neg Have(Cake)$
 - all ways of achieving them are pairwise mutex
ex.: (S_1): $Have(Cake)$ in mutex with $Eaten(Cake)$
because persistence of $Have(Cake)$, $Eat(Cake)$ are mutex



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Building of the Planning Graph

Create initial layer S_0 :

- 1 insert into S_0 all literals in the initial state

Repeat for increasing values of $i = 0, 1, 2, \dots$:

Create action layer A_i :

- 1 for each action schema, for each way to unify its preconditions to **non-mutually exclusive** literals in S_i , enter an action node into A_i
- 2 for every literal in S_i , enter a no-op action node into A_i
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Create state layer S_{i+1} :

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 - add to S_{i+1} the fluents in his Add list, linking them to a
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Planning Graphs: Properties

- Literals and actions increase monotonically and are finite
⇒ we eventually reach a level where they stabilize
 - Mutexes decrease monotonically (and cannot become less than zero)
⇒ they too eventually must level off
- ⇒ When we reach this stable state, if one of the goal literals is missing or is mutex with another goal literal, then it will remain so
⇒ we can stop

Planning Graphs: Complexity

- A planning graph is polynomial in the size of the problem:
 - a graph with n levels, a actions, l literals, has size $O(n(a + l)^2)$
 - time complexity is also $O(n(a + l)^2)$

⇒ The process of constructing the planning graph is very fast

- does not require choosing among actions

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Planning Graphs for Heuristic Estimation

Information provided by Planning Graphs

- Each level S_i represents a set of possible belief states
 - two literals connected by a mutex belong to different belief state
- A literal not appearing in the final level of the graph cannot be achieved by any plan
 - ⇒ if a goal literal is not in the final level, the problem is unsolvable
- The level S_j a literal l appears first is never greater than the level it can be achieved in a plan
 - j is called the level cost of literal l
- the level cost of a literal g_i in the graph constructed starting from state s , is an estimate of the cost to achieve it from s (i.e. $h(g)$)
 - this estimate is admissible
 - ex: from s_0 Have(cake) has cost 0 and Eaten(cake) has cost 1
- Planning graph admits several actions per level
 - ⇒ inaccurate estimate
- **Serialization**: enforcing only one action per level (adding mutex)
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 - j is called the level cost of literal l
- the level cost of a literal g_j in the graph constructed starting from state s , is an estimate of the cost to achieve it from s (i.e. $h(g)$)
 - this estimate is admissible
 - ex: from s_0 Have(cake) has cost 0 and Eaten(cake) has cost 1
- Planning graph admits several actions per level
 - ⇒ inaccurate estimate
- Serialization: enforcing only one action per level (adding mutex)
 - ⇒ better estimate

Planning Graphs for Heuristic Estimation

Information provided by Planning Graphs

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Planning Graphs for Heuristic Estimation [cont.]

Estimating the heuristic cost of a conjunction of goal literals

- **Max-level heuristic**: the maximum level cost of the sub-goals
 - admissible
- **Level-sum heuristic**: the sum of the level costs of the goals
 - inadmissible only if goals are independent,
 - it may work well in practice
- **Set-level heuristic**: the level at which all goal literals appear together, without pairwise mutexes
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The Graphplan Algorithm

- A strategy for extracting a plan from the planning graph
- Repeatedly adds a level to a planning graph (EXPAND-GRAPH)
- If all the goal literals occur in last level and are non-mutex
 - search for a plan that solves the problem (EXTRACT-SOLUTION)
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- If graph and nogoods have both leveled off then return failure
- Depends on EXPAND-GRAPH & EXTRACT-SOLUTION

function GRAPHPLAN(*problem*) **returns** solution or failure

graph $\leftarrow$ INITIAL-PLANNING-GRAPH(*problem*)

goals $\leftarrow$ CONJUNCTS(*problem*.GOAL)

nogoods $\leftarrow$ an empty hash table

for $t = 0$ **to** ∞ **do**

type in
AIMA
book **if** *goals* all non-mutex in S_t of *graph* **then**

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[Recall] Example: Spare Tire Problem

$Init(Tire(Flat) \wedge Tire(Spare) \wedge At(Flat, Axle) \wedge At(Spare, Trunk))$
 $Goal(At(Spare, Axle))$
 $Action(Remove(obj, loc),$
 PRECOND: $At(obj, loc)$
 EFFECT: $\neg At(obj, loc) \wedge At(obj, Ground)$)
 $Action(PutOn(t, Axle),$
 PRECOND: $Tire(t) \wedge At(t, Ground) \wedge \neg At(Flat, Axle)$
 EFFECT: $\neg At(t, Ground) \wedge At(t, Axle)$)
 $Action(LeaveOvernight,$
 PRECOND:
 EFFECT: $\neg At(Spare, Ground) \wedge \neg At(Spare, Axle) \wedge \neg At(Spare, Trunk)$
 $\wedge \neg At(Flat, Ground) \wedge \neg At(Flat, Axle) \wedge \neg At(Flat, Trunk)$)

(© S. Russell & P. Norwig, AIMA)

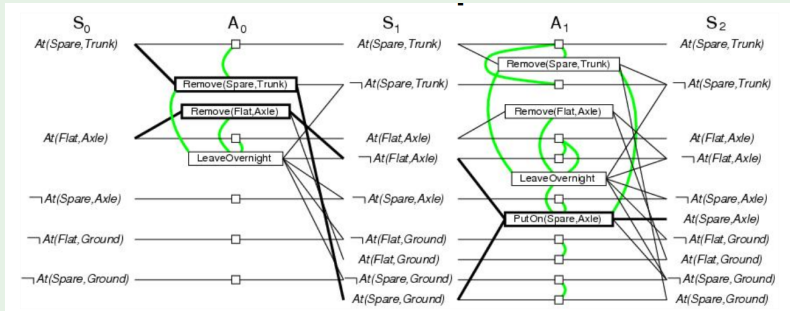
(We assume that the car is parked in a particularly bad neighborhood, so that the effect of leaving it overnight is that the tires disappear.)

One solution: $[Remove(Flat, Axle), Remove(Spare, Trunk), PutOn(Spare, Axle)]$

Graphplan: Example

Spare Tire problem

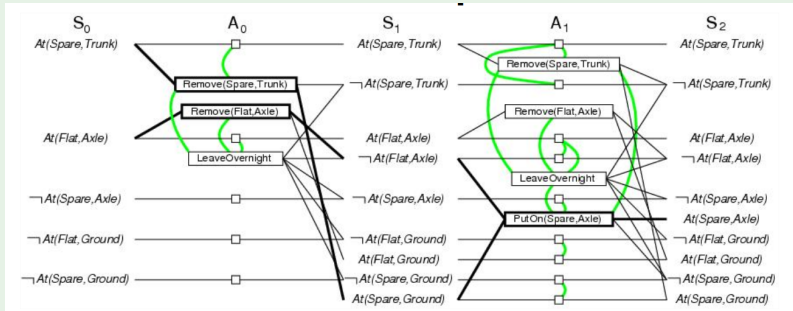
- Initial plan 5 literals from initial state and the Closed-World-Assumption literals (S_0).
 - fixed literals (e.g. *Tire(Flat)*) ignored here
 - irrelevant literals ignored here
- Goal *At(Spare, Axle)* not present in S_0
 $\Rightarrow$ no need to call EXTRACT-SOLUTION
- Graph and nogoods not leveled off $\Rightarrow$ invoke EXPAND-GRAPH



Graphplan: Example [cont.]

Spare Tire problem

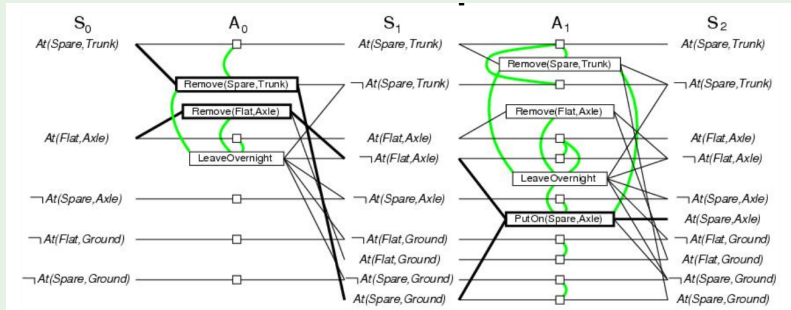
- Invoke EXPAND-GRAPH
 - add actions A_0 , persistence actions and mutexes
 - add fluents S_1 and mutexes
- Goal $At(Spare, Axle)$ not present in S_1
⇒ no need to call EXTRACT-SOLUTION
- Graph and nogoods not leveled off ⇒ invoke EXPAND-GRAPH



Graphplan: Example [cont.]

Spare Tire problem

- Invoke EXPAND-GRAPH
 - add actions A_1 , persistence actions and mutexes
 - add fluents S_2 and mutexes
- Goal $At(Spare, Axle)$ present in S_2
 - call EXTRACT-SOLUTION
- **Solution found!**



Exercise

- Consider the following variant of the Spare Tire problem:
add $At(Flat, Trunk)$ to the goal
- Write the (non-serialized) planning graph
- Extract a plan from the graph
- Do the same with the serialized planning graph

The Graphplan Algorithm [cont.]

Graphplan “family” of algorithms, depending on approach used in EXTRACT-SOLUTION(...)

About EXTRACT-SOLUTION(...)

- Can be formulated as an (incremental) SAT problem
 - one proposition for each ground action and fluent
 - clauses represent preconditions, effects, no-ops and mutexes
- Can be formulated as a backward search problem
- Planning problem restricted to planning graph
 - mutexes found by EXPAND-GRAPH prune paths in the search tree

⇒ much faster than unrestricted planning
- (if P.G. not serialized) may produce partial order plans
 - ⇒ may be later serialized into a total-order plan

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Partial-Order Plans

Partial-Order vs. Total-Order Plans

- **Total-order plans:** strictly linear sequences of actions
 - disregards the fact that some action are mutually independent
- **Partial-order plans:** set of precedence constraints between action pairs
 - form a directed acyclic graph
 - longest path to goal may be much shorter than total-order plan
 - easily converted into (possibly many) distinct total-order plans
(any possible interleaving of independent actions)

Partial-Order Plans

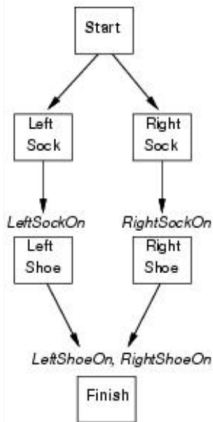
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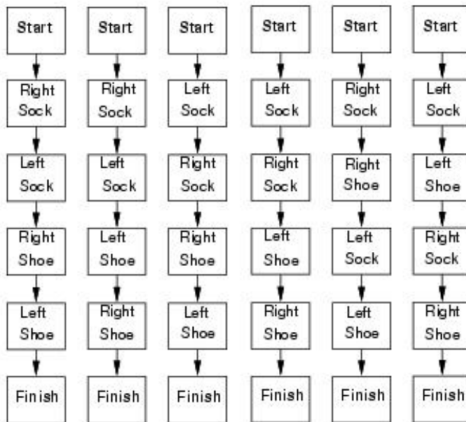
Partial-Order Plans: Example

Socks & Shoes Examples

Partial Order Plan:



Total Order Plans:



Termination of Graphplan

- Theorem: If the graph and the no-goods have both leveled off, and no solution is found we can safely terminate with failure
- Intuition (proof sketch):
 - Literals and actions increase monotonically and are finite
⇒ we eventually reach a level where they stabilize
 - Mutexes and no-goods decrease monotonically (and cannot become less than zero)
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- ⇒ When we reach this stable state, if one of the goal literals is missing or is mutex with another goal literal, then it will remain so
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Exercise

- Socks & Shoes example:
 - 1 Formalize the Socks & Shoes example in PDDL
 - 2 Write the non-serialized planning graph
 - 3 Compute the level cost for every fluent
 - 4 Choose some states, compute $h(s)$ using the three heuristics
 - 5 Extract a plan from the graph in (2)
 - 6 Compare $h(s)$ with the level they occur in the plan
 - 7 Write the serialized planning graph
 - 8 Repeat steps (3)-(6) with the serialized graph
- Do same steps (1)-(8) for the Air Cargo Transport example

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Planning as SAT Solving

- Encode bounded planning problem into a propositional formula

⇒ Solve it by (incremental) calls to a SAT solver

- A model for the formula (if any) is a plan of length t
- Many variants in the encoding
- Extremely efficient with many problems of interest

function SATPLAN(*init*, *transition*, *goal*, $T_{\max}$) **returns** solution or failure

inputs: *init*, *transition*, *goal*, constitute a description of the problem

$T_{\max}$, an upper limit for plan length

for $t = 0$ **to** $T_{\max}$ **do**

$cnf \leftarrow \text{TRANSLATE-TO-SAT}(init, transition, goal, t)$

$model \leftarrow \text{SAT-SOLVER}(cnf)$

if $model$ is not null **then**

return EXTRACT-SOLUTION($model$)

return *failure*

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Planning as SAT Solving [cont.]

- **TRANSLATE-TO-SAT**(INIT, TRANSITION, GOAL, T):

- ground fluents & actions at each step **are propositionalized**

- ex: $\langle At(P_1, SFO), 3 \rangle \implies At_P1_SFO_3$

- ex: $\langle Fly(P_1, SFO, JFK), 3 \rangle \implies Fly_P1_SFO_JFK_3$

- returns propositional formula: $Init^0 \wedge (\bigwedge_{i=1}^{t-1} Transition^{i,i+1}) \wedge Goal^t$

- $Init^0$ and $Goal^t$: conjunctions of literals at step 0 and t resp.

- ex: $Init^0: At_P1_SFO_0 \wedge At_P2_JFK_0 (\wedge \neg At_P1_JFK \wedge \neg At_P2_SFO \wedge \dots)$

- ex: $Goal^3: At_P1_JFK_3 \wedge At_P2_SFO_3$

- $Transition^{i,i+1}$: encodes transition from steps i to $i + 1$

- Actions: $Action^i \rightarrow (Precond^i \wedge Effects^{i+1})$

- ex: $Fly_P1_SFO_JFK_2 \rightarrow (At_P1_SFO_2 \wedge At_P1_JFK_3)$

- No-Ops: for each fluent F and step i :

$$F^{i+1} \leftrightarrow \bigvee_k ActionCausingF_k^i \vee (F^i \wedge \bigwedge_j \neg ActionCausingNotF_j^i)$$

- Mutex constraints: $\neg Action_1^i \vee \neg Action_2^i$

- ex: $\neg Fly_P1_SFO_JFK_2 \vee \neg Fly_P1_SFO_Newark_2$

- If serialized: add mutex between each pair of actions at each step

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- ground fluents & actions at each step **are propositionalized**

- ex: $\langle At(P_1, SFO), 3 \rangle \implies At_P1_SFO_3$

- ex: $\langle Fly(P_1, SFO, JFK), 3 \rangle \implies Fly_P1_SFO_JFK_3$

- returns propositional formula: $Init^0 \wedge (\bigwedge_{i=1}^{t-1} Transition^{i,i+1}) \wedge Goal^t$

- **Init**⁰ and **Goal**^t: conjunctions of literals at step 0 and t resp.

- ex: $Init^0: At_P1_SFO_0 \wedge At_P2_JFK_0 (\wedge \neg At_P1_JFK \wedge \neg At_P2_SFO \wedge \dots)$

- ex: $Goal^3: At_P1_JFK_3 \wedge At_P2_SFO_3$

- **Transition**^{i,i+1}: **encodes transition from steps i to i + 1**

- Actions: $Action^i \rightarrow (Precond^i \wedge Effects^{i+1})$

- ex: $Fly_P1_SFO_JFK_2 \rightarrow (At_P1_SFO_2 \wedge At_P1_JFK_3)$

- No-Ops: for each fluent F and step i:

$$F^{i+1} \leftrightarrow \bigvee_k ActionCausingF_k^i \vee (F^i \wedge \bigwedge_j \neg ActionCausingNotF_j^i)$$

- Mutex constraints: $\neg Action_1^i \vee \neg Action_2^i$

- ex: $\neg Fly_P1_SFO_JFK_2 \vee \neg Fly_P1_SFO_Newark_2$

- If serialized: add mutex between each pair of actions at each step

Exercise

Consider the socks & shoes example

- Translate it into SAT for $t=0,1,2$
 - non serialized
 - no need to propositionalize: treat ground atoms as propositions
 - no need to CNF-ize here (human beings don't like CNFs)
- Find a model for the formula
- Convert it back to a plan

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