

Course “**Fundamentals of Artificial Intelligence**”
EXAM TEXT

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1

Consider first-order-logic (FOL); let P, R, Q be predicates; let y, x be variables.
For each of the following statements, say if it is true or false.

- (a) $\forall y.P(y) \wedge \forall x.R(x) \models \forall y.(P(y) \wedge R(y))$
- (b) $\exists y\forall x\exists z.(P(y, x) \rightarrow R(y, z))$ is equivalent to $\neg P(C_1, x) \vee R(C_1, F_1(x))$ for some Skolem constant C_1 and function F_1
- (c) $\exists y.(P(y) \vee R(y))$ is equivalent to $(\exists y.P(y)) \vee (\exists x.R(x))$
- (d) $\forall y.\exists x.Q(y, x) \models \exists x.\forall y.Q(y, x)$

[SCORING [0...100]:

- +25pts for each correct answer
- -25pts for each incorrect answer
- 0pts for each unanswered question

]

2

For each of the following facts about conditional independence, say if it is true or false.

- (a) $\mathbf{P}(D \mid E) = \mathbf{P}(E \mid D)$ if and only if $\mathbf{P}(D) = \mathbf{P}(E)$.
- (b) If D and E are conditionally independent given F , then $\mathbf{P}(D, E, F) = \mathbf{P}(D)\mathbf{P}(E \mid F)\mathbf{P}(F)$
- (c) If D and E are conditionally independent given F , then $\mathbf{P}(D \mid E, F) = \mathbf{P}(D \mid F)$.
- (d) $\mathbf{P}(D, E) = \mathbf{P}(D \mid E)\mathbf{P}(E)$ always holds.

[SCORING [0...100]:

- +25pts for each correct answer
- -25pts for each incorrect answer
- 0pts for each unanswered question

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3

Given a generical search problem, assume time and space complexity are measured in terms of

b : maximum branching factor of the search tree

m : maximum depth of the state space (assume m is finite)

d : depth of the shallowest solution

Assume also that all steps cost are 1.

For each of the following facts about Iterative-Deepening Search, say if it is true or false.

- (a) Iterative-Deepening Search is optimal
- (b) Iterative-Deepening Search requires $O(b^d)$ memory to find a solution.
- (c) Iterative-Deepening Search is complete.
- (d) Iterative-Deepening Search requires $O(b^m)$ time to find a solution.

[SCORING [0...100]:

- +25pts for each correct answer
- -25pts for each incorrect answer
- 0pts for each unanswered question

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4

Consider propositional logic (PL); let F, C, D, E, B, A, G be atomic propositions
For each of the following statements, say if it is true or false.

- (a) The subformula $(\neg F \vee C)$ occurs only positively in the formula $\neg((\neg F \vee C) \rightarrow D)$
- (b) The subformula $\neg D$ occurs only positively in the formula $((\neg F \vee E) \rightarrow \neg D)$
- (c) The subformula $(F \rightarrow C)$ occurs only negatively in the formula $((F \rightarrow C) \rightarrow D)$
- (d) The subformula $(F \wedge \neg C)$ occurs only negatively in the formula $\neg(D \leftrightarrow (F \wedge \neg C))$

[SCORING [0...100]:

- +25pts for each correct answer
- -25pts for each incorrect answer
- 0pts for each unanswered question

]

5

Consider the semantics of “ $\alpha \rightarrow \beta$ ” (“ α implies β ”). Consider:

- the unary predicates *Italian()*, *French()*, *Russian()*, *American()*, denoting the corresponding nationalities
- the unary predicates *Odd()*, *Even()* denoting the corresponding mathematical concepts,
- the constants 0, 1, 2, 3, 4, 5, 6, 7, ... denoting the corresponding integer numbers
- the constants *SergioMattarella*, *EmmanuelMacron*, *VladimirPutin*, *DonaldTrump*, denoting names of famous presidents (of Italy, France, Russia and USA respectively).

For each of the following FOL ground formulas, say whether it is true or false under a model which gives the standard interpretation of the above predicates and constants (that is, the interpretation which maps the constant “VladimirPutin” into the Russian president, the predicate “Italian()” into the property of being Italian, etc.).

(a) $Even(4) \rightarrow American(SergioMattarella)$.

(b) $American(SergioMattarella) \rightarrow Even(4)$

(c) $American(SergioMattarella) \rightarrow Even(5)$.

(d) $French(EmmanuelMacron) \rightarrow Even(4)$.

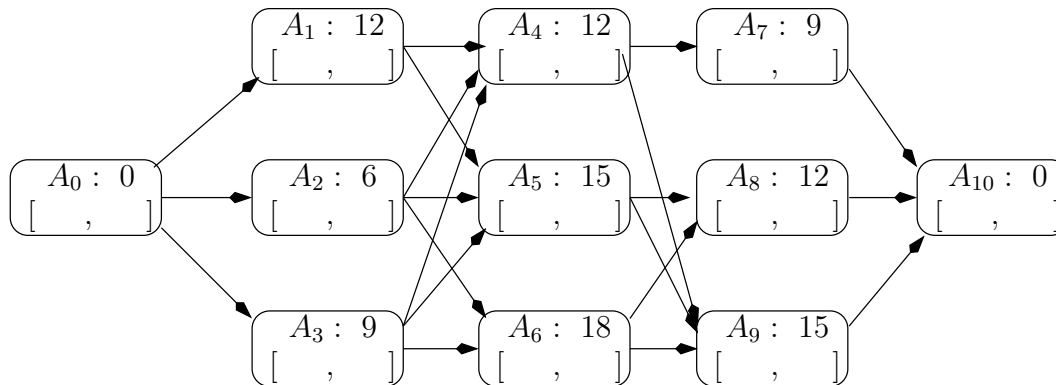
[SCORING [0...100]:

- +25pts for each correct answer
- -25pts for each incorrect answer
- 0pts for each unanswered question

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6

Consider the following actions with durations and dependencies:



Using the Critical Path method:

- Compute the earliest / latest possible start time [ES, LS] for each action.
(Fill them into the “[,]” fields in the nodes.).
- Indicate all the actions that are in (at least one) critical path and the minimum makespan.

[SCORING: [0...100], 75pts for (a), 25 pts for (b). -20pts for each error in the resolution process, although the final score cannot go lower than 0..]

7

An email provider is tuning a spam filter. From historical data, 25% of incoming emails are spam. For each incoming email, the filter checks three simple indicators:

Indicator #1: the email contains the word “free”;

Indicator #2: the email contains a hyperlink;

Indicator #3: the email comes from an unknown sender.

The filter treats the three indicators as a Naive Bayes model, i.e. it assumes they are conditionally independent given whether the email is spam or not, and from past data it has learned the following statistics: ¹

$p(\text{indicator \#1} \mid \text{spam})$	$= 0.55$
$p(\text{indicator \#1} \mid \neg \text{spam})$	$= 0.05$
$p(\text{indicator \#2} \mid \text{spam})$	$= 0.65$
$p(\text{indicator \#2} \mid \neg \text{spam})$	$= 0.15$
$p(\text{indicator \#3} \mid \text{spam})$	$= 0.75$
$p(\text{indicator \#3} \mid \neg \text{spam})$	$= 0.25$

A new email arrives: it contains a hyperlink and comes from an unknown sender, but it does **not** contain the word “free”. Compute the probability that this email is spam.

[SCORING: [0...100], 100pts for a fully correct solution, -10pts for each error in the resolution process. The score cannot go below 0..]

¹The data here are pure fantasy and are not supposed to correspond to the behaviour of any real spam filter.

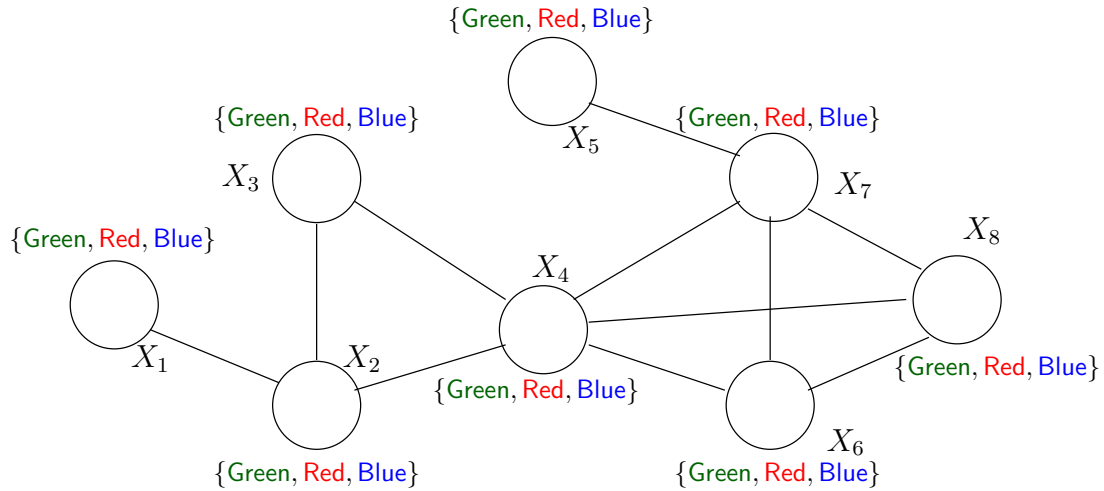
8

- (a) Describe as Pseudo-Code the Recursive Depth-first Tree-based And-Or-Search procedure for returning a conditional plan with non-deterministic actions.
- (b) If, for some current path and state, a recursive call to OR-Search returns “failure”, then this means that, starting from that state and path:
 - (i) there is no solution
 - (ii) if there is a non-cyclic solution, then it must be reachable from the earlier incarnation of the current state

[SCORING: [0...100], 75 pts for a correct answer to question (a), 25 pts for correct answer to question (b); no penalties for wrong answers.]

9

Consider the following constraint graph of a map coloring problem, with domain $D \stackrel{\text{def}}{=} \{\text{Green}, \text{Red}, \text{Blue}\}$.



Repeat the following steps:

1. Pick a node and a value according to the following criteria:
use the Minimum Remaining Values (MRV) heuristic; in suborder, as a tie breaker, use the degree (DH) heuristic; in suborder, as a tie breaker, pick the leftmost node.
Pick a value according to the Least Constraining Value (LCS) heuristic. In suborder, as a tie breaker, use the following order: $\{\text{Green}, \text{Red}, \text{Blue}\}$.
2. Apply the formal Forward Checking procedure.

until either you find one solution, or some node has an empty domain.

At every step, show the domain of the nodes X_i whose domain has been updated.

Notation: to represent the current domain of a node X_i , substitute with a blank “ ” any value in $\{\text{Green}, \text{Red}, \text{Blue}\}$ which cannot be assigned.

[SCORING: [0...100], 100 pts for correct answer. No penalties for wrong answers..]

10

Given the following set of propositional clauses Γ :

$$\begin{aligned} & (F \vee \neg B \vee G) \\ & (C \vee \neg D) \\ & (\neg D \vee G \vee B) \\ & (F \vee C) \\ & (D \vee H \vee \neg I) \\ & (\neg C \vee \neg D \vee E) \\ & (\neg F \vee \neg C) \\ & (C \vee \neg H \vee I) \\ & (F \vee \neg C \vee D) \\ & (F \vee \neg C \vee \neg G) \\ & (\neg F) \end{aligned}$$

Produce a PL-resolution proof that Γ is unsatisfiable, based on unit-resolution only.

Such proof must be written as a sequence of resolution steps in the form:

$$\begin{aligned} & [Clause_{11}, Clause_{12}] \implies Resolvent_Clause_1; \\ & [Clause_{21}, Clause_{22}] \implies Resolvent_Clause_2; \\ & \dots; \\ & [Clause_{k1}, Clause_{k2}] \implies Resolvent_Clause_k; \end{aligned}$$

s.t. $Resolvent_Clause_k$ is the empty clause, and each $Clause_{ij}$ is either in Γ or is a resolvent clause $Resolvent_Clause_m$ resulting from previous steps, i.e. s.t. $m < i$.

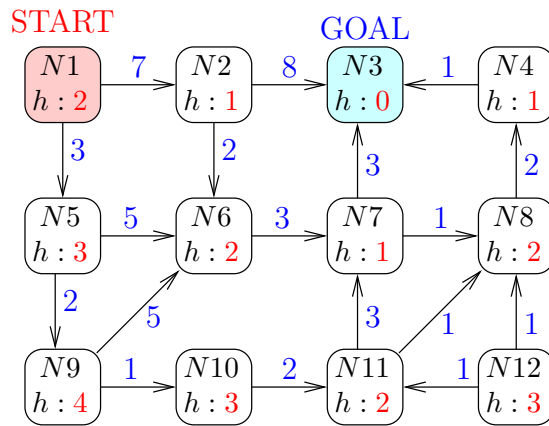
WRITE THE CLAUSES EXPLICITLY, NOT SIMPLY THEIR INDEXES!!

[SCORING: [0...100], 100 pts for a correct answer, no penalties for wrong answers.]

11

Apply A* to the following search problem, where

- the costs of the actions is denoted on the edges;
- the heuristic function is the Manhattan distance to the goal relative to the position in the grid (e.g., if the goal is in position (2,2), then the node n in position (1,0) has $h(n) = 3$);
- in case of same-score nodes, the oldest one is extracted from the frontier.



- (a) [10 pt. /100] Is the heuristic function admissible? (Motivate your answer).
- (b) [90 pt. /100] For each step of the execution, report:
- The node extracted from the priority queue.
 - The nodes which are added to the priority queue *after* expanding.

Then report the path returned by A*.

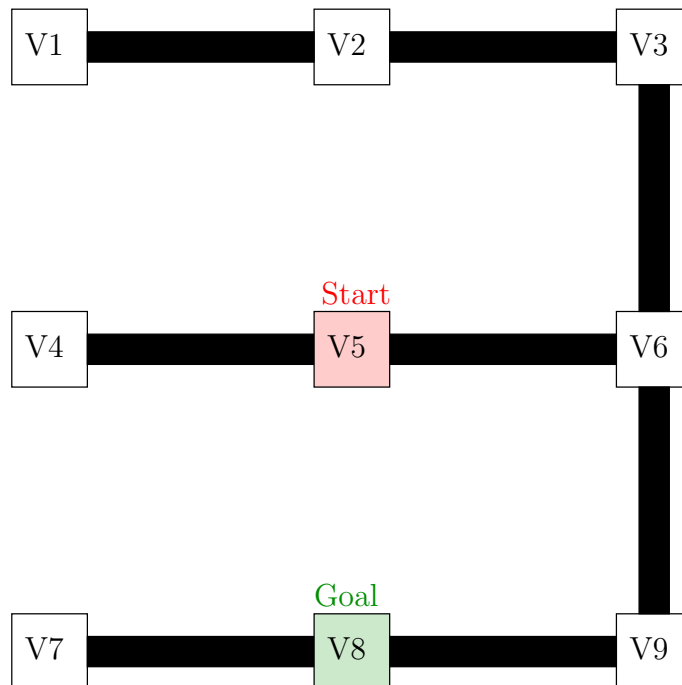
[SCORING: [0...100], 100pts for a fully correct solution, -10pts for each error in the resolution process. The score cannot go below 0..]

12

In the following state graph, apply *online DFS* and report the sequence of visited states, including repetitions (e.g. “ $V1 \rightarrow V2 \rightarrow V3 \Rightarrow V2 \dots$ ”).

The start is $V5$, the goal is $V8$.

The order of (untried) actions is $[up, left, down, right]$.



Important: in the reported sequence, you must distinguish exploration steps from backtracking steps.

- mark as a single-line arrow “ $\rightarrow$ ” an exploration step
- mark as a double-line arrow “ $\Rightarrow$ ” a backtracking step.

For instance, the sequence “ $V1 \rightarrow V2 \rightarrow V3 \Rightarrow V2$ ” means:

“Pass from $V1$ to $V2$ and then to $V3$ via exploration steps, and then backtrack to $V2$ ”.

[SCORING: [0...100], 100pts for a fully correct solution, -10pts for each error in the resolution process. The score cannot go below 0..]

13

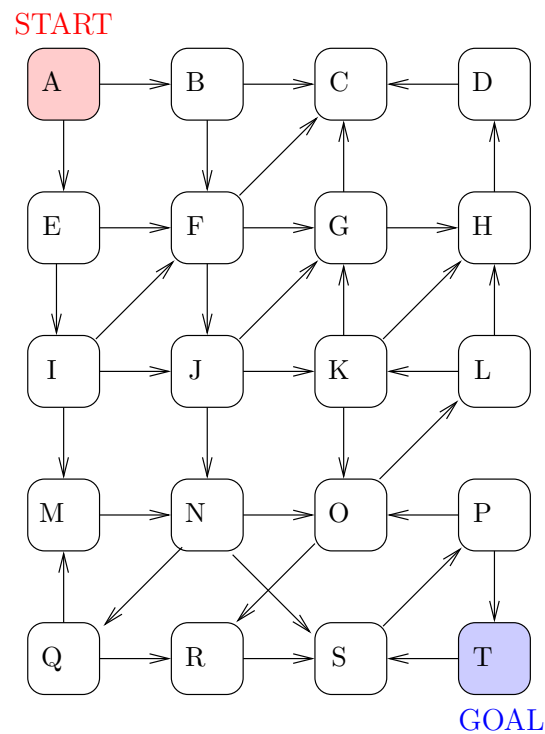
- (a) Describe as Pseudo-Code the CSP Min-Conflicts local-search heuristic procedure.
- (b) If the procedure fails to find a satisfying solution, this means that:
- 1 the input CSP problem has no solution
 - 2 we can conclude nothing about the existence of a solution for the input CSP problem.
- (Say if 1. or 2.)

[SCORING: [0...100], 75 pts for a correct answer to question (a), 25 pts for correct answer to question (b); no penalties for wrong answers.]

14

In the following state graph, apply *breadth-first search* (graph version) and report for each step:

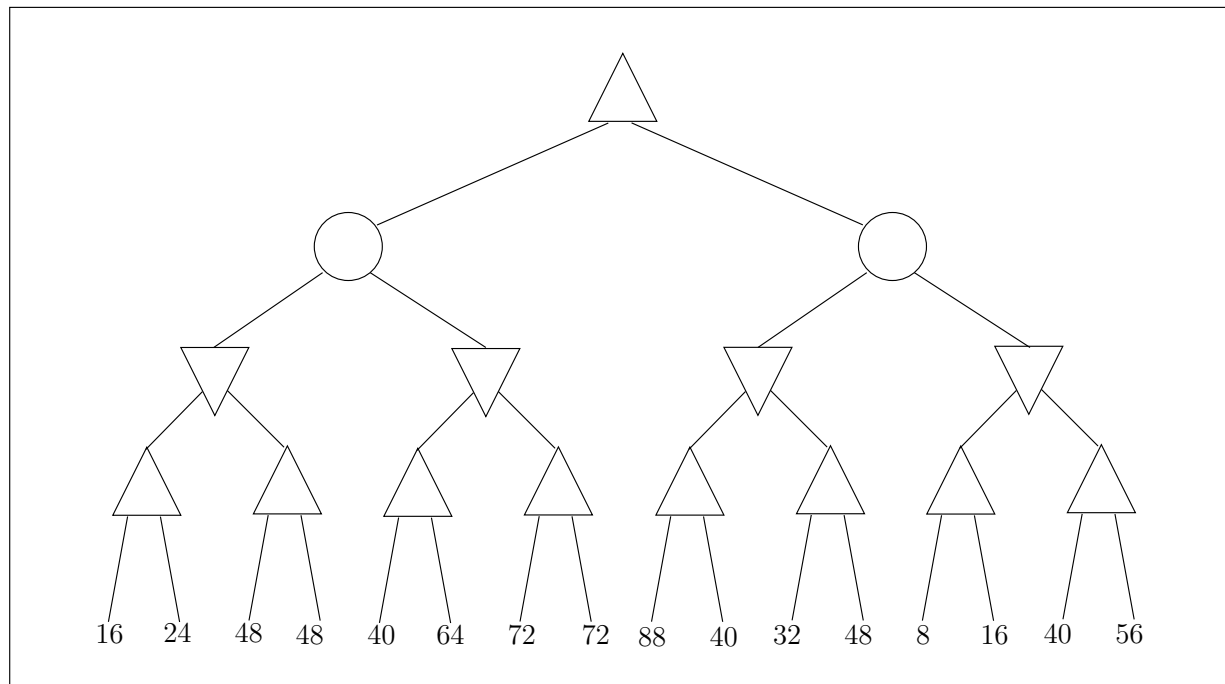
- the node extracted from the frontier;
- the nodes added to the frontier in the current step.



Actions are sorted according to the (ascending) alphabetical order of the destination.

[SCORING: [0...100], 100pts for a fully correct solution, -20pts for each error in the resolution process. The score cannot go below 0..]

15



Use *Expectiminimax* to propagate the expected utilities from the leaves to the root of the tree. Chance nodes assign uniform probabilities to their children.

[SCORING: [0...100], 100pts for a fully correct solution, -20pts for each error in the resolution process. The score cannot go below 0..]