Course "**Fundamentals of Artificial Intelligence**"
EXAM TEXT

Prof. Roberto Sebastiani
DISI, Università di Trento, Italy

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[COPY WITH SOLUTIONS]

1

For each of the following facts about conditional independence, say if it is true or false.

(a) If D and E are conditionally independent given F, then $\mathbf{P}(D, E|F) = \mathbf{P}(D, E)$
[Solution: false: $\mathbf{P}(D, E|F) = \mathbf{P}(D|F)\mathbf{P}(E|F) \neq \mathbf{P}(D, E)$]

(b) If D and E are conditionally independent given F, then $\mathbf{P}(D, F|E) = \mathbf{P}(D, F)$
[Solution: false: $\mathbf{P}(D, F|E) = \mathbf{P}(D|F, E)\mathbf{P}(F|E) \neq \mathbf{P}(D|F, E)\mathbf{P}(F) = \mathbf{P}(D|F)\mathbf{P}(F) = \mathbf{P}(D, F)$]

(c) If D and E are conditionally independent given F, then $\mathbf{P}(D, E, F) = \mathbf{P}(D|F)\mathbf{P}(E|F)\mathbf{P}(F)$
[Solution: true, $\mathbf{P}(D, E, F) = \mathbf{P}(D, E|F)\mathbf{P}(F) = \mathbf{P}(D|F)\mathbf{P}(E|F)\mathbf{P}(F)$]

(d) If D and E are conditionally independent given F, then $\mathbf{P}(D, E, F) = \mathbf{P}(D)\mathbf{P}(E)\mathbf{P}(F)$
[Solution: false: $\mathbf{P}(D, E, F) = \mathbf{P}(D, E|F)\mathbf{P}(F) = \mathbf{P}(D|F)\mathbf{P}(E|F)\mathbf{P}(F) \neq \mathbf{P}(D)\mathbf{P}(E)\mathbf{P}(F)$]

2

Given a generical search problem, assume time and space complexity are measured in terms of

b : maximum branching factor of the search tree

m : maximum depth of the state space (assume m is finite)

d : depth of the shallowest solution

Assume also that all steps cost are 1.

For each of the following facts, say if it is true or false

(a) Breadth-First Search is optimal
[Solution: true]

(b) Breadth-First Search requires $O(b^m)$ memory to find a solution.
[Solution: false, requires $O(b^d)$ memory]

(c) Depth-First Search with loop-prevention requires $O(b^d)$ memory to find a solution.
[Solution: false, requires $O(bm)$ memory]

(d) Depth-First Search with loop-prevention requires $O(b^d)$ steps to find a solution.
[Solution: false, requires $O(b^m)$ steps]

3

Consider propositional logic (PL); let C, D, E, F, G, A, B be atomic propositions
We adopt the set notation for resolution rules, s.t. Γ denotes a set of clauses.

For each of the following statements, say if it is true or false.

(a) The following is a correct application of the PL clause-subsumption rule:

$$\frac{\Gamma, \quad (\ A \vee \ C), \quad (\ A \vee \neg E \vee \ C)}{\Gamma, \quad (\ A \vee \neg E \vee \ C)}$$

[Solution: false]

(b) The following is a correct application of the PL unit-resolution rule:

$$\frac{\Gamma, \quad (\ E), \quad (\ A \vee \neg E \vee \ C)}{\Gamma, \quad (\ E), \quad (\ A \vee \ C)}$$

[Solution: true]

(c) The following is a correct application of the PL general resolution rule:

$$\frac{\Gamma, \quad (\ A \vee \neg E \vee \ C), \quad (\neg A \vee \neg C \vee \ F)}{\Gamma, \quad (\neg E \vee \ F)}$$

[Solution: False]

(d) The following is a correct application of the PL general resolution rule:

$$\frac{\Gamma, \quad (\ E \vee \neg F \vee \neg A), \quad (\neg C \vee \neg E \vee \neg F)}{\Gamma, \quad (\neg F \vee \neg C \vee \neg A)}$$

[Solution: true]

4

In the following FOL formulas, let R, Q, P, and $>$, $\leq$, $<$, $\geq$ denote predicates,
h, g, f, F_1, F_2, F_3 and $+, -, \cdot, /$ denote functions,
x, y, z, x_1, x_2, x_3 denote variables,
A, B, C, C_1, C_2, C_3 and $0, 1, 2, 3, 4$ denote constants.

For each of the following facts, say if it is true or false.

(*a*) The FOL formula $(\exists x_1.\neg R(x_1)) \leftrightarrow (\neg\forall x_1.R(x_1))$ is valid
[Solution: true]

(*b*) The FOL formula $(\forall x_2 \exists x_1.R(x_1, x_2)) \rightarrow (\exists x_1 \forall x_2.R(x_1, x_2))$ is valid.
[Solution: false]

(*c*) The FOL formula $(2 > 4)$ is unsatisfiable.
[Solution: false (in FOL "$>$", "2", "4" have no fixed interpretation)]

(*d*) The FOL formula $\forall x.((x > 2) \rightarrow (x > 1))$ is valid.
[Solution: false (in FOL "$>$", "2", "1" have no fixed interpretation)]

5

Given the following symbols, representing concept, relation and individual names in the alien language of the remote planet Sgotz:

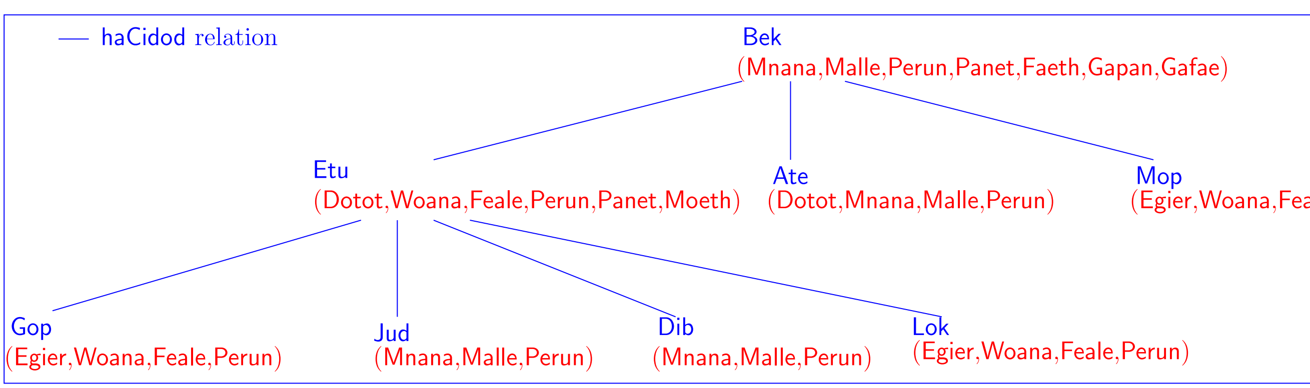
- a set of primitive $\mathcal{ALCQ}$ concept names: {Perun, Malle, Feale, Dotot, Egier}
- a set of $\mathcal{ALCQ}$ relation names: {haCidod}
- a set of $\mathcal{ALCQ}$ individual names: {Lok, Bek, Etu, Dib, Ate, Jud, Mop, Gop}

and the following $\mathcal{ALCQ}$ $\mathcal{T}$-box $\mathcal{T}$ and $\mathcal{A}$-box $\mathcal{A}$:

$\mathcal{T}$			$\mathcal{A}$
Perun		⟨*primitive concept*⟩	Etu : Woana; Mop : Woana; Gop : Woana; Lok : Woana;
Feale		⟨*primitive concept*⟩	
Malle		⟨*primitive concept*⟩	Bek : Mnana; Ate : Mnana; Jud : Mnana; Dib : Mnana;
Dotot		⟨*primitive concept*⟩	
Egier		⟨*primitive concept*⟩	Ate : Dotot; Etu : Dotot
Woana	$\equiv$	Perun $\sqcap$ Feale	
Mnana	$\equiv$	Perun $\sqcap$ Malle	Mop : Egier; Gop : Egier; Lok : Egier
Moeth	$\equiv$	Woana $\sqcap$ $\exists$haCidod.Perun	
Faeth	$\equiv$	Mnana $\sqcap$ $\exists$haCidod.Perun	⟨Bek, Etu⟩ : haCidod; ⟨Bek, Ate⟩ : haCidod;
Panet	$\equiv$	Perun $\sqcap$ haCidod.Perun	⟨Bek, Mop⟩ : haCidod;
Gamae	$\equiv$	Moeth $\sqcap$ haCidod.Panet	
Gafae	$\equiv$	Faeth $\sqcap$ haCidod.Panet	⟨Etu, Gop⟩ : haCidod; ⟨Etu, Jud⟩ : haCidod;
Gapan	$\equiv$	Panet $\sqcap$ haCidod.Panet	⟨Etu, Dib⟩ : haCidod; ⟨Etu, Lok⟩ : haCidod;

For each of the following $\mathcal{ALCQ}$ queries to $\mathcal{T} \cup \mathcal{A}$, say if it is true or false.

(*a*) Bek : Faeth $\sqcap$ (≥ 2)haCidod.Dotot [Solution: true]
(*b*) Etu : Moeth $\sqcap$ (≥ 3)haCidod.Egier [Solution: false]
(*c*) Bek : Gapan $\sqcap$ $\forall$haCidod.Moeth [Solution: false]
(*d*) Bek : Faeth $\sqcap$ $\exists$haCidod.Panet [Solution: true]

[Solution:


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6

An experienced doctor has to cope with an epidemic of covid19, where 40% of people of the area have been infected. She considers the following possible symptoms:

Symptom #1: fever;
Symptom #2: headache;
Symptom #3: nausea.

She models the cause-effect relation as a **Naive Bayes Model scenario**, s.t the effects are considered conditionally independent given the cause, and she knows from statistics the following data: [1]

$P(fever\|\ covid19)$	$= 0.7$
$P(fever\|\neg covid19)$	$= 0.1$
$P(headache\|\ covid19)$	$= 0.3$
$P(headache\|\neg covid19)$	$= 0.2$
$P(nausea\|\ covid19)$	$= 0.6$
$P(nausea\|\neg covid19)$	$= 0.2$

She is informed that one patient has headache and nausea but not fever. Compute the probability that such patient has contracted covid19.

Notice: *the problem must be solved my using the Naive Bayes Model scenario. Any attempt to use any other technique will be considered incorrect.*

[Solution: With a **Naive Bayes Model** scenario, we have that:

$$\mathbf{P}(Cause|Effect_1, ..., Effect_n) = \alpha \mathbf{P}(Cause) * \prod_{i=1}^{n} \mathbf{P}(Effect_i|Cause)).$$

for some normalization constant α. Thus, we have:

$$\begin{aligned}
& P(\ covid19|\neg fever \wedge headache \wedge nausea) \\
= \ & \alpha * P(\ covid19) * P(\neg fever|\ covid19) * P(\ headache|\ covid19) * P(\ nausea|\ covid19) \\
= \ & \alpha * 0.4 * (1 - 0.7) * 0.3 * 0.6 \\
= \ & \alpha * 0.0216 \\
& P(\neg covid19|\neg fever \wedge headache \wedge nausea) \\
= \ & \alpha * P(\neg covid19) * P(\neg fever|\neg covid19) * P(\ headache|\neg covid19) * P(\ nausea|\neg covid19) \\
= \ & \alpha * (1 - 0.4) * (1 - 0.1) * 0.2 * 0.2 \\
= \ & \alpha * 0.0216
\end{aligned}$$

Thus, after normalization:

$$\begin{aligned}
P(\ covid19|\neg fever \wedge headache \wedge nausea) = & \ 0.5 \\
P(\neg covid19|\neg fever \wedge headache \wedge nausea) = & \ 0.5
\end{aligned}$$

]

[1]The data here are pure fantasy and are not supposed to correspond to actual medical data.

7

(*a*) Describe as Pseudo-Code the Depth-Limited Search and Iterative-Deepening procedures.

[Solution:

```
function DEPTH-LIMITED-SEARCH(problem, limit) returns a solution, or failure/cutoff
  return RECURSIVE-DLS(MAKE-NODE(problem.INITIAL-STATE), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns a solution, or failure/cutoff
  if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
  else if limit = 0 then return cutoff
  else
      cutoff_occurred? ← false
      for each action in problem.ACTIONS(node.STATE) do
          child ← CHILD-NODE(problem, node, action)
          result ← RECURSIVE-DLS(child, problem, limit − 1)
          if result = cutoff then cutoff_occurred? ← true
          else if result ≠ failure then return result
      if cutoff_occurred? then return cutoff else return failure

function ITERATIVE-DEEPENING-SEARCH(problem) returns a solution, or failure
   for depth = 0 to ∞ do
      result ← DEPTH-LIMITED-SEARCH(problem, depth)
      if result ≠ cutoff then return result
```

or any schema equivalent to the above one (from AIMA book).]

(*b*) calling B the branching factor and S the depth of the shallowest solution,

- what is the time complexity of the Iterative-Deepening procedure? [Solution: $O(B^S)$]
- what is the memory complexity of the Iterative-Deepening procedure? [Solution: $O(BS)$]

8

(a) Describe as Pseudo-Code the specialized solving procedure for tree-structured CSPs.

[Solution:

function TREE-CSP-SOLVER(*csp*) **returns** a solution, or failure
  inputs: *csp*, a CSP with components $X,\ D,\ C$

  $n \leftarrow$ number of variables in X
  assignment ← an empty assignment
  root ← any variable in X
  $X \leftarrow$ TOPOLOGICALSORT(X, *root*)
  for $j = n$ **down to** 2 **do**
    MAKE-ARC-CONSISTENT(PARENT(X_j), X_j)
    if it cannot be made consistent **then return** *failure*
  for $i = 1$ **to** n **do**
    assignment[X_i] ← any consistent value from D_i
    if there is no consistent value **then return** *failure*
  return *assignment*

or any schema equivalent to the above one (from AIMA book).]

(b) Say if the following sentence is true or false, and briefly explain why.

- It requires polynomial time in worst-case if the input constraint graph has no loops.

[Solution: True. It requires $O(nd^2)$ steps in worst case.]

9

Given the following Sudoku scenario:

1	2	3	4	5	6	7	8	9	
4		1							A
3				6					B
2						8	7		C
1								7	D
5								8	E
6								9	F
7	1	4							G
	5	3		9					H
		2							I

(a) Apply the AC-3 algorithm. Describe in the right sequence the domains of unassigned nodes whose domains become unary after one run of AC-3. (E.g.:
$D_{A1} := \{3\}$,
$D_{B1} := \{7\}$,
...)

(b) Can AC-3 reduce to unary the domains of nodes $B3$ and $C3$?

(c) After one run of AC-3, is the resulting graph arc-consistent?

[Solution:

(a) We have, in sequence
$D_{H1} := \{8\}$,
$D_{I1} := \{9\}$,
$D_{I2} := \{6\}$,
$D_{C2} := \{9\}$

(b) No. Decent Sudoku players can infer also $A2 = B2 = \{7, 8\}$, so that, by path-consistency (not arc-consistency!), we have $B3 := \{5\}$, $C3 := \{6\}$, but this cannot be performed by AC-3.

(c) Yes. AC-3 always makes a graph arc-consistent.

]

10

Consider the following CNF formula in PL:

$$
\begin{array}{l}
(\neg E \quad \vee \neg B \quad \vee \neg N) \wedge \\
(\ \ A \quad \vee \ \ H \quad \vee \ \ C) \wedge \\
(\neg H \quad \vee \ \ I \quad \vee \ \ A) \wedge \\
(\neg L \quad \vee \ \ C \quad \vee \neg M) \wedge \\
(\neg G \quad \vee \neg A \quad \vee \ \ E) \wedge \\
(\neg E \quad \vee \neg G \quad \vee \ \ A) \wedge \\
(\neg E \quad \vee \neg F \quad \vee \neg A) \wedge \\
(\ \ I \quad \vee \ \ L \quad \vee \ \ M) \wedge \\
(\ \ N \quad \vee \ \ L \quad \vee \ \ M)
\end{array}
$$

Consider the WalkSAT algorithm, with probability parameter $p = 0.2$. Suppose at a given step the current assignment is

$$\{ \ A, \ B, \ C, \ D, \ E, \neg F, \ G, \neg H, \ I, \neg L, \neg M, \ N\}.$$

Assuming the most-likely event happens, describe what the assignment is after the next step.

[Solution:

The current assignments makes only the 1^{st} clause unsatisfied. Since $p = 0.2$, the most likely event is that the algorithm flips the symbol in the first clause which maximizes the number of satisfied clause at next step.

We notice that flipping B would cause no other clause to become unsatisfied, whereas flipping either of E, N would cause other clauses to become unsatisfied. Thus WalkSAT flips B, obtaining the assignment

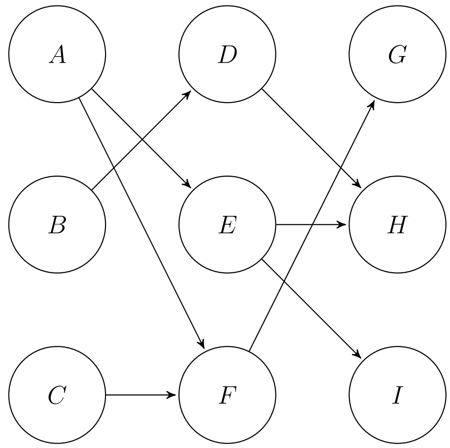
$$\{ \ A, \neg B, \ C, \ D, \ E, \neg F, \ G, \neg H, \ I, \neg L, \neg M, \ N\}.$$

which satisfies all clauses.]

11

Consider the following actions with durations / dependencies:

a	duration(a)
A	1
B	2
C	1
D	2
E	2
F	3
G	3
H	1
I	2



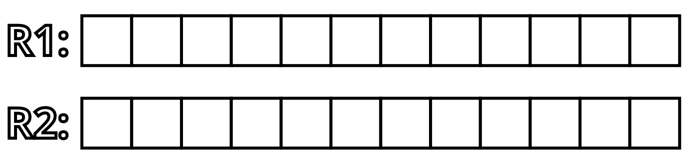
1) Compute the earliest / latest possible start time (ES/LS) for each action using the Critical Path method. [40pts.]

Assume that completing each action requires one unit of a reusable resource:

2) Report an optimal schedule given 2 reusable resources in the grid below. [40pts.]

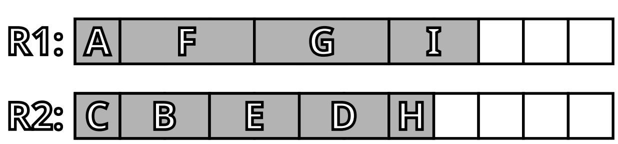
3) What would be the optimal execution time given 1 reusable resource? [10pts.]

4) What would be the optimal execution time given infinite resources? [10pts.]



[Solution:

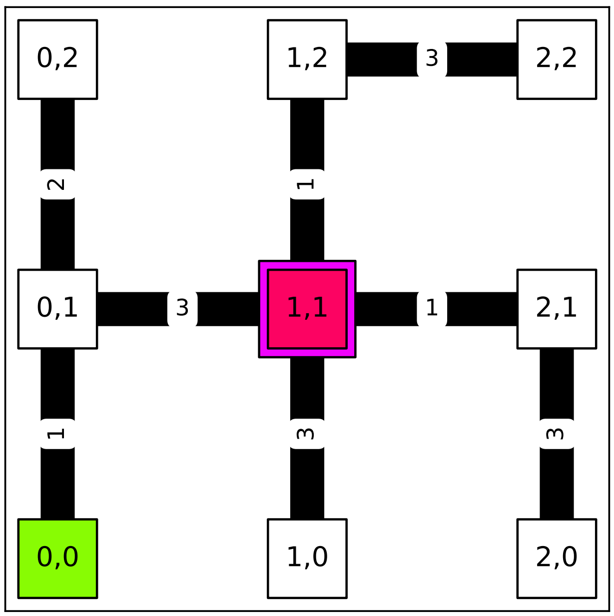
a	ES	LS
A	0	0
B	0	2
C	0	0
D	2	4
E	1	3
F	1	1
G	4	4
H	4	6
I	3	5



3) 17.
4) 7.]

12

In the following state graph, apply *LRTA** and report the list of visited states, including repetitions (e.g. $(0,1) \rightarrow (0,0) \rightarrow (0,1) \rightarrow ...$). The order of (untried) actions is $[up, right, down, left]$.



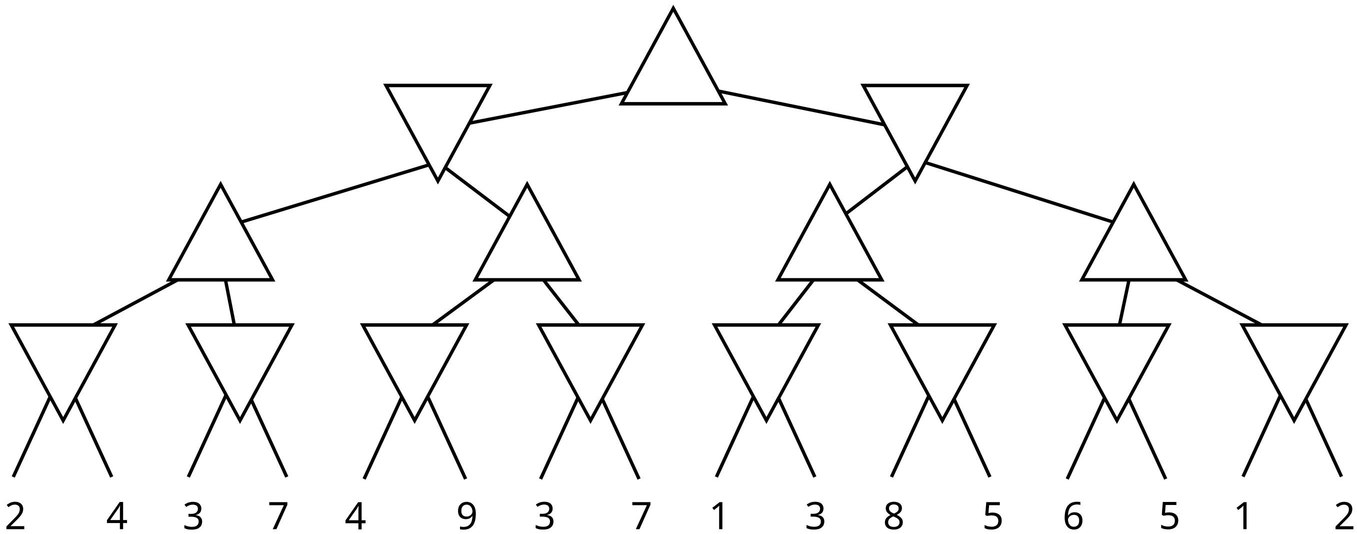
Start: $(1,1)$ - Goal: $(0,0)$

[Solution:

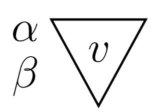
$(1,1)$, $(1,2)$, $(2,2)$, $(1,2)$, $(1,1)$, $(2,1)$, $(2,0)$, $(2,1)$, $(1,1)$, $(1,0)$, $(1,1)$, $(0,1)$, $(0,2)$, $(0,1)$, $(1,1)$, $(1,2)$, $(1,1)$, $(2,1)$, $(1,1)$, $(0,1)$, $(0,0)$
]

13

Use *Minimax with α, β-pruning* to propagate the utilities from the leaves to the root of the tree.

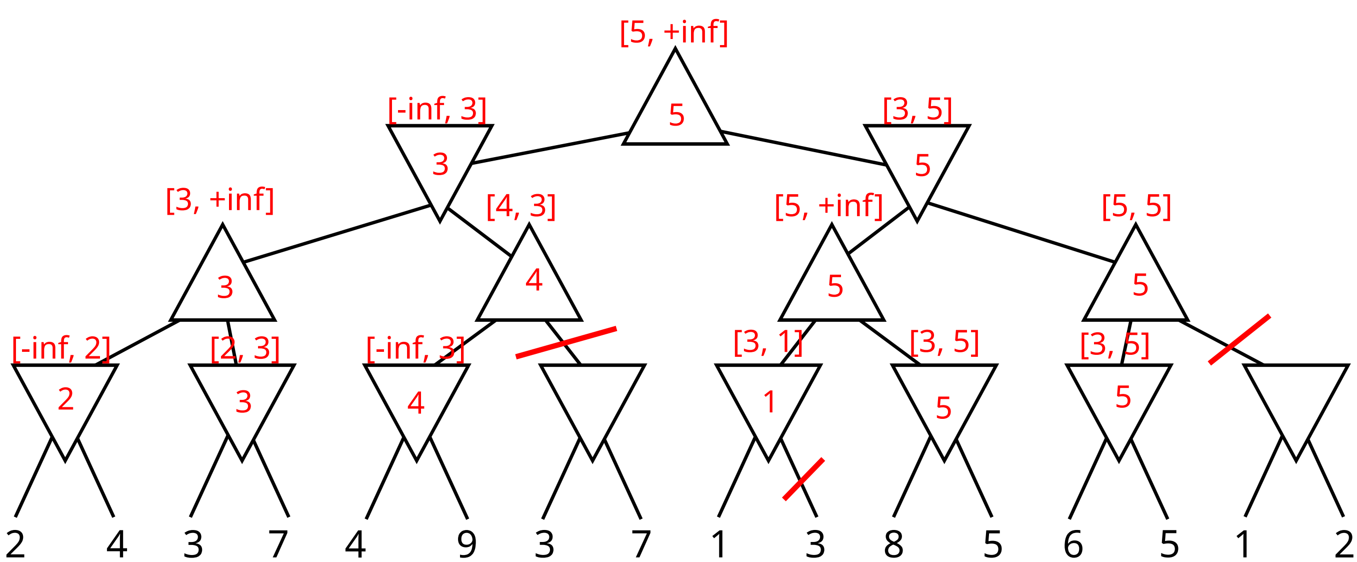


For each node report the values of α, β, as well as its returned value v **when the recursive calls ends**. Use the following format:



Additionally, clearly mark in the tree the pruned branches.

[Solution:



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14

Use *hill climbing* for solving the maximization problem over (x, y) with the following objective function:

y								
7	**0**	**2**	**3**	**4**	**4**	**2**	**1**	**0**
6	**1**	**1**	**4**	**5**	**5**	**3**	**2**	**1**
5	**2**	**3**	**5**	**8**	**6**	**4**	**2**	**3**
4	**3**	**5**	**6**	**9**	**10**	**6**	**5**	**1**
3	**3**	**4**	**7**	**10**	**9**	**6**	**4**	**1**
2	**2**	**2**	**4**	**6**	**7**	**5**	**4**	**3**
1	**1**	**1**	**3**	**5**	**4**	**3**	**1**	**1**
0	**0**	**1**	**2**	**3**	**4**	**2**	**2**	**0**
	0	1	2	3	4	5	6	7 x

The next state is selected among the neighbors with strictly higher objective function value with *probability proportional to their objective function value*. Neighbors of (x, y) are sorted as follows:

$$(x-1, y-1), (x, y-1), (x+1, y-1), (x-1, y), (x+1, y), (x-1, y+1), (x, y+1), (x+1, y+1)$$

The choice vector is: $[1/4, 3/4]$.

The *initial state* is: $(7, 7)$.
For each step in the resolution process report (1) the current state; (2) the list of candidate next states.

[Solution:

it: 1, curr: (7, 7), candidates = [(6, 6) : 1/2, (7, 6) : 1/4, (6, 7) : 1/4], choice: 1/4 → (6, 6)
it: 2, curr: (6, 6), candidates = [(5, 5) : 4/10, (7, 5) : 3/10, (5, 6) : 3/10], choice: 3/4 → (5, 6)
it: 3, curr: (5, 6), candidates = [(4, 5) : 6/15, (5, 5) : 4/15, (4, 6) : 5/15], choice: 1/4 → (4, 5)
it: 4, curr: (4, 5), candidates = [(3, 4) : 9/27, (4, 4) : 10/27, (3, 5) : 8/27], choice: 3/4 → (3, 5)
it: 5, curr: (3, 5), candidates = [(3, 4) : 9/19, (4, 4) : 10/19], choice: 1/4 → (3, 4)
it: 6, curr: (3, 4), candidates = [(3, 3) : 10/20, (4, 4) : 10/20], choice: 3/4 → (4, 4)
it: 7, curr: (4, 4), candidates = [], STOP

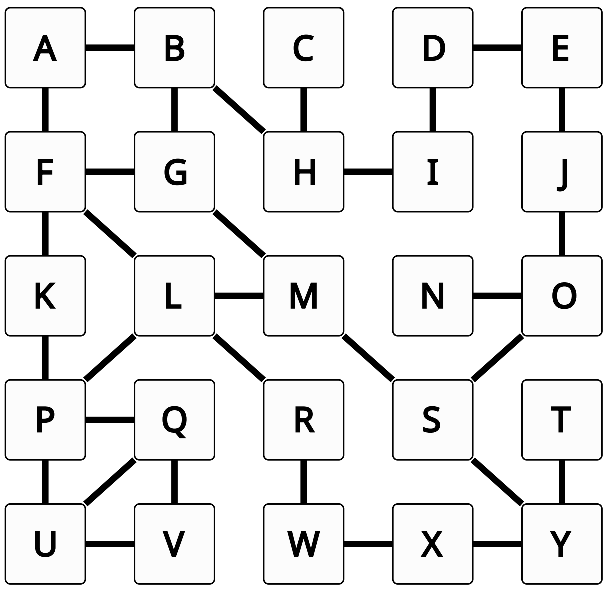
]

15

In the following state graph, apply *breadth-first search* and report for each step:

- the node extracted from the frontier;
- the nodes added to the frontier in the current step.

Actions are sorted according to the (ascending) alphabetical order of the destination.



Start: **M** - Goal: **V**

[Solution:
M - [G, L, S]
G - [B, F]
L - [P, R]
S - [O, Y]
B - [A, H]
F - [K]
P - [Q, U]
R - [W]
O - [J, N]
Y - [T, X]
A - []
H - [C, I]
K - []
Q - stop
]