Course "**Fundamentals of Artificial Intelligence**" EXAM TEXT

Prof. Roberto Sebastiani
DISI, Università di Trento, Italy

2026.02.19

842917

Name (please print):

Surname (please print):

Matriculation Number:

1

For each of the following facts about conditional independence, say if it is true or false.

(a) If D and E are conditionally independent given F, then $\mathbf{P}(D, E|F) = \mathbf{P}(D, E)$

(b) If D and E are conditionally independent given F, then $\mathbf{P}(D, F|E) = \mathbf{P}(D, F)$

(c) If D and E are conditionally independent given F, then $\mathbf{P}(D, E, F) = \mathbf{P}(D|F)\mathbf{P}(E|F)\mathbf{P}(F)$

(d) If D and E are conditionally independent given F, then $\mathbf{P}(D, E, F) = \mathbf{P}(D)\mathbf{P}(E)\mathbf{P}(F)$

[SCORING [0...100]:

- +25pts for each correct answer
- -25pts for each incorrect answer
- 0pts for each unanswered question

]

2

Given a generical search problem, assume time and space complexity are measured in terms of

b : maximum branching factor of the search tree

m : maximum depth of the state space (assume m is finite)

d : depth of the shallowest solution

Assume also that all steps cost are 1.
For each of the following facts, say if it is true or false

(a) Breadth-First Search is optimal

(b) Breadth-First Search requires $O(b^m)$ memory to find a solution.

(c) Depth-First Search with loop-prevention requires $O(b^d)$ memory to find a solution.

(d) Depth-First Search with loop-prevention requires $O(b^d)$ steps to find a solution.

[SCORING [0...100]:

- +25pts for each correct answer
- -25pts for each incorrect answer
- 0pts for each unanswered question

]

3

Consider propositional logic (PL); let C, D, E, F, G, A, B be atomic propositions
We adopt the set notation for resolution rules, s.t. Γ denotes a set of clauses.

For each of the following statements, say if it is true or false.

(a) The following is a correct application of the PL clause-subsumption rule:

$$\frac{\Gamma,\quad (\quad A \vee \quad C),\quad (\quad A \vee \neg E \vee \quad C)}{\Gamma,\quad (\quad A \vee \neg E \vee \quad C)}$$

(b) The following is a correct application of the PL unit-resolution rule:

$$\frac{\Gamma,\quad (\quad E),\quad (\quad A \vee \neg E \vee \quad C)}{\Gamma,\quad (\quad E),\quad (\quad A \vee \quad C)}$$

(c) The following is a correct application of the PL general resolution rule:

$$\frac{\Gamma,\quad (\quad A \vee \neg E \vee \quad C),\quad (\neg A \vee \neg C \vee \quad F)}{\Gamma,\quad (\neg E \vee \quad F)}$$

(d) The following is a correct application of the PL general resolution rule:

$$\frac{\Gamma,\quad (\quad E \vee \neg F \vee \neg A),\quad (\neg C \vee \neg E \vee \neg F)}{\Gamma,\quad (\neg F \vee \neg C \vee \neg A)}$$

[SCORING [0...100]:

- +25pts for each correct answer
- -25pts for each incorrect answer
- 0pts for each unanswered question

]

4

In the following FOL formulas, let R, Q, P, and $>, \leq, <, \geq$ denote predicates,
h, g, f, F_1, F_2, F_3 and $+, -, \cdot, /$ denote functions,
x, y, z, x_1, x_2, x_3 denote variables,
A, B, C, C_1, C_2, C_3 and $0, 1, 2, 3, 4$ denote constants.
For each of the following facts, say if it is true or false.

(a) The FOL formula $(\exists x_1.\neg R(x_1)) \leftrightarrow (\neg\forall x_1.R(x_1))$ is valid

(b) The FOL formula $(\forall x_2 \exists x_1.R(x_1, x_2)) \rightarrow (\exists x_1 \forall x_2.R(x_1, x_2))$ is valid.

(c) The FOL formula $(2 > 4)$ is unsatisfiable.

(d) The FOL formula $\forall x.((x > 2) \rightarrow (x > 1))$ is valid.

[SCORING [0...100]:

- +25pts for each correct answer
- -25pts for each incorrect answer
- 0pts for each unanswered question

]

5

Given the following symbols, representing concept, relation and individual names in the alien language of the remote planet Sgotz:

- a set of primitive $\mathcal{ALCQ}$ concept names: {Perun, Malle, Feale, Dotot, Egier}
- a set of $\mathcal{ALCQ}$ relation names: {haCidod}
- a set of $\mathcal{ALCQ}$ individual names: {Lok, Bek, Etu, Dib, Ate, Jud, Mop, Gop}

and the following $\mathcal{ALCQ}$ $\mathcal{T}$-box $\mathcal{T}$ and $\mathcal{A}$-box $\mathcal{A}$:

$\mathcal{T}$			$\mathcal{A}$
Perun		$\langle primitive\ concept \rangle$	Etu : Woana; Mop : Woana; Gop : Woana; Lok : Woana;
Feale		$\langle primitive\ concept \rangle$	
Malle		$\langle primitive\ concept \rangle$	Bek : Mnana; Ate : Mnana; Jud : Mnana; Dib : Mnana;
Dotot		$\langle primitive\ concept \rangle$	
Egier		$\langle primitive\ concept \rangle$	Ate : Dotot; Etu : Dotot
Woana	$\equiv$	Perun $\sqcap$ Feale	
Mnana	$\equiv$	Perun $\sqcap$ Malle	Mop : Egier; Gop : Egier; Lok : Egier
Moeth	$\equiv$	Woana $\sqcap \exists$haCidod.Perun	
Faeth	$\equiv$	Mnana $\sqcap \exists$haCidod.Perun	$\langle$Bek, Etu$\rangle$: haCidod; $\langle$Bek, Ate$\rangle$: haCidod;
Panet	$\equiv$	Perun $\sqcap$ haCidod.Perun	$\langle$Bek, Mop$\rangle$: haCidod;
Gamae	$\equiv$	Moeth $\sqcap$ haCidod.Panet	
Gafae	$\equiv$	Faeth $\sqcap$ haCidod.Panet	$\langle$Etu, Gop$\rangle$: haCidod; $\langle$Etu, Jud$\rangle$: haCidod;
Gapan	$\equiv$	Panet $\sqcap$ haCidod.Panet	$\langle$Etu, Dib$\rangle$: haCidod; $\langle$Etu, Lok$\rangle$: haCidod;

For each of the following $\mathcal{ALCQ}$ queries to $\mathcal{T} \cup \mathcal{A}$, say if it is true or false.

(a) Bek : Faeth $\sqcap (\geq 2)$haCidod.Dotot
(b) Etu : Moeth $\sqcap (\geq 3)$haCidod.Egier
(c) Bek : Gapan $\sqcap \forall$haCidod.Moeth
(d) Bek : Faeth $\sqcap \exists$haCidod.Panet

[SCORING [0...100]:

- +25pts for each correct answer
- -25pts for each incorrect answer
- 0pts for each unanswered question

]

6

An experienced doctor has to cope with an epidemic of covid19, where 40% of people of the area have been infected. She considers the following possible symptoms:

Symptom #1: fever;
Symptom #2: headache;
Symptom #3: nausea.

She models the cause-effect relation as a **Naive Bayes Model scenario**, s.t the effects are considered conditionally independent given the cause, and she knows from statistics the following data: [1]

$P(fever\|\ \ covid19)$	$= 0.7$
$P(fever\|\neg covid19)$	$= 0.1$
$P(headache\|\ \ covid19)$	$= 0.3$
$P(headache\|\neg covid19)$	$= 0.2$
$P(nausea\|\ \ covid19)$	$= 0.6$
$P(nausea\|\neg covid19)$	$= 0.2$

She is informed that one patient has headache and nausea but not fever. Compute the probability that such patient has contracted covid19.

Notice: *the problem must be solved my using the Naive Bayes Model scenario. Any attempt to use any other technique will be considered incorrect.*

[SCORING: [0...100], 100 pts for correct answer, no penalties for wrong answers.]

[1] The data here are pure fantasy and are not supposed to correspond to actual medical data.

7

(a) Describe as Pseudo-Code the Depth-Limited Search and Iterative-Deepening procedures.

(b) calling B the branching factor and S the depth of the shallowest solution,

- what is the time complexity of the Iterative-Deepening procedure?
- what is the memory complexity of the Iterative-Deepening procedure?

[SCORING: [0...100], 75 pts for a correct answer to question (a), 25 pts for correct answer to question (b); no penalties for wrong answers.]

8

(a) Describe as Pseudo-Code the specialized solving procedure for tree-structured CSPs.

(b) Say if the following sentence is true or false, and briefly explain why.

- It requires polynomial time in worst-case if the input constraint graph has no loops.

[SCORING: [0...100], 75 pts for a correct answer to question (a), 25 pts for correct answer to question (b); no penalties for wrong answers.]

9

Given the following Sudoku scenario:

1	2	3	4	5	6	7	8	9	
4		1							A
3				6					B
2						8	7		C
1								7	D
5								8	E
6								9	F
7	1	4							G
	5	3		9					H
		2							I

(a) Apply the AC-3 algorithm. Describe in the right sequence the domains of unassigned nodes whose domains become unary after one run of AC-3. (E.g.:
$D_{A1} := \{3\}$,
$D_{B1} := \{7\}$,
...)

(b) Can AC-3 reduce to unary the domains of nodes $B3$ and $C3$?

(c) After one run of AC-3, is the resulting graph arc-consistent?

[SCORING: [0...100], 50pts for correct answer (a), 25pts each for correct answer (b) and (c). No penalties for wrong answers..]

10

Consider the following CNF formula in PL:

$$\begin{array}{l}
(\neg E \vee \neg B \vee \neg N) \wedge \\
(A \vee H \vee C) \wedge \\
(\neg H \vee I \vee A) \wedge \\
(\neg L \vee C \vee \neg M) \wedge \\
(\neg G \vee \neg A \vee E) \wedge \\
(\neg E \vee \neg G \vee A) \wedge \\
(\neg E \vee \neg F \vee \neg A) \wedge \\
(I \vee L \vee M) \wedge \\
(N \vee L \vee M)
\end{array}$$

Consider the WalkSAT algorithm, with probability parameter $p = 0.2$. Suppose at a given step the current assignment is

$$\{A, B, C, D, E, \neg F, G, \neg H, I, \neg L, \neg M, N\}.$$

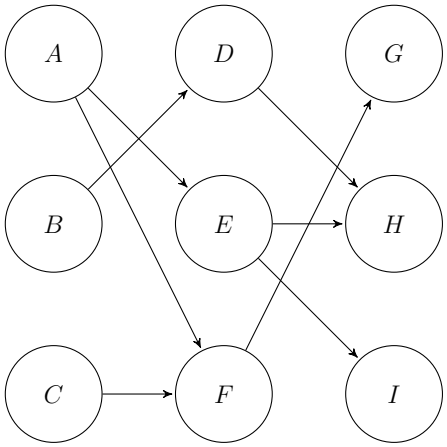
Assuming the most-likely event happens, describe what the assignment is after the next step.

[SCORING: [0...100], 100 pts for a correct answer, no penalties for wrong answers.]

11

Consider the following actions with durations / dependencies:

a	duration(a)
A	1
B	2
C	1
D	2
E	2
F	3
G	3
H	1
I	2



1) Compute the earliest / latest possible start time (ES/LS) for each action using the Critical Path method. [40pts.]

Assume that completing each action requires one unit of a reusable resource:

2) Report an optimal schedule given 2 reusable resources in the grid below. [40pts.]

3) What would be the optimal execution time given 1 reusable resource? [10pts.]

4) What would be the optimal execution time given infinite resources? [10pts.]

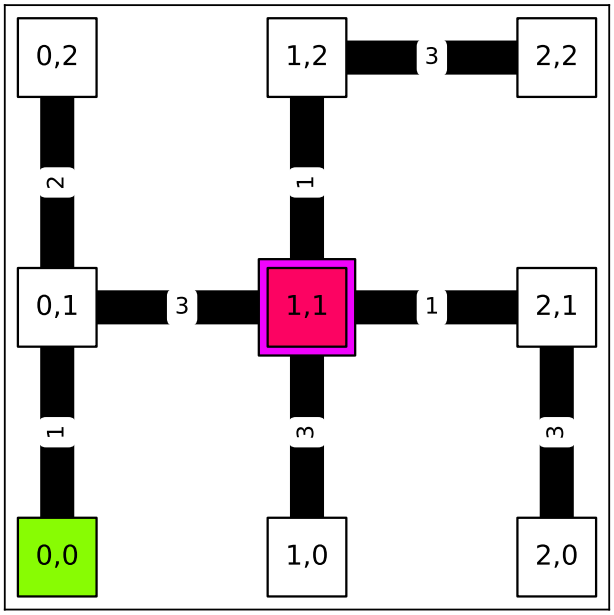
R1:

R2:

[SCORING: [0...100], 100pts for a fully correct solution. The score cannot go below 0..]

12

In the following state graph, apply *LRTA** and report the list of visited states, including repetitions (e.g. $(0,1) \rightarrow (0,0) \rightarrow (0,1) \rightarrow ...$). The order of (untried) actions is $[up, right, down, left]$.

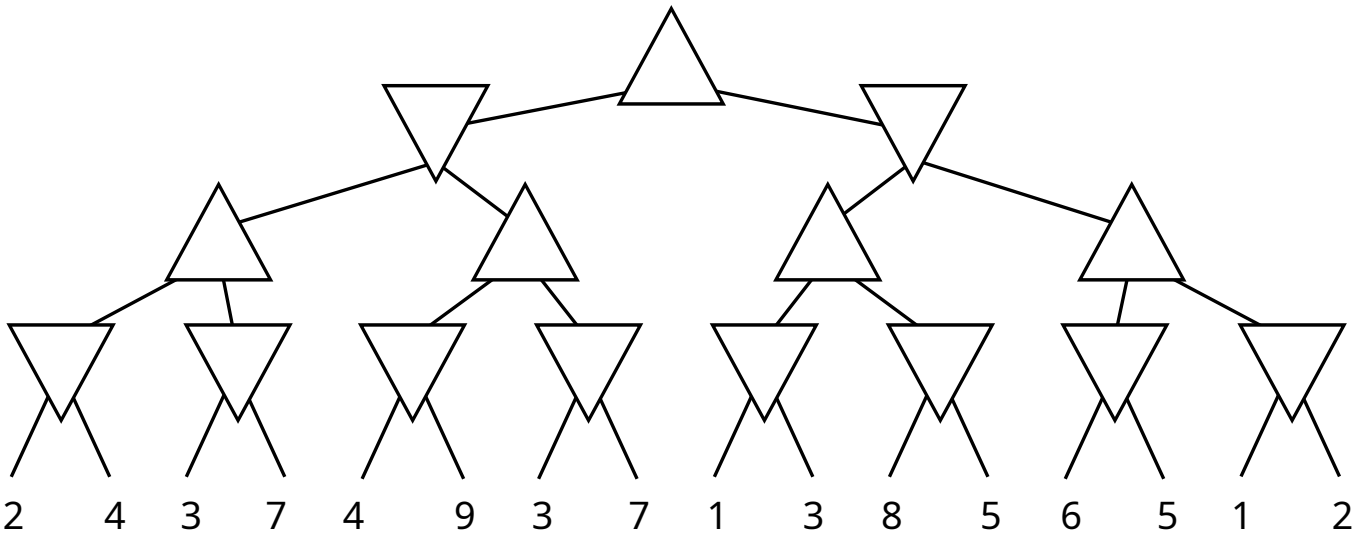


Start: $(1,1)$ - Goal: $(0,0)$

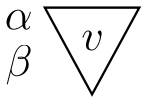
[SCORING: [0...100], 100pts for a fully correct solution, -10pts for each error in the resolution process. The score cannot go below 0..]

13

Use *Minimax with α, β-pruning* to propagate the utilities from the leaves to the root of the tree.



For each node report the values of α, β, as well as its returned value v **when the recursive calls ends**. Use the following format:



Additionally, clearly mark in the tree the pruned branches.

[SCORING: [0...100], 100pts for a fully correct solution, -20pts for each error in the resolution process. The score cannot go below 0..]

14

Use *hill climbing* for solving the maximization problem over (x, y) with the following objective function:

y								
7	**0**	**2**	**3**	**4**	**4**	**2**	**1**	**0**
6	**1**	**1**	**4**	**5**	**5**	**3**	**2**	**1**
5	**2**	**3**	**5**	**8**	**6**	**4**	**2**	**3**
4	**3**	**5**	**6**	**9**	**10**	**6**	**5**	**1**
3	**3**	**4**	**7**	**10**	**9**	**6**	**4**	**1**
2	**2**	**2**	**4**	**6**	**7**	**5**	**4**	**3**
1	**1**	**1**	**3**	**5**	**4**	**3**	**1**	**1**
0	**0**	**1**	**2**	**3**	**4**	**2**	**2**	**0**
	0	1	2	3	4	5	6	7 x

The next state is selected among the neighbors with strictly higher objective function value with *probability proportional to their objective function value*. Neighbors of (x, y) are sorted as follows:

$$(x-1, y-1), (x, y-1), (x+1, y-1), (x-1, y), (x+1, y), (x-1, y+1), (x, y+1), (x+1, y+1)$$

The choice vector is: $[1/4, 3/4]$.

The *initial state* is: $(7, 7)$.
For each step in the resolution process report (1) the current state; (2) the list of candidate next states.

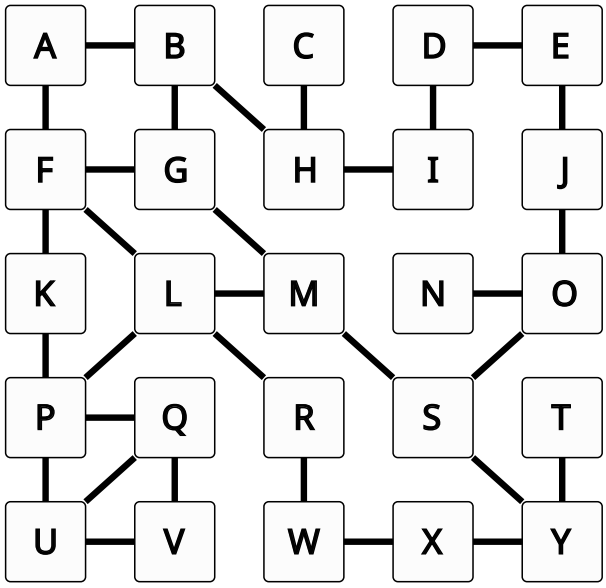
[SCORING: [0...100], 100pts for a fully correct solution, -20pts for each error in the resolution process. The score cannot go below 0..]

15

In the following state graph, apply *breadth-first search* and report for each step:

- the node extracted from the frontier;
- the nodes added to the frontier in the current step.

Actions are sorted according to the (ascending) alphabetical order of the destination.



Start: **M** - Goal: **V**

[SCORING: [0...100], 100pts for a fully correct solution, -10pts for each error in the resolution process. The score cannot go below 0..]