Course "**Fundamentals of Artificial Intelligence**"
EXAM TEXT

Prof. Roberto Sebastiani
DISI, Università di Trento, Italy

2026.01.13

862917

Name (please print):

Surname (please print):

Matriculation Number:

1

Given a generical search problem, assume time and space complexity are measured in terms of

b : maximum branching factor of the search tree

m : maximum depth of the state space (assume m is finite)

s : depth of the shallowest solution

Assume also that all steps cost are 1.

For each of the following facts about Iterative-Deepening Search, say if it is true or false.

(a) Iterative-Deepening Search is complete.

(b) Iterative-Deepening Search requires $O(b^m)$ time to find a solution.

(c) Iterative-Deepening Search is optimal

(d) Iterative-Deepening Search requires $O(b^s)$ memory to find a solution.

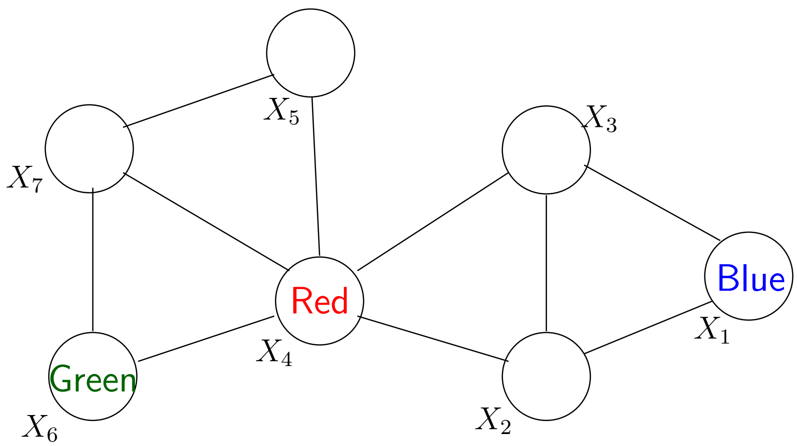
[SCORING [0...100]:

- +25pts for each correct answer
- -25pts for each incorrect answer
- 0pts for each unanswered question

]

2

Consider the following constraint graph of a map coloring problem, with domain $D \stackrel{\text{def}}{=} \{$Red, Green, Blue$\}$, and consider the partial value assignment induced by the following unary constraints:
$\{X_1 =$ Blue, $X_4 =$ Red, $X_6 =$ Green, $\}$ (see figure).



For each of the following facts, say if it is true or false

(a) Forward Checking reduces the domain of X_5 to { , Green, }

(b) AC-3 reduces the domain of X_5 to { , Green, }

(c) Forward checking reduces the domain of some variable to the empty set.

(d) AC-3 reduces the domain of some variable to the empty set.

[SCORING [0...100]:

- +25pts for each correct answer
- -25pts for each incorrect answer
- 0pts for each unanswered question

]

3

Consider propositional logic (PL); let A, B, C, D, E, F, G be atomic propositions
For each of the following statements, say if it is true or false.

(a) The subformula $(A \rightarrow B)$ occurs only positively in the formula $(C \rightarrow (A \rightarrow B))$

(b) The subformula $(A \wedge \neg B)$ occurs only positively in the formula $((\neg C \vee D) \leftrightarrow (A \wedge \neg B))$

(c) The subformula $(\neg A \wedge B)$ occurs only negatively in the formula $\neg((\neg A \wedge B) \rightarrow C)$

(d) The subformula $(A \vee \neg B)$ occurs both positively and negatively in the formula $\neg(C \leftrightarrow (A \vee \neg B))$

[SCORING [0...100]:

- +25pts for each correct answer
- -25pts for each incorrect answer
- 0pts for each unanswered question

]

4

Given:

- the constant symbols, representing proper names: $\{Charles\}$
- the unary predicate symbols, representing adjectives or categories: $\{Expensive, Pen, Prince, Princess, Handsome, Pretty, Man, Woman, Person\}$
- the binary predicate symbols, representing verbs: $\{Owns, Marries, Beats, Loves, Respects, Appreciate\}$

with their standard interpretation.

For each of the following pairs ⟨[English sentence], [FOL formula]⟩, say if [English sentence] is correctly encoded into [FOL formula].

(*a*) ⟨ "Every prince owns some expensive pen", $\forall x.(Prince(x) \rightarrow \exists y.(Pen(y) \land Expensive(y) \land Owns(x, y)))$ ⟩

(*b*) ⟨ "Every man who respects all persons is appreciated by some person." , $\forall x.\forall y.\{[Man(x) \land Person(y) \land Respects(x, y)] \rightarrow [\exists z.(Person(z) \land Appreciate(z, x))]\}$ ⟩

(*c*) ⟨ "Every man is loved by some woman" , $\exists y.\{Woman(y) \land \forall x.[Man(x) \rightarrow Loves(x, y)]\}$ ⟩

(*d*) ⟨ "Some pretty woman loves Charles", $\exists x.(Woman(x) \land Pretty(x) \land Loves(x, Charles)$ ⟩

[SCORING [0...100]:

- +25pts for each correct answer
- -25pts for each incorrect answer
- 0pts for each unanswered question

]

[SCORING: [0...100], 25 pts for each correct answer, no penalties for wrong answers.]

5

Consider (normal) modal logics. Let $\mathsf{IsIdle}(\mathsf{Pump}), \mathsf{IsOff}(\mathsf{Light})$ be possible facts, let $Paul, Claire$ be agents and let $\mathbf{K}_{Paul}, \mathbf{K}_{Claire}$ denote the modal operators "Paul knows that..." and "Claire knows that..." respectively.
For each of the following facts, say if it is true or false.

(a) If $\neg\mathbf{K}_{Paul}(\mathsf{IsIdle}(\mathsf{Pump}) \wedge \mathsf{IsOff}(\mathsf{Light}))$ holds, then $\neg\mathbf{K}_{Paul}\mathsf{IsIdle}(\mathsf{Pump}) \wedge \neg\mathbf{K}_{\mathsf{Paul}}\mathsf{IsOff}(\mathsf{Light})$ holds

(b) If $\neg\mathbf{K}_{Paul}\mathsf{IsIdle}(\mathsf{Pump}) \wedge \neg\mathbf{K}_{\mathsf{Paul}}\mathsf{IsOff}(\mathsf{Light})$ holds, then $\neg\mathbf{K}_{Paul}(\mathsf{IsIdle}(\mathsf{Pump}) \wedge \mathsf{IsOff}(\mathsf{Light}))$ holds

(c) If $\mathsf{IsIdle}(\mathsf{Pump}) \leftrightarrow \mathsf{IsOff}(\mathsf{Light})$ and $\mathbf{K}_{Claire}\mathsf{IsIdle}(\mathsf{Pump})$ hold, then $\mathbf{K}_{Claire}\mathsf{IsOff}(\mathsf{Light})$ holds

(d) If $\mathbf{K}_{Paul}(\mathsf{IsIdle}(\mathsf{Pump}) \rightarrow \mathbf{K}_{\mathsf{Claire}}\mathsf{IsOff}(\mathsf{Light}))$ and $\mathbf{K}_{Paul}\mathsf{IsIdle}(\mathsf{Pump})$ hold, then $\mathbf{K}_{Paul}\mathbf{K}_{Claire}\mathsf{IsOff}(\mathsf{Light}))$ holds

[SCORING [0...100]:

- +25pts for each correct answer
- -25pts for each incorrect answer
- 0pts for each unanswered question

]

6

(a) Describe as Pseudo-Code the generic procedure for minimization via Simulated Annealing, assuming a function Schedule(t) returning a "temperature" parameter which slowly decreases with time step t, and a function Value(s) that measures the value of a state s to be minimized.

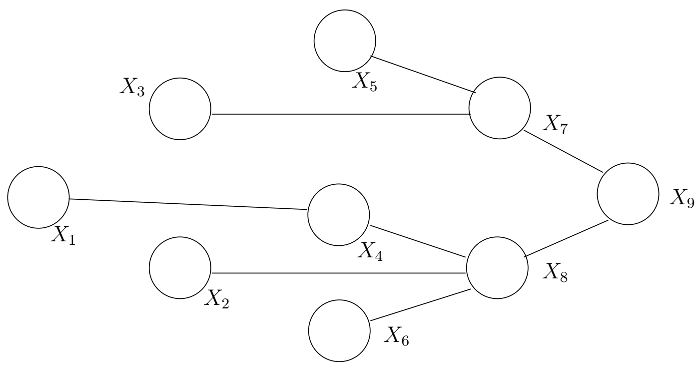
(b) Say if the following sentence is true or false, and briefly explain why.

- The probability of picking a "bad move" decreases as the "temperature" decreases.

[SCORING: [0...100], 75 pts for a correct answer to question (a), 25 pts for correct answer to question (b); no penalties for wrong answers.]

7

Consider the following tree-structured map-coloring problem, with domain $D \stackrel{\text{def}}{=} \{\text{Red}, \text{Green}, \text{Blue}\}$, with the following initial domain restrictions:



$X_1 = \{ \text{Red} \;, \quad , \quad \}$
$X_2 = \{ \quad , \text{Green} \;, \quad \}$
$X_3 = \{ \quad , \quad , \text{Blue} \}$
$X_4 = \{ \text{Red} \;, \text{Green} \;, \quad \}$
$X_5 = \{ \quad , \quad , \text{Blue} \}$
$X_6 = \{ \quad , \quad , \text{Blue} \}$
$X_7 = \{ \quad , \text{Green} \;, \text{Blue} \}$
$X_8 = \{ \text{Red} \;, \text{Green} \;, \text{Blue} \}$
$X_9 = \{ \text{Red} \;, \text{Green} \;, \text{Blue} \}$

Using the Tree-Structured Algorithm, and considering the following node ordering: $\{X_9, X_7, X_5, X_3, X_8, X_6, X_4, X_2, X_1\}$.

(a) Show the ordered progression of domain updates induced by the algorithm.
(b) As a consequence of the above process, state if the problem is solvable or not.
If yes, show one solution. (Produce the values following the ordering above.)
If not, explain why it is not.
(c) Let n, d be ths number of nodes and the (maximum) domain size respectively. What is the worst-case complexity of this algorithm?

Note: Solving the problem by using any other algorithm or any other ordering will be considered incorrect.
Notation: to represent the current domain of a node X_i, substitute with a blank " " any value in $\{\text{Red}, \text{Green}, \text{Blue}\}$ which cannot be assigned. (Ex: in current graph: $X_5 : \{ \;\;, \;\;, \text{Blue}\}$)

[SCORING: [0...100], 50 pts for correct answer to question (a), 25pts each for correct answer to (b) and (c). No penalties for wrong answers..]

8

Consider the following Horn formula in PL:

$$
\begin{array}{llll}
(\neg H & \vee \neg I & \vee \;\; A) & \wedge \\
(\;\; I & \vee \neg L & \vee \neg M) & \wedge \\
(\neg L & \vee \;\; C & \vee \neg M) & \wedge \\
(\neg A & \vee \;\; D &) & \wedge \\
(\neg E & \vee \neg F & \vee \;\; A) & \wedge \\
(\;\; G & \vee \neg A & \vee \neg E) & \wedge \\
(\neg L & \vee \;\; N & \vee \neg H) & \wedge \\
(\neg I & \vee \;\; L & \vee \neg N) & \wedge \\
(\;\; E & \vee \neg A & \vee \neg D) & \wedge \\
(\;\; A &) & & \wedge \\
(\neg E & \vee \neg A & \vee \;\; F) & \wedge \\
(\neg A & \vee \;\; H & \vee \neg C) & \wedge \\
(\;\; E & \vee \neg G & \vee \neg A) & \wedge \\
(\neg A & \vee \neg L & \vee \;\; M) &
\end{array}
$$

Using the simple polynomial procedure for Horn formulas,

(a) decide if the formula is satisfiable or not

(b) if satisfiable, return the satisfying total truth assignment;
if unsatisfiable, return the falsified clause.

Note: Solving the problem by using any other procedure will be considered incorrect.

[SCORING: [0...100], 75 pts for a correct answer to (a), 25 pts for correct answer to (b), no penalties for wrong answers.]

9

Let $P()$, $Q()$, $R()$ denote predicates, $f()$, $g()$, $h()$ denote functions, A, B denote constants, x, y, w, z denote variables, each possibly with suffixes.

For each of the following pair of clauses C_1, C_2:

- compute the most-general unifier (mgu) θ of their resolvent, or say "NULL" if none exists (represent the unifier as a list $\{variable_1/term_1, variable_2/term_2, ...\}$)
- compute the clause C resulting from resolving C_1 and C_2 with mgu θ, or say "NO SOLUTION" if resolution is not applicable

(a) $C_1 \stackrel{\text{def}}{=} P(h(x), B) \vee R(f(x), A)$
$C_2 \stackrel{\text{def}}{=} \neg P(A, y) \vee \neg R(f(g(B)), y)$

(b) $C_1 \stackrel{\text{def}}{=} P(x, h(x)) \vee Q(x, f(z))$
$C_2 \stackrel{\text{def}}{=} \neg Q(g(z), A) \vee P(g(B), y)$

(c) $C_1 \stackrel{\text{def}}{=} P(f(x), y) \vee Q(f(y), g(y))$,
$C_2 \stackrel{\text{def}}{=} P(f(A), h(z)) \vee \neg P(z, g(B))$

(d) $C_1 \stackrel{\text{def}}{=} R(f(A), f(z)) \vee \neg P(A, f(z))$,
$C_2 \stackrel{\text{def}}{=} P(y, z) \vee R(x, h(y))$

[SCORING: [0...100], 25 pts for each correct answer, no penalties for wrong answers.]

10

Given the random propositional variables a, b, c and their joint probability distribution $\mathbf{P}(\mathbf{A}, \mathbf{B}, \mathbf{C})$ described as follows:

A	B	C	$\mathbf{P}(\mathbf{A}, \mathbf{B}, \mathbf{C})$
T	T	T	0.056
F	T	T	0.024
T	F	T	0.192
F	F	T	0.128
T	T	F	0.023
F	T	F	0.097
T	F	F	0.048
F	F	F	0.432

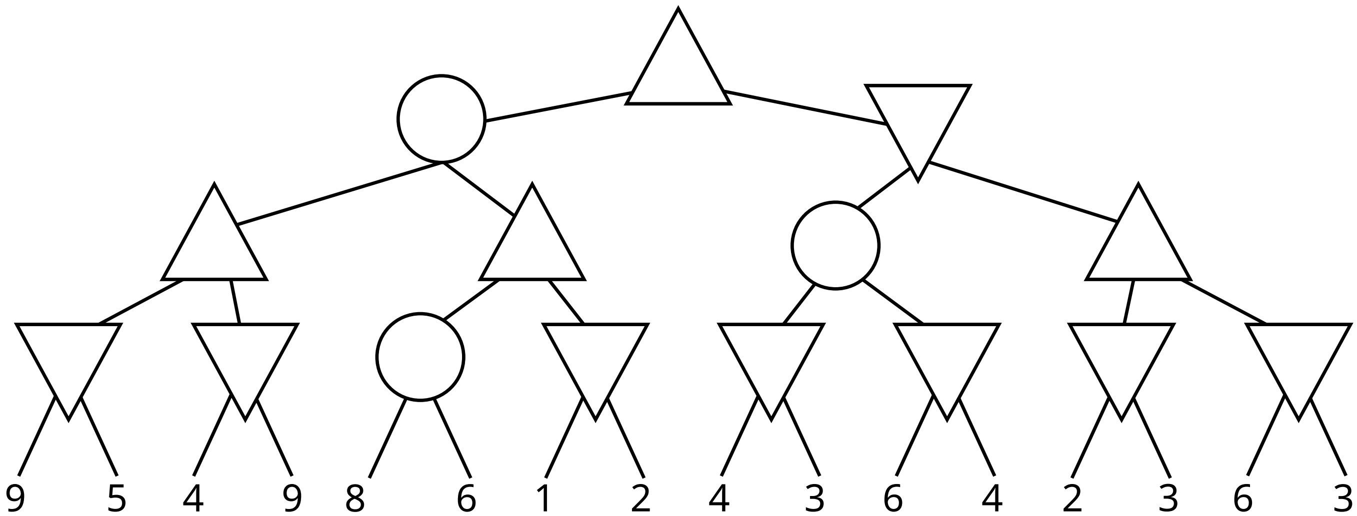
(a) Using marginalization, compute the probability $P(c)$

(b) Using normalization, compute the probability distribution $\mathbf{P}(\mathbf{A}|\neg b)$.
Note: Solving the problem by using any other technique will be considered incorrect.

[SCORING: [0...100], 50 pts for each correct answer, no penalties for wrong answers.]

11

Use *ExpectiMinimax* to propagate the expected utilities from the leaves to the root of the tree. Chance nodes assign uniform probabilities to their children.

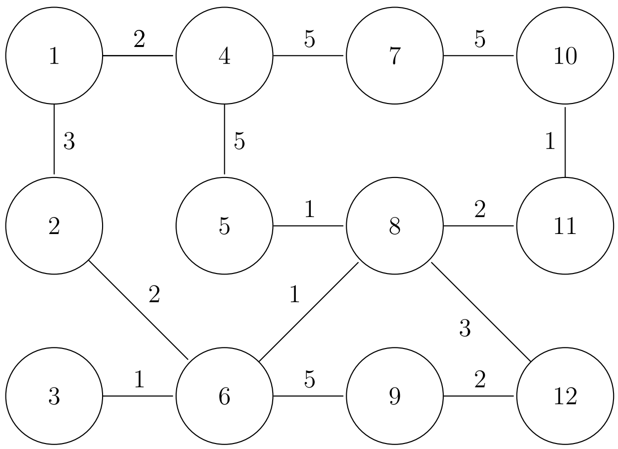


For each MIN/MAX/CHANCE node, write inside the expected utility.

[SCORING: [0...100], 100pts for a fully correct solution, -20pts for each error in the resolution process. The score cannot go below 0..]

12

Apply A* to the following search problem:



Initial state: 1
Goal state: 10

All actions can be reverted and their cost is denoted on the edges.

- Actions are sorted according to the destination state with ascending numerical order

$$\text{e.g. } next(s, a) = 7 \wedge next(s, a') = 3 \Rightarrow a' < a$$

- The heuristic function is the Manhattan distance from the goal relative to the position in the grid (e.g., if the goal is in position $(2, 2)$, then the node n in position $(1, 0)$ has $h(n) = 3$).

[10 pt. /100] Is the heuristic function admissible? (Motivate your answer).
[90 pt. /100] For each step of the execution, report:

- The node extracted from the priority queue.
- The elements added to the priority queue *after* expanding.

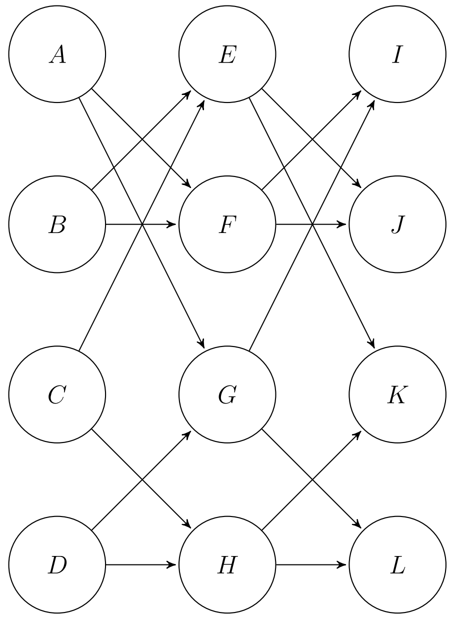
Then report the the path returned by A*.

[SCORING: [0...100], 100pts for a fully correct solution, -10pts for each error in the resolution process. The score cannot go below 0..]

13

Consider the following actions with durations / dependencies:

a	duration(a)
A	3
B	1
C	1
D	1
E	2
F	3
G	3
H	2
I	3
J	2
K	1
L	2



Using the Critical Path method:

- Compute the earliest / latest possible start time (ES/LS) for each action.
- Indicate all the actions that are in a critical path and the minimum makespan.

[SCORING: [0...100], 100pts for a fully correct solution, -20pts for each error in the resolution process. The score cannot go below 0..]

14

Apply a *genetic algorithm* to the following optimization problem on ternary ($p_i \in \{0, 1, 2\}$) strings of length 4. The fitness function of individuals $\mathbf{p} = p_1p_2p_3p_4$ is:

$$f(\mathbf{p}) = \sum_{i=1}^{4} p_i \qquad \text{(e.g. } f(1210) = 4 \text{)}$$

Parents are selected from the available individuals with *probability directly proportional to their fitness value*.
There are 3 possible split points:

$$S1 = p_1|p_2p_3p_4$$
$$S2 = p_1p_2|p_3p_4$$
$$S3 = p_1p_2p_3|p_4$$

selected with *equal probability* 1/3. Mutations happen with probability 0.4 to each element independently.

$$[m(p_i) = p_i : 0.6, \qquad m(p_i) = (p_i + 1) \texttt{ mod } 3 : 0.4]$$

Apply a genetic algorithm until 4 new individuals are generated, report every step of the generation.
The *initial population* (iteration 0) is:

$$1201, 2000, 1021, 0002$$

The *choice vector* is:

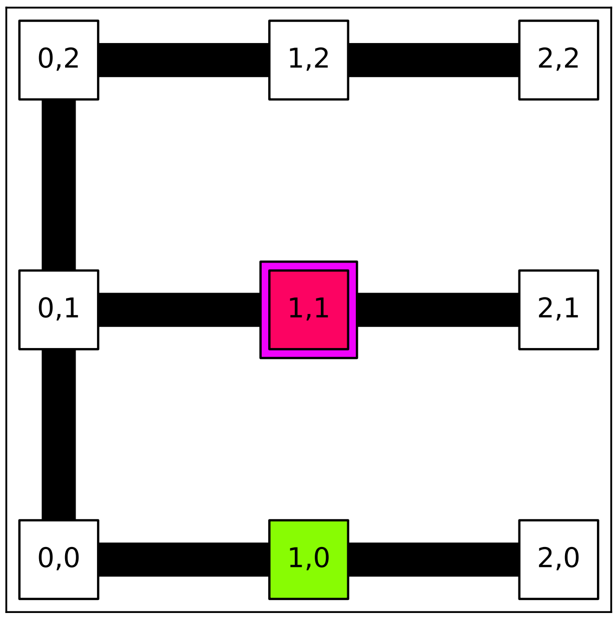
$$[3/4, 1/100, 1/2]$$

Report the all the individuals of the population resulting from one full iteration of the algorithm, as well as the intermediate steps (i.e., for each new individual, how the indices of the parents and split point were selected).

[SCORING: [0...100], 100pts for a fully correct solution, -15pts for each error in the resolution process. The score cannot go below 0..]

15

In the following state graph, apply *online DFS* and report the list of visited states, including repetitions (e.g. $(0,1) \rightarrow (0,0) \rightarrow (0,1) \rightarrow ...$). The order of (untried) actions is $[up, right, down, left]$.



Start: $(1,1)$ - Goal: $(1,0)$

[SCORING: [0...100], 100pts for a fully correct solution, -10pts for each error in the resolution process. The score cannot go below 0..]