# Course "**Fundamentals of Artificial Intelligence**"
# EXAM TEXT

Prof. Roberto Sebastiani
DISI, Università di Trento, Italy

2021.09.07

13276914

[COPY WITH SOLUTIONS]

# 1

Consider propositional logic (PL);
let $\varphi, \psi, \phi$ be generic propositional formulas;
let $A$, $B$, $C$, $D$, $E$, $F$, $G$ be atomic propositions which do not occur in $\varphi, \psi, \phi$.
For each of the following statements, say if it is true or false.

($a$) $(\varphi \wedge \psi) \models \phi$ if and only if $\varphi \models (\psi \rightarrow \phi)$
[ Solution: True ]

($b$) $\neg\varphi \vee (\psi \wedge \phi)$ is equivalent to $(\neg\varphi \vee A) \wedge (\neg A \vee \psi) \wedge (\neg A \vee \phi) \wedge (A \vee \neg\psi \vee \neg\phi)$
[ Solution: False (it is equi-satisfiable to it) ]

($c$) $(\varphi \wedge \neg\psi) \rightarrow \neg\phi$ is valid if and only if $(\varphi \wedge \neg\psi \wedge \phi)$ is unsatisfiable
[ Solution: True ]

($d$) $\varphi \models (\psi \wedge \phi)$ if and only if $\varphi \models \psi$ and $\varphi \models \phi$
[ Solution: true ]

# 2

Consider (normal) modal logics. Let $\mathsf{IsOn}(\mathsf{Switch}), \mathsf{IsRunning}(\mathsf{Engine})$ be possible facts, let $Susy, Bill$ be agents and let $\mathbf{K}_{Susy}, \mathbf{K}_{Bill}$ denote the modal operators "Susy knows that..." and "Bill knows that..." respectively.
For each of the following facts, say if it is true or false.

($a$) If $\mathbf{K}_{Susy}(\mathsf{IsOn}(\mathsf{Switch}) \wedge \mathsf{IsRunning}(\mathsf{Engine}))$ holds, then $\mathbf{K}_{Susy}\mathsf{IsOn}(\mathsf{Switch}) \wedge \mathbf{K}_{\mathsf{Susy}}\mathsf{IsRunning}(\mathsf{Engine})$ holds
[ Solution: true ]

($b$) If $\mathbf{K}_{Susy}(\mathsf{IsOn}(\mathsf{Switch}) \vee \mathsf{IsRunning}(\mathsf{Engine}))$ holds, then $\mathbf{K}_{Susy}\mathsf{IsOn}(\mathsf{Switch}) \vee \mathbf{K}_{\mathsf{Susy}}\mathsf{IsRunning}(\mathsf{Engine})$ holds
[ Solution: false ]

($c$) If $\mathbf{K}_{Bill}\mathsf{IsOn}(\mathsf{Switch})$ and $\mathsf{IsOn}(\mathsf{Switch}) \leftrightarrow \mathsf{IsRunning}(\mathsf{Engine})$ hold, then $\mathbf{K}_{Bill}\mathsf{IsRunning}(\mathsf{Engine})$ holds
[ Solution: false ]

($d$) If $\mathbf{K}_{Susy}\mathsf{IsOn}(\mathsf{Switch})$ and $\mathbf{K}_{Susy}(\mathsf{IsOn}(\mathsf{Switch}) \rightarrow \mathbf{K}_{\mathsf{Bill}}\mathsf{IsOn}(\mathsf{Switch}))$ hold, then $\mathbf{K}_{Susy}\mathbf{K}_{Bill}\mathsf{IsOn}(\mathsf{Switch}))$ holds
[ Solution: true ]

# 3

Let us consider the PEAS description of the task environment for an automated taxi. For each of the following facts, say if it is true or false.

($a$) Safety is a Actuator
[ Solution: false ]

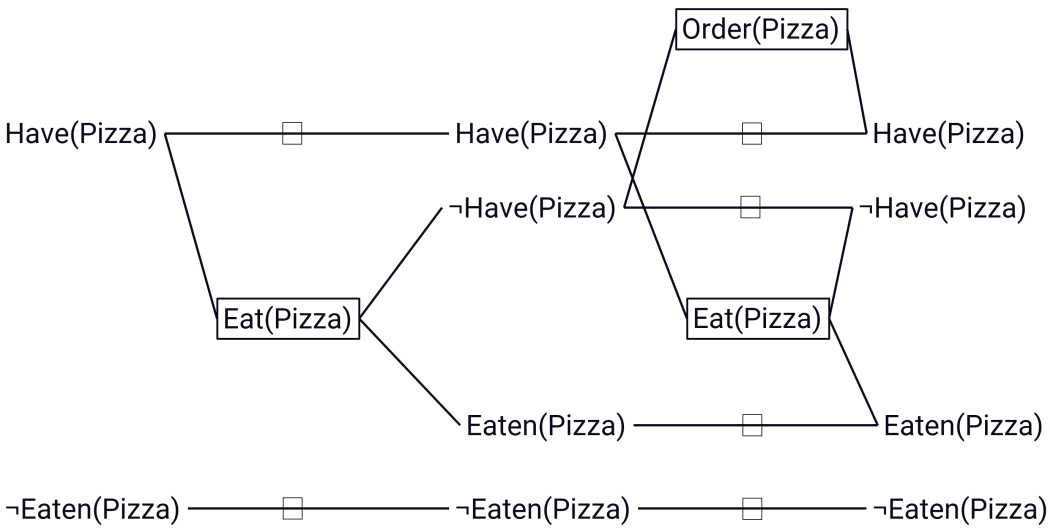
($b$) Traffic is a Environment
[ Solution: true ]

($c$) Brake is a Actuator
[ Solution: true ]

($d$) Cameras is a Performance Measure
[ Solution: false ]

# 4

Consider the graph shown below.




For each of the following facts, say if it is true or false.

($a$) At level $A_0$ there are no mutex between Eat and the persistence of Have
[ Solution: false ]

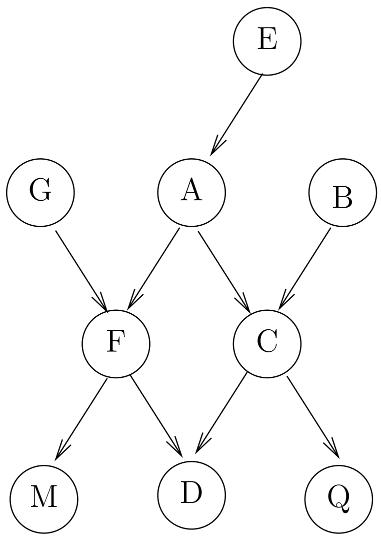
($b$) At level $A_1$ the mutex between Eat and Order is a competing needs.
[ Solution: true ]

($c$) At level $S_1$ there are no mutex between ¬ Have and Eaten
[ Solution: true ]

($d$) At level $A_1$ the mutex between Eat and the persistence of Have is an interference
[ Solution: true ]

# 5

Consider the following DAG of a Bayesian network.




For each of the following facts, say if it is true or false.

(a) $\mathbf{P}(F|GAM) = \mathbf{P}(C|GA)$
[ Solution: false ]

(b) $\mathbf{P}(F|GACB) = \mathbf{P}(F|GA)$
[ Solution: true, due to local semantics ]

(c) $\mathbf{P}(C|ABE) = \mathbf{P}(C|AB)$
[ Solution: true, due to local semantics ]

(d) $\mathbf{P}(F|GAMDC) = \mathbf{P}(F|GAMDCEBQ)$
[ Solution: true, due to Markov blanket rule ]

# 6

Consider the following Horn formula in PL:

$$
\begin{array}{llll}
( \; G & \lor \neg A & \lor \neg E) & \land \\
(\neg L & \lor \; N & \lor \neg H) & \land \\
(\neg A & \lor \; D & ) & \land \\
(\neg I & \lor \; L & \lor \neg N) & \land \\
( \; E & \lor \neg A & \lor \neg D) & \land \\
(\neg E & \lor \neg A & \lor \; F) & \land \\
(\neg A & \lor \; H & \lor \neg C) & \land \\
(\neg H & \lor \neg I & \lor \; A) & \land \\
( \; I & \lor \neg L & \lor \neg M) & \land \\
(\neg L & \lor \; C & \lor \neg M) & \land \\
(\neg E & \lor \neg F & \lor \neg A) & \land \\
( \; E & \lor \neg G & \lor \neg A) & \land \\
(\neg A & \lor \neg L & \lor \; M) & \land \\
( \; A & ) & &
\end{array}
$$

Using the simple polynomial procedure for Horn formulas,

($a$) decide if the formula is satisfiable or not

($b$) if satisfiable, return the satisfying truth assignment;
if unsatisfiable, return the falsified clause.

[ Solution:

($a$) (i) run unit propagation: $A$, $D$, $E$, $G$, $F$. The resulting formula is:

$$
\begin{array}{lllll}
( \; G & \lor & \lor) & \land & \\
(\neg L & \lor \; N & \lor \neg H) & \land & \\
( & \lor \; D & ) & \land & \\
(\neg I & \lor \; L & \lor \neg N) & \land & \\
( \; E & \lor & \lor) & \land & \\
( & \lor & \lor \; F) & \land & \\
( & \lor \; H & \lor \neg C) & \land & \\
(\neg H & \lor \neg I & \lor \; A) & \land & \\
( \; I & \lor \neg L & \lor \neg M) & \land & \\
(\neg L & \lor \; C & \lor \neg M) & \land & \\
( & \lor & \lor) & \land & \times \times \times \\
( \; E & \lor & \lor) & \land & \\
( & \lor \neg L & \lor \; M) & \land & \\
( \; A & ) & & &
\end{array}
$$

(ii) the resulting formula contains an empty clause, thus the original formula is unsatisfiable

($b$) the falsified clause is $(\neg E \lor \neg F \lor \neg A)$

]

# 7

Given:

- the constant symbols: $\{Charles\}$
- the unary predicate symbols: $\{Pen, Artisan, Expensive, Man, Woman\}$
- the binary predicate symbols: $\{Possesses, Builds, Loves, IsFriendOf\}$ (usage: "Verb(Subject,Object)")
- the standard binary equality predicate "="

Translate the following English sentences into FOL formulas

($a$) All pens Charles possesses are expensive
[ Solution: $\forall x.(Pen(x) \wedge Possesses(Charles, x)) \rightarrow Expensive(x))$
or any formula which is logically equivalent to it ]

($b$) All pens Charles possesses are built by some artisan
[ Solution: $\forall x.(Pen(x) \wedge Possesses(Charles, x)) \rightarrow \exists y.(Artisan(y) \wedge Builds(x, y))$
or any formula which is logically equivalent to it ]

($c$) Charles loves a woman no other man loves
[ Solution: $\exists x.(Woman(x) \wedge Loves(Charles, x) \wedge \forall y.((Man(y) \wedge Loves(y, x)) \rightarrow (y = Charles))$
or any formula which is logically equivalent to it ]

($d$) Not all friends of Charles' friends are Charles' friends.
[ Solution: $\neg\forall x.\forall y.(IsFriendOf(x, Charles) \wedge IsFriendOf(y, x)) \rightarrow IsFriendOf(y, Charles)$
or, equivalently,
$\exists x.\exists y.(IsFriendOf(x, Charles) \wedge IsFriendOf(y, x) \wedge \neg IsFriendOf(y, Charles))$ or any formula which is logically equivalent to it ]

# 8

For each of the following FOL formulas, compute its CNF-Ization.
Use symbols $C_1, C_2, C_3, ...$ for Skolem constants and symbols $F_1, F_2, F_3, ...$ for Skolem functions.

$(a)$ $\neg\forall x.\exists y.\forall z.P(x, y, z)$
[ Solution:
$\neg\forall x.\exists y.\forall z.P(x, y, z)$
$\exists x.\forall y.\exists z.\neg P(x, y, z)$
$\neg P(C_1, y, F_1(y))$ ]

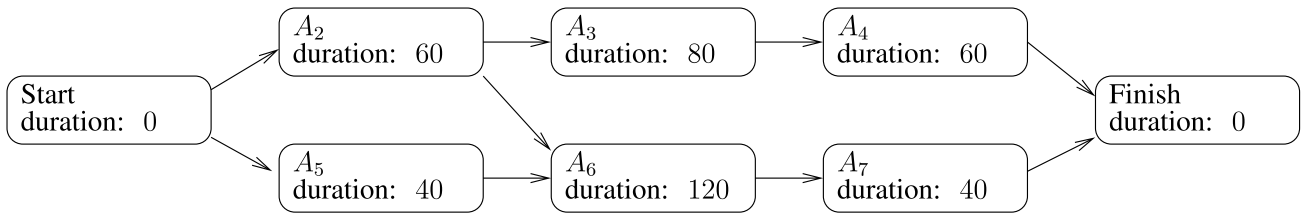
$(b)$ $\exists x.(\forall y.P(x, y) \rightarrow \exists z.Q(x, z))$
[ Solution:
$\exists x.(\forall y.P(x, y) \rightarrow \exists z.Q(x, z))$
$\exists x.(\neg\forall y.P(x, y) \vee \exists z.Q(x, z))$
$\exists x.(\exists y.\neg P(x, y) \vee \exists z.Q(x, z))$
$\neg P(C_1, C_2) \vee Q(C_1, C_3)$ ]

$(c)$ $\forall x.(\forall y.P(x, y) \leftrightarrow \exists z.Q(x, z))$
[ Solution:
$\forall x.(\forall y.P(x, y) \leftrightarrow \exists z.Q(x, z))$
$\forall x.((\neg\forall y.P(x, y) \vee \exists z.Q(x, z)) \wedge (\forall y.P(x, y) \vee \neg\exists z.Q(x, z)))$
$\forall x.((\exists y.\neg P(x, y) \vee \exists z.Q(x, z)) \wedge (\forall y.P(x, y) \vee \forall z.\neg Q(x, z)))$
$((\neg P(x, F_1(x)) \vee Q(x, F_2(x))) \wedge (P(x, y) \vee \neg Q(x, z)))$ ]

$(d)$ $(\forall x.\exists y.\forall z.P(x, y, z)) \rightarrow (\forall x.\exists y.\forall z.Q(x, y, z))$
[ Solution:
$(\forall x.\exists y.\forall z.P(x, y, z)) \rightarrow (\forall x.\exists y.\forall z.Q(x, y, z))$
$(\neg\forall x.\exists y.\forall z.P(x, y, z)) \vee (\forall x.\exists y.\forall z.Q(x, y, z))$
$(\exists x.\forall y.\exists z.\neg P(x, y, z)) \vee (\forall x.\exists y.\forall z.Q(x, y, z))$
$\neg P(C_1, y, F_1(y)) \ \vee \ Q(x, F_2(x), z)$ ]

# 9

Consider the following partial-order plan, with respective predicted duration of each action.




(a) Find the critical path in the form $\{Start, A_{i_1}, ..., A_{i_k}, Finish\}$ and its duration.
[ Solution: Critical path: $\{Start, A_2, A_6, A_7, Finish\}$
Duration: 60+120+40=220 ]

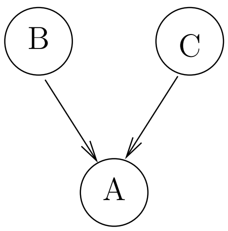
Suppose that, due to some technical problem, $A_5$ takes 60 time units longer than expected. If so:

(b) How much the start of $A_6$ is delayed of?
[ Solution: 40+60-60=40 time units ]

(c) what would be critical path and the global duration then?
[ Solution: Critical path: $\{Start, A_5, A_6, A_7, Finish\}$
Duration: (40+60)+120+40=260 time units ]

# 10

Consider the following simple Bayesian network:




$P(b) \stackrel{\text{def}}{=} 0.5,\ P(c) \stackrel{\text{def}}{=} 0.3,\ P(a|B,C) \stackrel{\text{def}}{=}$

| $B$ | $C$ | $P(a\|B,C)$ |
|---|---|---|
| $T$ | $T$ | 0.7 |
| $F$ | $T$ | 0.3 |
| $T$ | $F$ | 0.2 |
| $F$ | $F$ | 0.1 |

($a$) Compute the joint probability distribution $\mathbf{P(A, B, C)}$

[ Solution:

| $A$ | $B$ | $C$ | $\mathbf{P(A, B, C)} \stackrel{\text{def}}{=} \mathbf{P(A\|B, C)} * \mathbf{P(B)} * \mathbf{P(C)}$ |
|---|---|---|---|
| $T$ | $T$ | $T$ | $0.7 * 0.5 * 0.3 = 0.105$ |
| $T$ | $T$ | $F$ | $0.2 * 0.5 * 0.7 = 0.07$ |
| $T$ | $F$ | $T$ | $0.3 * 0.5 * 0.3 = 0.045$ |
| $T$ | $F$ | $F$ | $0.1 * 0.5 * 0.7 = 0.035$ |
| $F$ | $T$ | $T$ | $0.3 * 0.5 * 0.3 = 0.045$ |
| $F$ | $T$ | $F$ | $0.8 * 0.5 * 0.7 = 0.28$ |
| $F$ | $F$ | $T$ | $0.7 * 0.5 * 0.3 = 0.105$ |
| $F$ | $F$ | $F$ | $0.9 * 0.5 * 0.7 = 0.315$ |

]

($b$) Using marginalization, compute the probability $P(a)$

[ Solution:

$P(a)$

$= P(a, b, c) + P(a, b, \neg c) + P(a, \neg b, c) + P(a, \neg b, \neg c) =$

$= 0.105 + 0.070 + 0.045 + 0.035 = 0.255$ ]

($c$) Using normalization, compute the probability $P(a|b, c)$ and verify that the result matches the $P(a|B, C)$ matrix above. [ Solution:

$$P(a|b,c) = \alpha \cdot P(a,b,c) = \overbrace{\frac{1}{P(a,b,c) + P(\neg a, b, c)}}^{\alpha} \cdot P(a,b,c) =$$

$1/(0.105 + 0.045) * 0.105 = 0.7$ ]

Notation: uppercase letters are used for propositional variables representing random events, whereas the corresponding lowercase letters represent truth assignments to such propositional variables. Ex: $a \stackrel{\text{def}}{=} (A = \mathsf{true})$, $\neg a \stackrel{\text{def}}{=} (A = \mathsf{false})$.

# 11

Given the tree provided within the included paper, please do the following tasks:

- Report the alpha, beta, and node values for each MIN and MAX nodes. Each value has to be provided directly within the paper behind the $\infty$ symbol for the alpha and beta values, and within each MIN and MAX node.
- Mark the pruned branches. When a pruning operation is performed, each element under the pruned branch (including the pruned branch) has to be marked with the X.

[ Solution:

The solutions is available within the file 2021-09-07-alphabetapruning-v1-withsolutions.pptx in your Google Drive folder.
]

# 12

The graph contained within the included paper represents the states space of a hypothetical search problem where:

- States are denoted by letters.
- Arcs are labeled with the cost of traversing them.
- The estimated cost to a goal (i.e., the $h$ function) is reported inside nodes (so that lower scores are better).

Considering $ST$ as the initial state and $EN$ the goal state, please apply the A* search algorithm and report each step of the resolution process. Then, explain if the heuristic adopted is admissible or not. The solution format has to be provided as shown in the example provided.

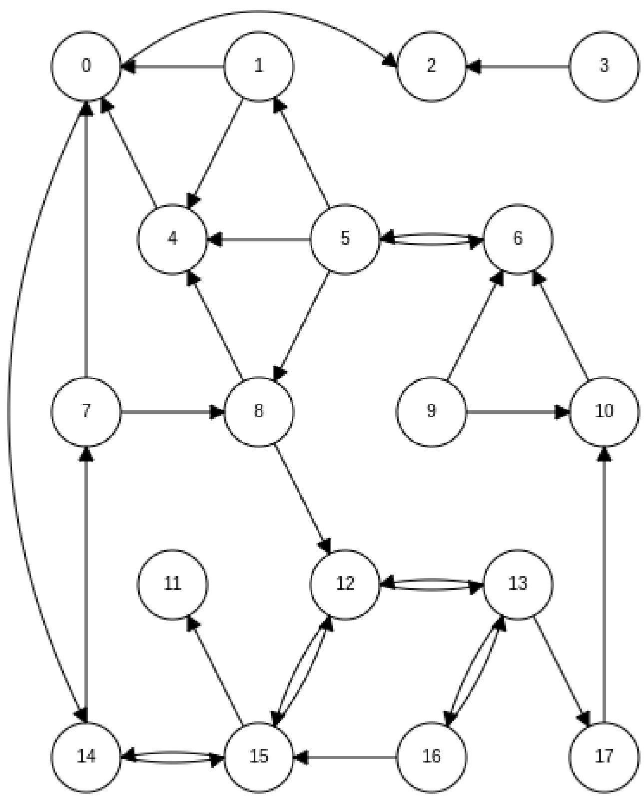
[ Solution:

The solutions is available within the file 2021-09-07-astar-v1-withsolutions.pptx in your Google Drive folder.

]

# 13

Consider the graph shown below.




Please complete the following tasks.

- Provide the list of nodes explored by the BFS algorithm performing the goal test after the generation of a new node.
- Provide the list of nodes explored by the DFS algorithm.

For both BFS and DFS algorithm, the starting node is 0 and the goal state is 16. Nodes are explored in numerical descending order.

[ Solution:

Exploration of the BFS algorithm:
0, 14, 2, 15, 7, 12, 11, 8, 13, 4, 17, 10, 16

Exploration of the DFS algorithm:
0, 14, 15, 12, 13, 17, 16
]

# 14

Consider the following constraint network.
Variables: $X_1, X_2, X_3, X_4, X_5$
Domains: $D_1 = \{3, 4, 5, 6, 7, 9\}, D_2 = \{2, 3, 4, 6, 8, 9\}, D_3 = \{1, 2, 6, 8, 9\}, D_4 = \{1, 2, 3, 5, 8, 9\}, D_5 = \{2, 5, 6, 7, 8\}$ Constraints:

$$X_1 > X_2 \text{ } or \text{ } X_2 - X_1 = 2$$

$$X_2 < X_3$$

$$X_2 < X_4 \text{ } or \text{ } X_2 - X_4 = 1$$

$$X_3 < X_5$$

Please complete the following tasks.

(a) Is the network arc-consistent? If not, compute the arc-consistent network.

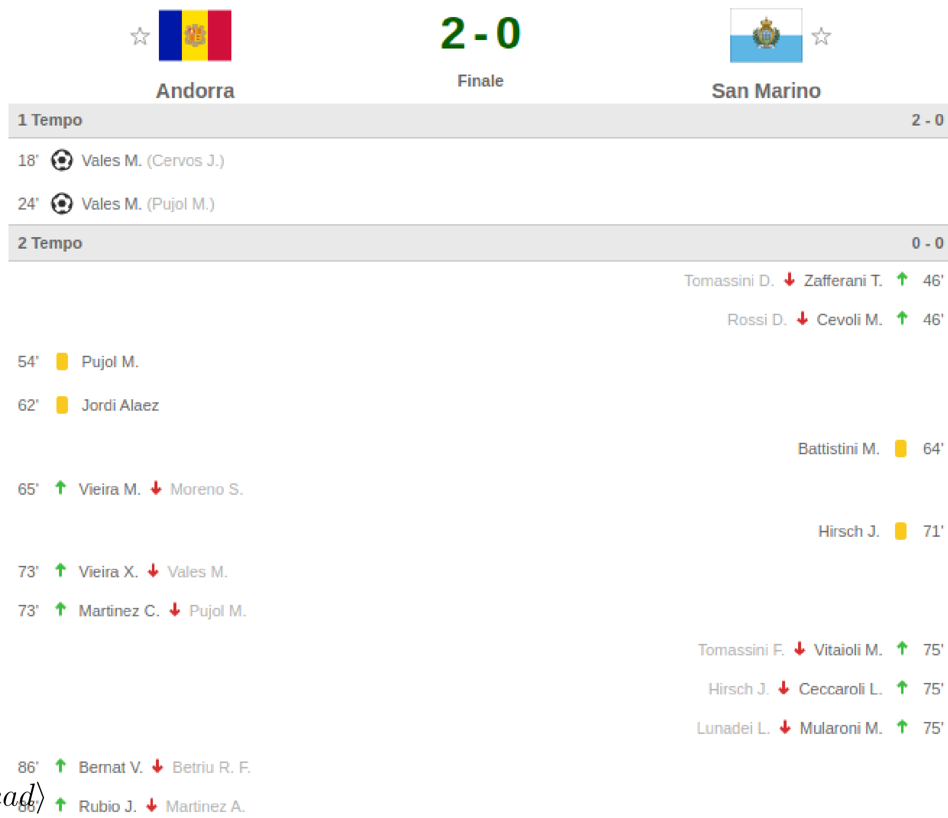
(b) If the consistency holds, provide the first admissible solution by exploring the domains from $D_1$ to $D_5$ and the values in descending order.

[ Solution:
Enforce arc consistency between $X_2$ and $X_3$ led to $D_2 = \{2, 3, 4, 6, 8\}$ and $D_3 = \{6, 8, 9\}$
Enforce arc consistency between $X_3$ and $X_5$ led to $D_3 = \{6\}$ and $D_5 = \{7, 8\}$
Enforce arc consistency between $X_2$ and $X_3$ has to be done again and it leads to $D_2 = \{2, 3, 4\}$

One possible solution: $X_1 = 9,\ X_2 = 4,\ X_3 = 6,\ X_4 = 9,\ X_5 = 8$ ]

# 15

Given the image below, please state the interval-algebra relations that hold between the provided pairs related to the described real-world event:




Pairs:
$\langle FirstHalf, HomeTeamAhead\rangle$
$\langle PujolWarned, ValesScored\rangle$
$\langle MartinezPlayed, CeccaroliPlayed\rangle$
$\langle BattistiniWarned, HomeTeamAhead\rangle$
$\langle ValesScored, SecondHalf\rangle$
$\langle ValesScored, MularoniPlayed\rangle$
$\langle FirstHalf, RubioPlayed\rangle$
$\langle HirschWarned, TieResult\rangle$

Notice and notations:

- events like goals, have not to be intended as instantaneous events, but like events during a certain (small) amount of time;
- a player is intended to be warned from the moment in which he received the yellow card, until the end of the match or until the moment in which he is substituted;
- you have to assume that the halftime break exists.
- the list of relations have to be provided by using the format: $Relation(Event1, Event2)$

[ Solution:

$Overlap(FirstHalf, HomeTeamAhead)$
$After(ValesScored, PujolWarned)$
$Finishes(MartinezPlayed, CeccaroliPlayed)$
$Finishes(BattistiniWarned, HomeTeamAhead)$
$Before(ValesScored, SecondHalf)$
$Before(ValesScored, MularoniPlayed)$
$Before(FirstHalf, RubioPlayed)$
$After(HirschWarned, TieResult)$

]