

Fundamentals of Artificial Intelligence

Chapter 03: Problem Solving as Search

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Outline

- 1 Problem-Solving Agents
- 2 Example Problems
- 3 Search Generalities
 - Tree Search
 - Graph Search
 - Implementation Issues & Strategies
- 4 Uninformed Search Strategies
 - Breadth-First Search
 - Uniform-cost Search
 - Depth-First Search
 - Depth-Limited Search & Iterative Deepening
- 5 Informed Search Strategies
 - Greedy Best-First Search
 - A* Search
 - Memory-bounded Heuristic Search (hints)
 - Heuristic Functions

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Problem Solving as Search

One of the dominant approaches to AI problem solving:
formulate a problem/task as search in a state space.

Main Paradigm

- Goal formulation: define the successful states
 - Ex: a set of states, a Boolean test function ...
- Problem formulation:
 - define a representation for states
 - define legal actions and transition functions
- Search: find a solution by means of a search process
 - solutions are sequences of actions
- Execution: given the solution, perform the actions

⇒ Problem-solving agents are (a kind of) goal-based agents

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Problem Solving as Search: Example

Example: Traveling in Romania

- **Informal description:** On holiday in Romania; currently in Arad. Flight leaves tomorrow from Bucharest
- **Formulate goal:** (Be in) Bucharest
- **Formulate problem:**
 - States: various cities
 - Actions: drive between cities
 - Initial state: Arad
- **Search for a solution:** sequence of cities from Arad to Bucharest
 - e.g. Arad, Sibiu, Fagaras, Bucharest
 - explore a search tree/graph

Note

The agent is assumed to have no heuristic knowledge about traveling in Romania to exploit.

Problem Solving as Search: Example

Example: Traveling in Romania

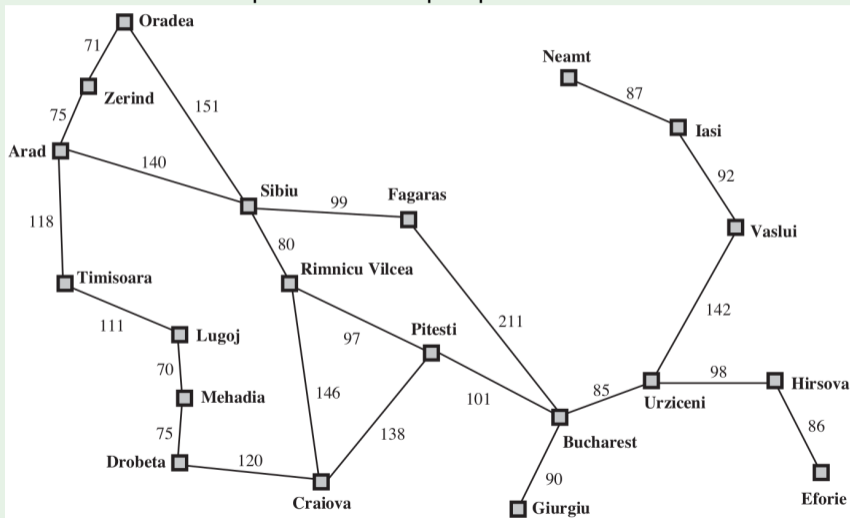
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Problem Solving as Search: Example [cont.]

A simplified road map of part of Romania.



Problem Solving as Search [cont.]

Assumptions for Problem-solving Agents (this chapter only)

- state representations are **atomic**
 - ⇒ world states are considered as wholes, with no internal structure
 - Ex: Arad, Sibiu, Zerind, Bucharest,... (shortcut for In(Arad), In(Sibiu), ...)
- the environment is **fully observable**
 - ⇒ the agent always knows the current state
 - Ex: Romanian cities & roads have signs
- the environment is **discrete**
 - ⇒ at any state there are only finitely many actions to choose from
 - Ex: from Arad, (go to) Sibiu, or Zerind, or Timisoara (see map)
- the environment is **known**
 - ⇒ the agent knows which states are reached by each action
 - ex: the agent has the map
- the environment is **deterministic**
 - ⇒ each action has exactly one outcome
 - Ex: from Arad choose go to Sibiu ⇒ next step in Sibiu

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Problem Solving as Search [cont.]

Remarks about search

- Search happens **inside the agent**
 - a **planning** stage before acting
 - different from **searching in the world**
- An agent is given a description of what to achieve, not an algorithm to solve it
⇒ the only possibility is to search for a solution
- Searching can be computationally very demanding (NP-hard)
- Can be driven with benefits by knowledge of the problem (heuristic knowledge)
⇒ **informed/heuristic search**

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Problem-solving Agent: Schema

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function SIMPLE-PROBLEM-SOLVING-AGENT(percept) returns an action
  persistent: seq, an action sequence, initially empty
               state, some description of the current world state
               goal, a goal, initially null
               problem, a problem formulation

  state ← UPDATE-STATE(state, percept)
  if seq is empty then
    goal ← FORMULATE-GOAL(state)
    problem ← FORMULATE-PROBLEM(state, goal)
    seq ← SEARCH(problem)
    if seq = failure then return a null action
  action ← FIRST(seq)
  seq ← REST(seq)
  return action
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While executing the solution sequence the agent ignores its percepts when choosing an action since it knows in advance what they will be (“open loop system”)

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Well-defined problems and solutions

Problem Formulation: Components

- the **initial state** the agent starts in
 - Ex: $In(Arad)$
- the set of **applicable actions** available in a state ($ACTIONS(s)$)
 - Ex: if s is $In(Arad)$, then the applicable actions are $\{Go(Sibiu), Go(Timisoara), Go(Zerind)\}$
- a description of what each action does (aka **transition model**)
 - $RESULT(s,A)$: state resulting from applying action A in state s
 - Ex: $RESULT(IN(ARAD), GO(ZERIND))$ is $IN(ZERIND)$
- the **goal test** determining if a given state is a goal state
 - Explicit (e.g.: $\{In(Bucharest)\}$)
 - Implicit (e.g. (Ex: $CHECKMATE(x)$))
- the **path cost** function assigns a numeric cost to each path
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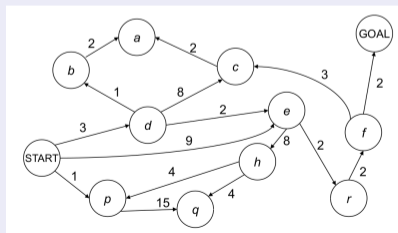
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Well-defined problems and solutions [cont.]

State Space, Graphs, Paths, Solutions and Optimal Solutions

Initial state, **actions**, and **transition model** implicitly define the **state space** of the problem

- the state space forms a **directed graph** (e.g. the Romania map)
 - typically too big to be created explicitly and be stored in full
 - in a state space graph, each state occurs only once
- a **path** is a sequence of states connected by actions
- a **solution** is a path from the initial state to a goal state
- an **optimal solution** is a solution with the lowest path cost

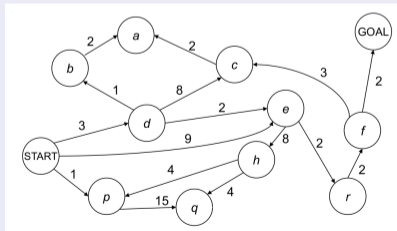


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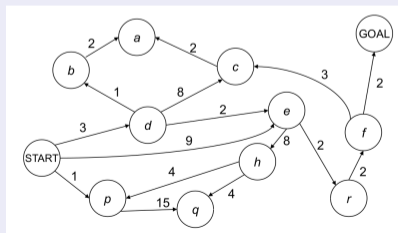


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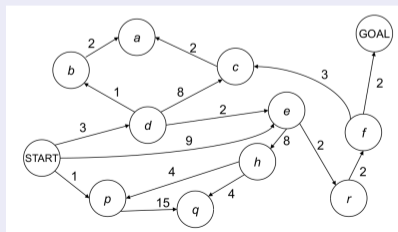


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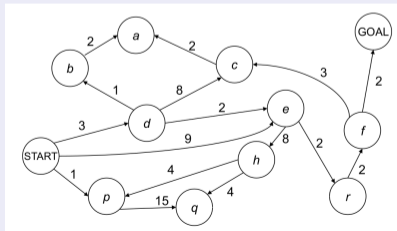


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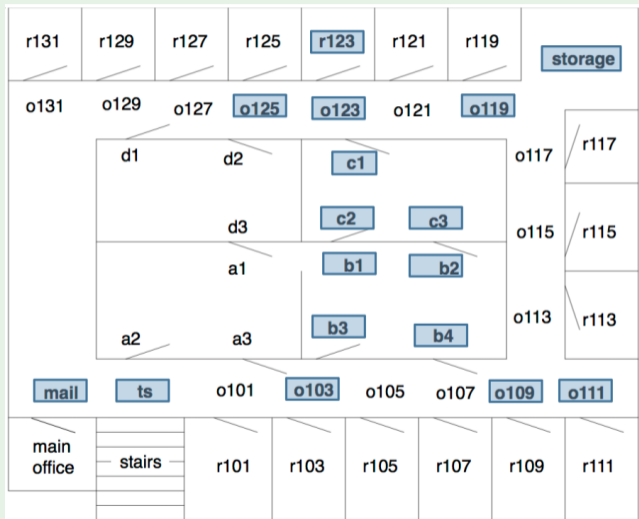


Example: Path finding for a Delivery Robot

Task: move from o103 to r123

- States

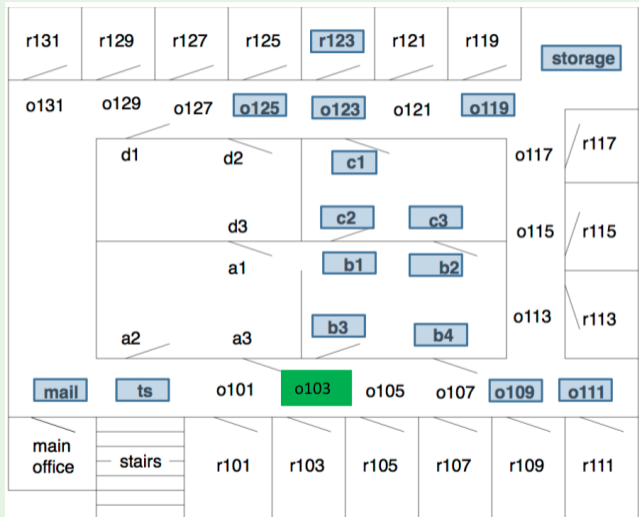
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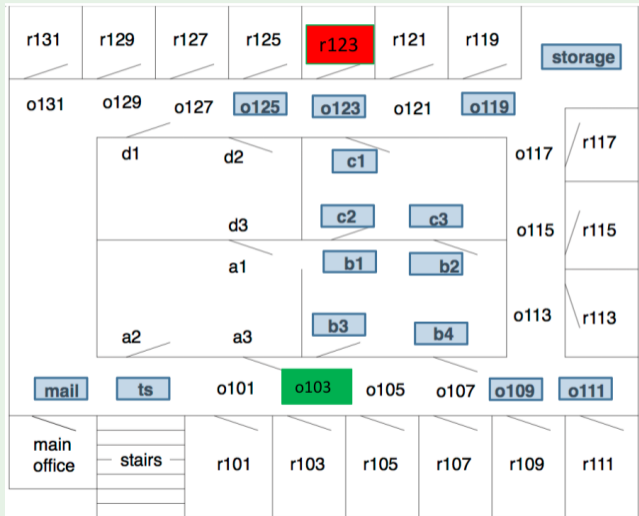
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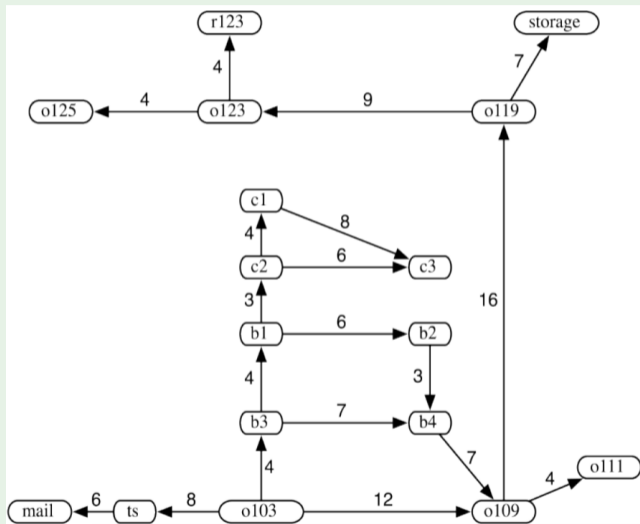
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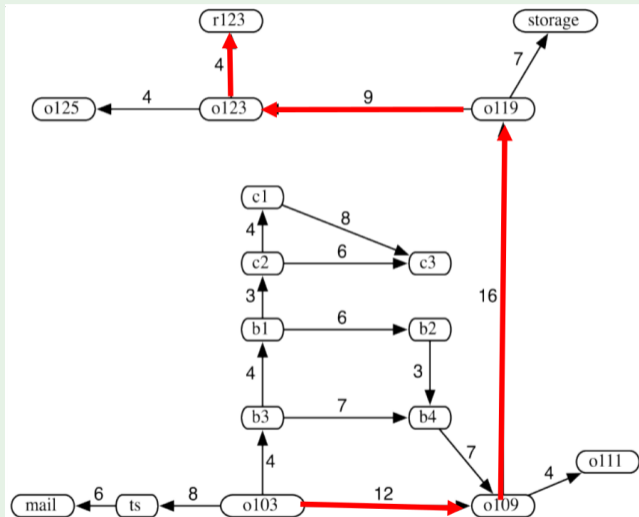
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Abstraction

Problem formulations are **models** of reality (i.e. **abstract descriptions**)

- real world is absurdly complex
 - ⇒ state space must be **abstracted** for problem solving
- lots of details removed because irrelevant to the problem
 - Ex: **exact position**, “**turn steering wheel to the left by 20 degree**”, ...
- **abstraction**: the process of removing detail from representations
 - **abstract state** represents many real states
 - **abstract action** represents complex combination of real actions
- **valid abstraction**: can expand any abstract solution into a solution in the detailed world
- **useful abstraction**: if carrying out each of the actions in the solution is easier than in the original problem

The **choice of a good abstraction** involves **removing as much detail as possible**, while **retaining validity** and **ensuring that the abstract actions are easy to carry out**.

Well-defined problems and solutions [cont.]

Abstraction

Problem formulations are **models** of reality (i.e. **abstract descriptions**)

- real world is absurdly complex
 - ⇒ state space must be **abstracted** for problem solving
- lots of details removed because irrelevant to the problem
 - Ex: **exact position**, “**turn steering wheel to the left by 20 degree**”, ...
- **abstraction**: the process of removing detail from representations
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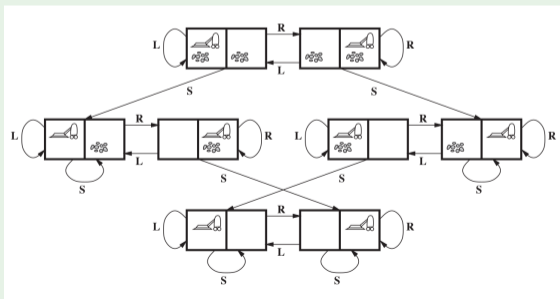
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Toy Example: Simple Vacuum Cleaner

- **States:** 2 locations, each $\{clean, dirty\}$: $2 \cdot 2^2 = 8$ states
- **Initial State:** any
- **Actions:** $\{Left, Right, Suck\}$
- **Transition Model:** (...), Left [Right] if A [B], Suck if clean \implies no effect
- **Goal Test:** check if squares are clean
- **Path Cost:** each step costs 1 \implies path cost is # of steps in path



Toy Example: The 8-Puzzle

- **States:** Integer location of each tile $\implies 9!/2$ reachable states
- **Initial State:** any
- **Actions:** moving $\{Left, Right, Up, Down\}$ the empty space
- **Transition Model:** empty space switched with the tile in target location
- **Goal Test:** checks state corresponds with goal configuration
- **Path Cost:** each step costs 1 \implies path cost is # of steps in path

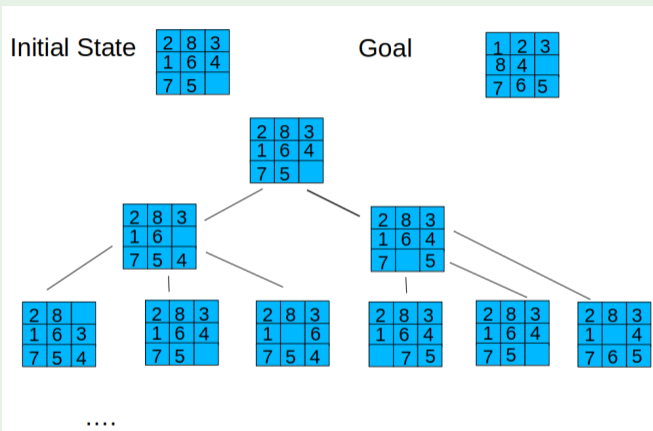
7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

Toy Example: The 8-Puzzle [cont.]

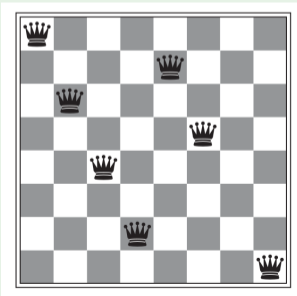


(Courtesy of Michela Milano, UniBO)

NP-complete: N-Puzzle ($N = k^2 - 1$): $N!/2$ reachable states

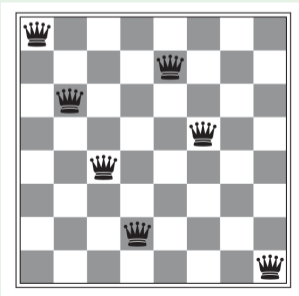
Toy Example: 8-Queens Problem

- **States:** any arrangement of 0 to 8 queens on the board
⇒ $64 \cdot 63 \cdot \dots \cdot 57 \approx 1.8 \cdot 10^{14}$ possible sequences
- **Initial State:** no queens on the board
- **Actions:** add a queen to any empty square
- **Transition Model:** returns the board with a queen added
- **Goal Test:** 8 queens on the board, none attacked by other queen
- **Path Cost:** none



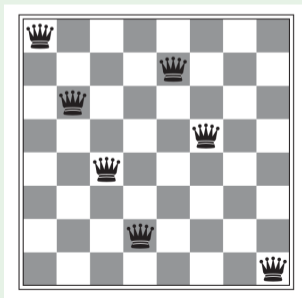
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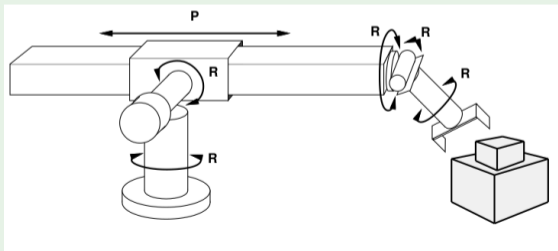
Toy Example: 8-Queens Problem (incremental)

- **States:** $n \leq 8$ queens on board, one per column in the n leftmost columns, **no queen attacking another**
⇒ 2057 possible sequences
- **Actions:** Add a queen to any square in the leftmost empty column such that it is not attacked by any other queen.
- ...



Real-World Example: Robotic Assembly

- **States:** real-valued coordinates of robot joint angles, and of parts of the object to be assembled
- **Initial State:** any arm position and object configuration
- **Actions:** continuous motions of robot joints
- **Transition Model:** position resulting from motion
- **Goal Test:** complete assembly (without robot)
- **Path Cost:** time to execute



Other Real-World Examples

- Airline travel planning problems
- Touring problems
- VLSI layout problem
- Robot navigation
- Automatic assembly sequencing
- Protein design
- ...

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Searching for Solutions

Search: Generate sequences of actions.

- **Expansion:** one starts from a state, and applying the operators (or successor function) will generate new states
- **Search strategy:** at each step, choose which state to expand.
- **Search Tree/DAG:** represents the expansion of all states starting from the initial state (the **root** of the tree/DAG)
- The **leaves** of the tree/DAG represent either:
 - states to expand
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Tree Search Algorithms

Tree Search: Basic idea

- **Off-line, simulated exploration of state space**
 - start from initial state
 - pick one leaf node, and generate its successors (a.k.a. **expanding** a node)
 - set of current leaves called **frontier** (a.k.a. **fringe**, **open list**)
 - strategy for picking leaves critical (**search strategy**)
 - ends when either **a goal state is reached**, or **no more candidates to expand are available** (or time-out/memory-out occur)

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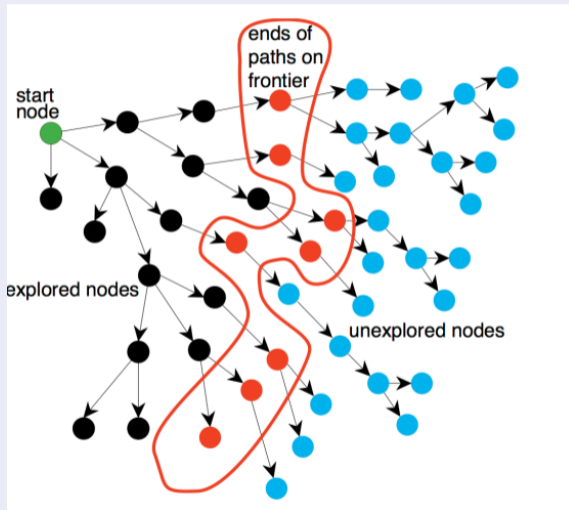
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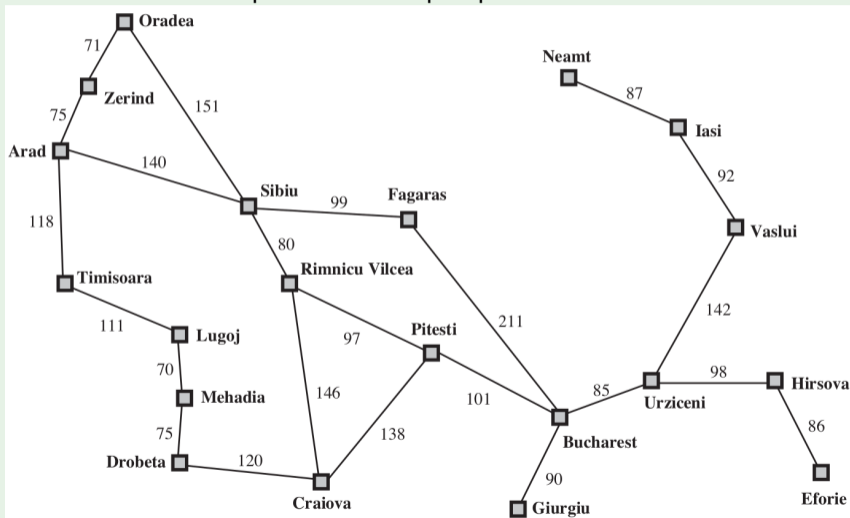
Tree Search Algorithms [cont.]



(Courtesy of Maria Simi, UniPI)

Tree-Search Example: Trip from Arad to Bucharest

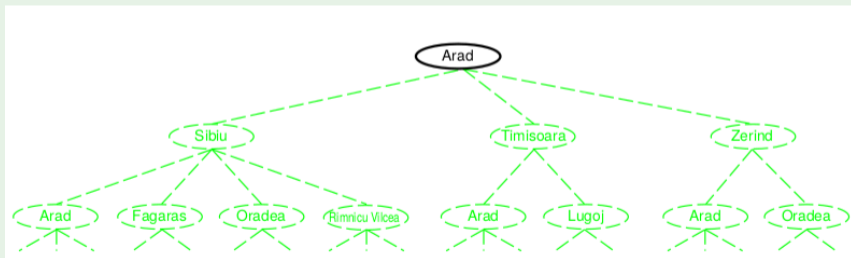
A simplified road map of part of Romania.



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Expanding the search tree

- Initial state: {*Arad*}
- Expand initial state \Rightarrow {*Sibiu*, *Timisoara*, *Zerind*}
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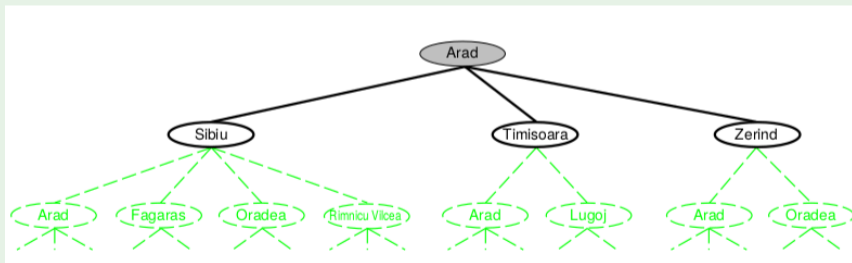
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Beware: Arad \mapsto Sibiu \mapsto Arad (repeated state \Rightarrow **loopy path!**)

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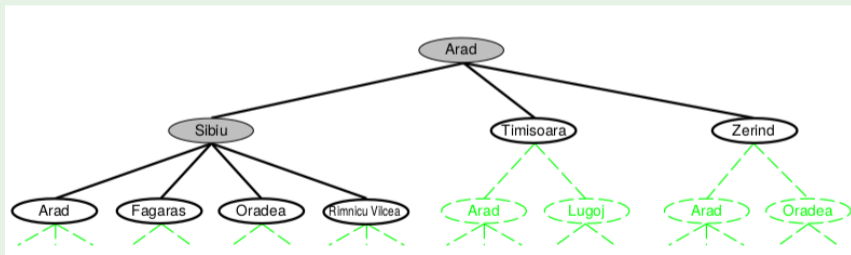
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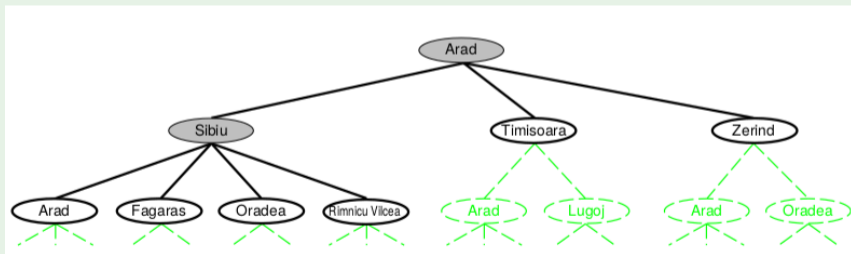
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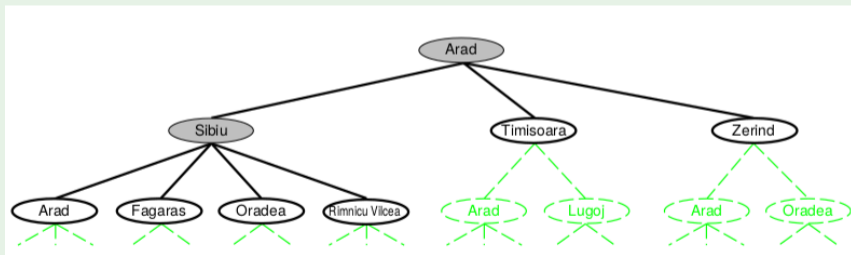
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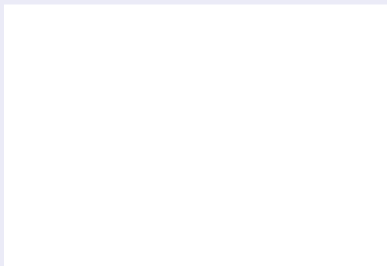


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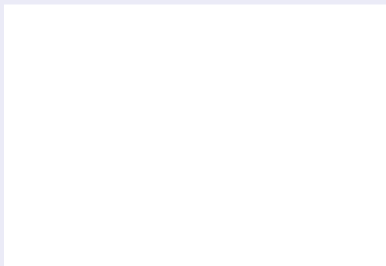
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- **Redundant paths** occur when there is more than one way to get from one state to another
 - ⇒ same state & subtree explored more than once
 - ex: Arad, Sibiu (*subtree*) vs. Arad, Zerind, Oradea, Sibiu (*same subtree*)
- Failure to detect repeated states can:
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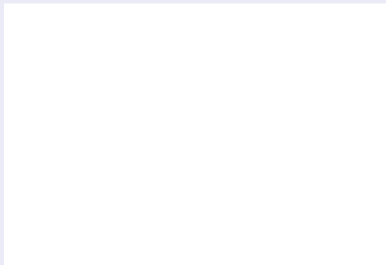
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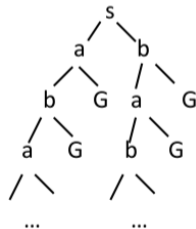
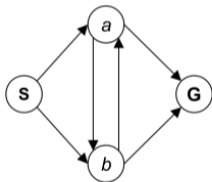
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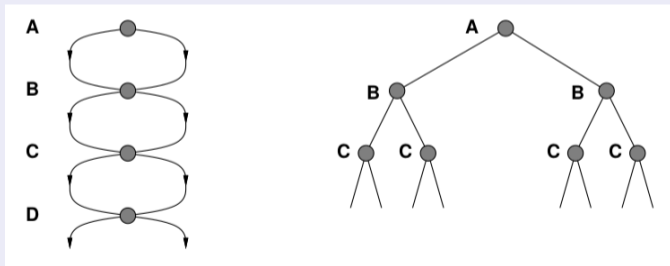


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Moral: Algorithms that forget their history are doomed to repeat it!

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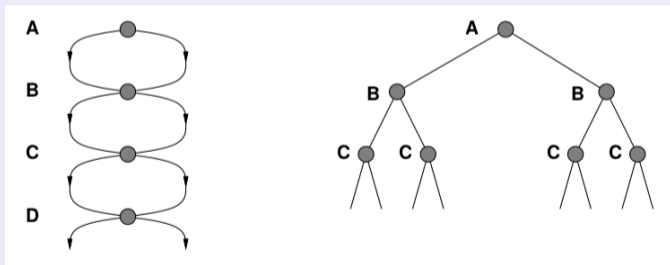


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Graph Search Algorithms

Graph Search: Basic idea

- Add a data structure which remembers every expanded node
 - a.k.a. explored set or closed list
 - typically a hash table (access $O(1)$)
- Do not expand a node if it already occurs in explored set

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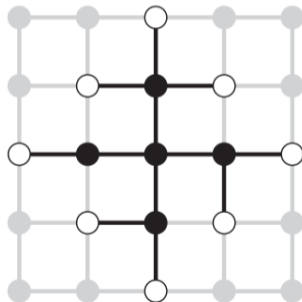
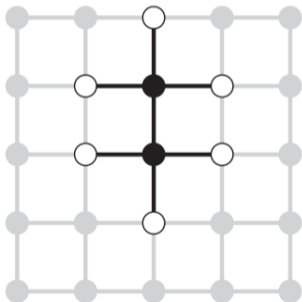
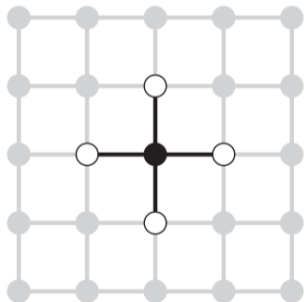
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Graph Search Algorithms: Example

Separation Property of graph search:

The frontier separates the state-space graph into the **explored region** and the **unexplored region**

Ex: Graph search on a rectangular-grid problem



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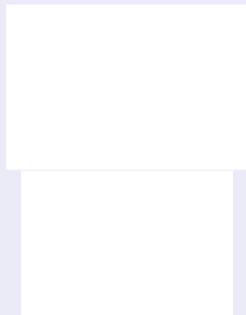
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Implementation: States vs. Nodes

- A **state** is a **representation of a physical configuration**
- A **node** is a **data structure** constituting part of a search tree
 - includes fields: **state**, **parent**, **action**, **path cost $g(x)$**

⇒ **node** \neq **state**

- Within a given problem, it should be easy to compute a child node from its parent and the action performed

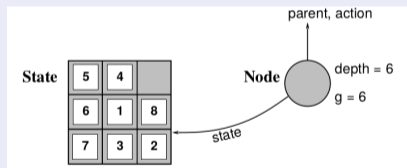


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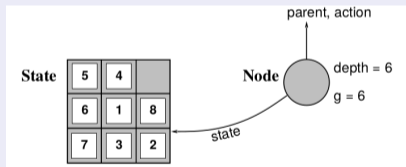


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- Within a given problem, it should be easy to compute a child node from its parent and the action performed



function CHILD-NODE(*problem*, *parent*, *action*) **returns** a node
return a node with

STATE = *problem*.RESULT(*parent*.STATE, *action*),

PARENT = *parent*, ACTION = *action*,

PATH-COST = *parent*.PATH-COST + *problem*.STEP-COST(*parent*.STATE, *action*)

Implementation: Frontier and Explored

Frontier/Fringe

- Implemented as a Queue:
 - First-in-First-Out, FIFO (aka “queue”): $O(1)$ access
 - Last-in-First-Out, LIFO (aka “stack”): $O(1)$ access
 - Best-First-out (aka “priority queue”): $O(\log(n))$ access
- Three primitives:
 - `ISEMPTY(QUEUE)`: returns true iff there are no more elements
 - `POP(QUEUE)`: removes and returns the first element of the queue
 - `INSERT(ELEMENT,QUEUE)`: inserts an element into queue

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- Implemented as a Hash Table: $O(1)$ access
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 - `ISTHERE(ELEMENT,HASH)`: returns true iff element is in the hash
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Implementation: general tree search

```
function TREE-SEARCH(problem, fringe) returns a solution, or failure  
fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)  
loop do  
  if fringe is empty then return failure  
  node ← REMOVE-FRONT(fringe)  
  if GOAL-TEST(problem, STATE(node)) then return node  
  fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

```
function EXPAND(node, problem) returns a set of nodes  
successors ← the empty set  
for each action, result in SUCCESSOR-FN(problem, STATE[node]) do  
  s ← a new NODE  
  PARENT-NODE[s] ← node; ACTION[s] ← action; STATE[s] ← result  
  PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)  
  DEPTH[s] ← DEPTH[node] + 1  
  add s to successors  
return successors
```

Implementation: general graph search

```
function GRAPH-SEARCH(problem, fringe) returns a solution, or failure  
  closed ← an empty set  
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)  
  loop do  
    if fringe is empty then return failure  
    node ← REMOVE-FRONT(fringe)  
    if GOAL-TEST(problem, STATE[node]) then return node  
    if STATE[node] is not in closed then  
      add STATE[node] to closed  
      fringe ← INSERTALL(EXPAND(node, problem), fringe)  
  end
```

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Uninformed vs. Informed Search Strategies

Strategies: Two possibilities

- Uninformed strategies (a.k.a. blind strategies)
 - do not use any domain knowledge
 - apply rules arbitrarily and do an exhaustive search strategy
 - ⇒ impractical for some complex problems.
- Informed strategies
 - use domain knowledge
 - apply rules following heuristics (driven by domain knowledge)
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Evaluating Search Strategies

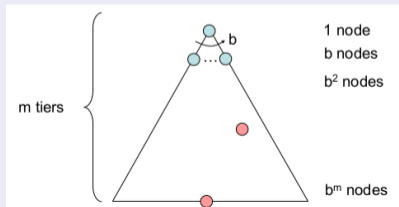
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- Time and space complexity are measured in terms of

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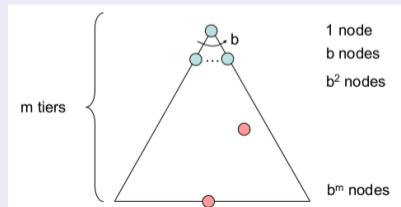
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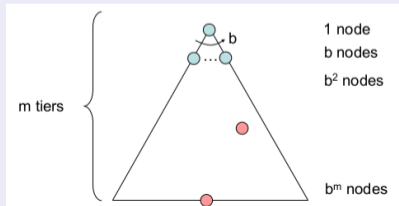
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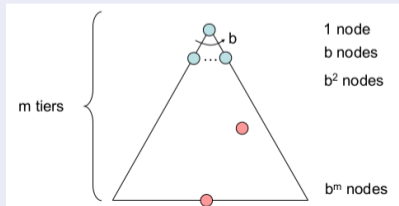
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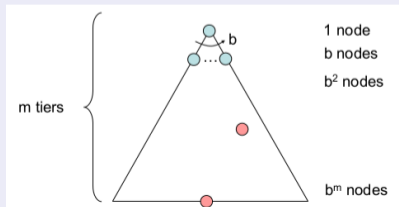
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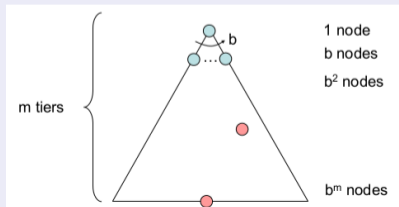
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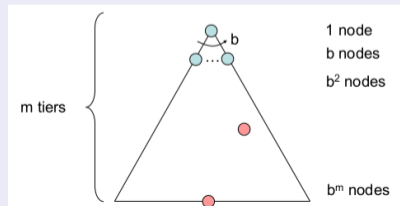
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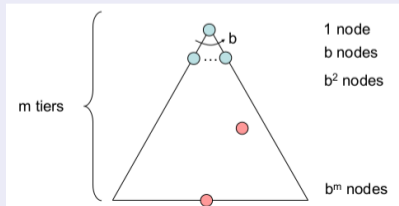
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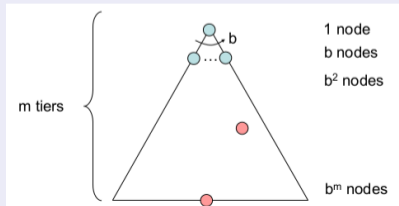
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Uninformed Search Strategies

Uninformed strategies

Use only the information available in the problem definition

- Different uninformed search strategies
 - Breadth-first search
 - Uniform-cost search
 - Depth-first search
 - Depth-limited search & Iterative-deepening search
- Defined by **the access strategy of the frontier/fringe** (i.e. the order of node expansion)
 - **goal test strategy** may vary as well

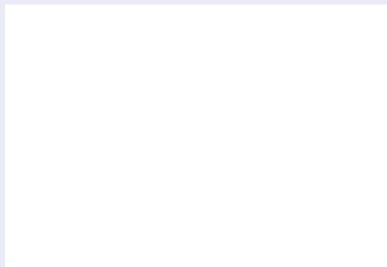
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Breadth-First Search Strategy (BFS)

Breadth-First Search

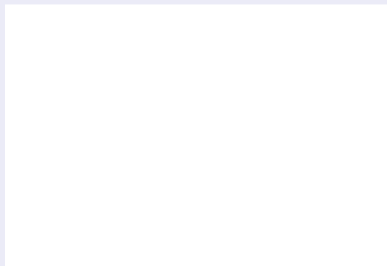
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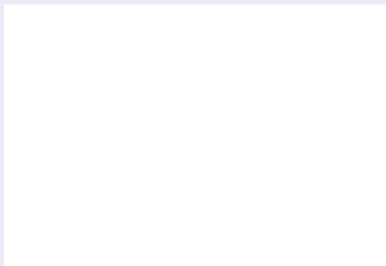
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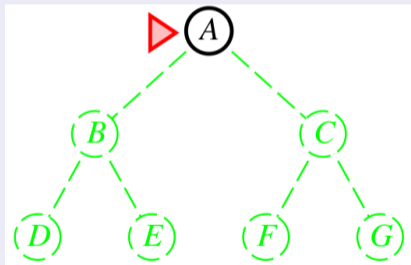
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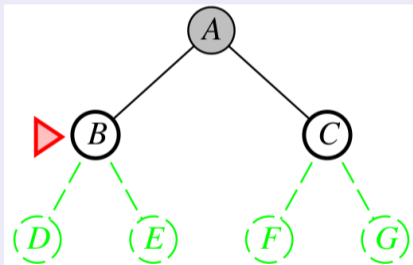
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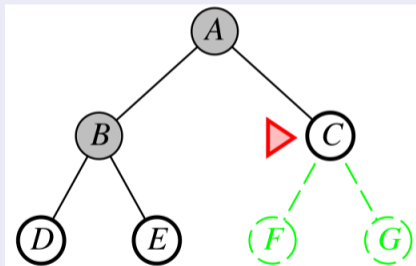
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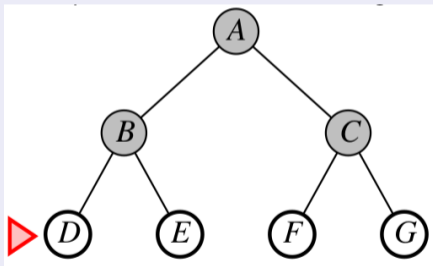
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Breadth-First Search Strategy (BFS) [cont.]

BFS, Graph version (Tree version without “explored”)

```
function BREADTH-FIRST-SEARCH(problem) returns a solution, or failure
  node ← a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
  if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
  frontier ← a FIFO queue with node as the only element
  explored ← an empty set
  loop do
    if EMPTY?(frontier) then return failure
    node ← POP(frontier) /* chooses the shallowest node in frontier */
    add node.STATE to explored
    for each action in problem.ACTIONS(node.STATE) do
      child ← CHILD-NODE(problem, node, action)
      if child.STATE is not in explored or frontier then
        if problem.GOAL-TEST(child.STATE) then return SOLUTION(child)
        frontier ← INSERT(child, frontier)
```

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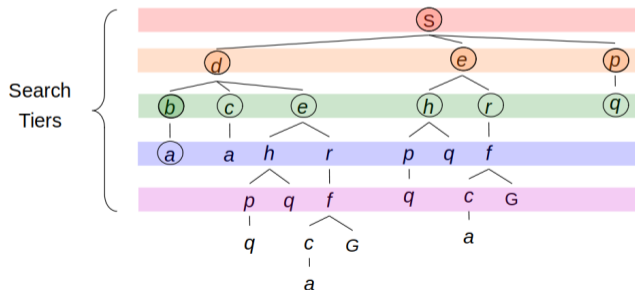
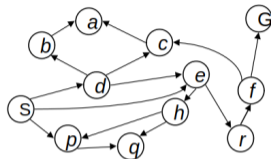
Note: the goal test is applied to each node **when it is generated**,
rather than when **it is selected for expansion**
⇒ solution detected 1 layer earlier

Breadth-First Search: Tiers

State space is explored by tiers (tree version, children expanded in alphabetical order)

Strategy: expand a shallowest node first

Implementation: Fringe is a FIFO queue

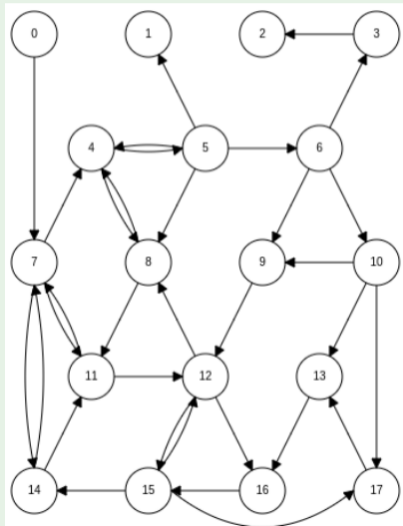


Exercises

- 1 Run previous example, with BFS **graph** search.
- 2 Consider the following graph, initial state 0, goal state 17:
 - explore it using BFS, tree version
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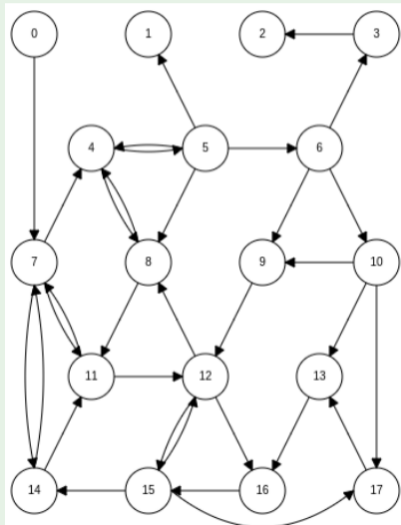
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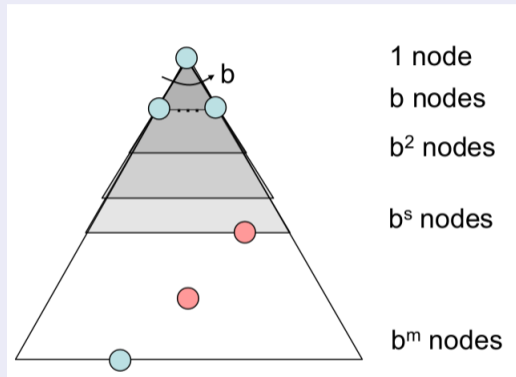
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Breadth-First Search (BFS): Properties

d : depth of shallowest solution

- How many steps?
 - processes all nodes above shallowest solution
 \implies takes $O(b^d)$ time
- How much memory?
 - max frontier size: b^d nodes
 $\implies O(b^d)$ memory size
- Is it complete?
 - if solution exists, b^d finite
 \implies Yes
- Is it optimal?
 - if and only if all costs are 1
 \implies shallowest solution



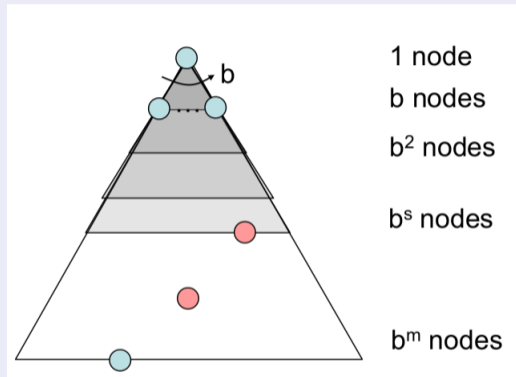
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Memory requirement is a major problem for breadth-first search

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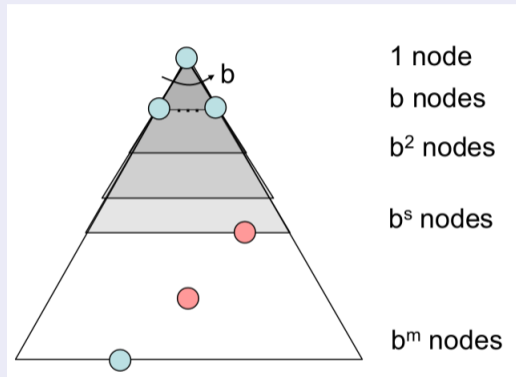
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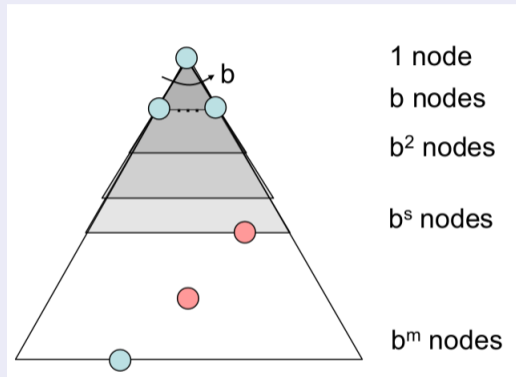
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 - processes all nodes above shallowest solution
 \implies takes $O(b^d)$ time
- How much memory?
 - max frontier size: b^d nodes
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- Is it complete?
 - if solution exists, b^d finite
 \implies Yes
- Is it optimal?
 - if and only if all costs are 1
 \implies shallowest solution



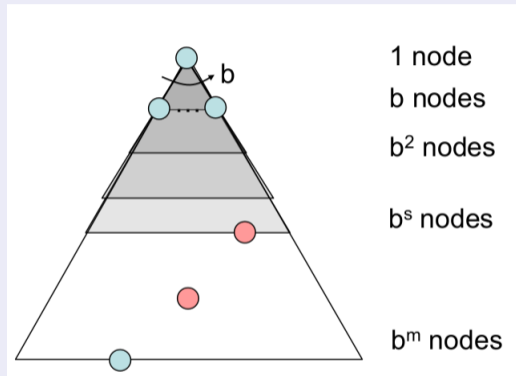
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Memory requirement is a major problem for breadth-first search

Breadth-First Search (BFS): Properties

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Memory requirement is a major problem for breadth-first search

Breadth-First Search (BFS): Time and Memory

- Assume:
 - 1 million nodes generated per second
 - 1 node requires 1000 bytes of storage
 - branching factor $b = 10$

Depth	Nodes	Time	Memory
2	110	.11 milliseconds	107 kilobytes
4	11,110	11 milliseconds	10.6 megabytes
6	10^6	1.1 seconds	1 gigabyte
8	10^8	2 minutes	103 gigabytes
10	10^{10}	3 hours	10 terabytes
12	10^{12}	13 days	1 petabyte
14	10^{14}	3.5 years	99 petabytes
16	10^{16}	350 years	10 exabytes

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Memory requirements is a bigger problem for BFS than execution time

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Uniform-Cost Search Strategy (UCS)

Uniform-Cost Search

- Idea: Expand first the node with lowest path cost $g(n)$
- Implementation: frontier/fringe implemented as a priority queue ordered by $g()$
 - ⇒ novel nearest successors pushed to the top of the queue
- similar to BFS if step costs are all equal
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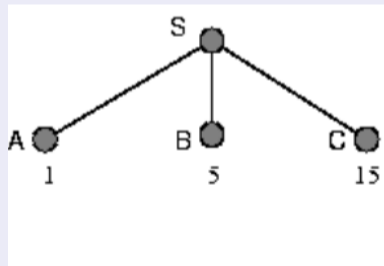
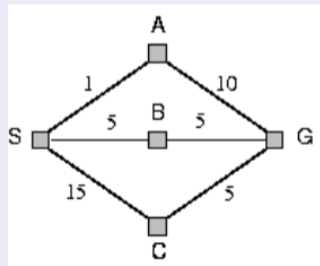
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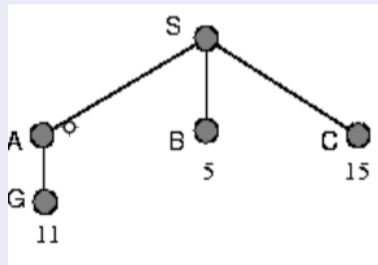
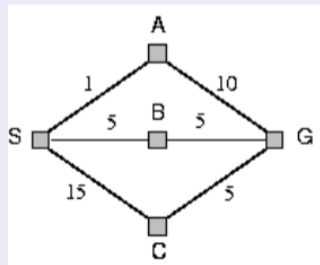


(Courtesy of Michela Milano, UNIBO)

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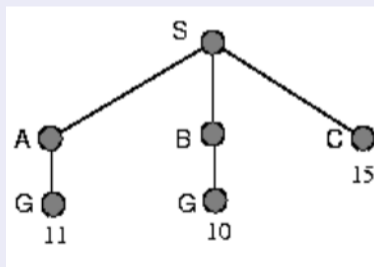
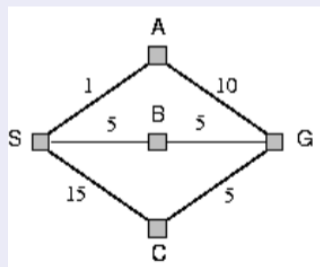


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(Courtesy of Michela Milano, UniBO)

Uniform-Cost Search Strategy (UCS) [cont.]

UCS, Graph version (Tree version: without “explored”)

```
function UNIFORM-COST-SEARCH(problem) returns a solution, or failure
  node ← a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
  frontier ← a priority queue ordered by PATH-COST, with node as the only element
  explored ← an empty set
  loop do
    if EMPTY?(frontier) then return failure
    node ← POP(frontier) /* chooses the lowest-cost node in frontier */
    if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
    add node.STATE to explored
    for each action in problem.ACTIONS(node.STATE) do
      child ← CHILD-NODE(problem, node, action)
      if child.STATE is not in explored or frontier then
        frontier ← INSERT(child, frontier)
      else if child.STATE is in frontier with higher PATH-COST then
        replace that frontier node with child
```

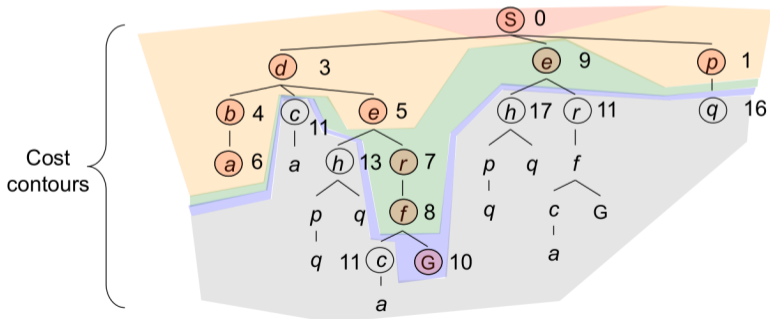
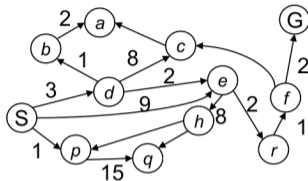
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- apply the goal test to a node **when it is selected for expansion** rather than **when it is first generated**
 - replace in the frontier a node with same state but worse path cost
- ⇒ **avoid generating suboptimal paths** (see previous example)

Uniform-Cost Search

Strategy: expand a cheapest node first:

Fringe is a priority queue (priority: cumulative cost)

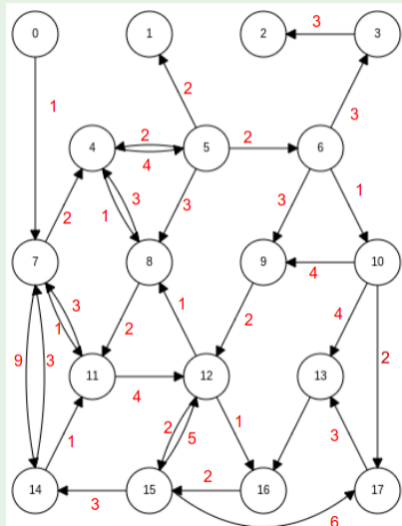


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- 1 Apply UCS to the Romania-map Example
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Uniform-Cost Search (UCS): Properties

C^* : cost of cheapest solution; ϵ : minimum arc cost

$\Rightarrow 1 + \lfloor C^*/\epsilon \rfloor$ “effective depth”

- How many steps?

- processes all nodes costing less than cheapest solution

\Rightarrow takes $O(b^{1+\lfloor C^*/\epsilon \rfloor})$ time

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- max frontier size: $b^{1+\lfloor C^*/\epsilon \rfloor}$

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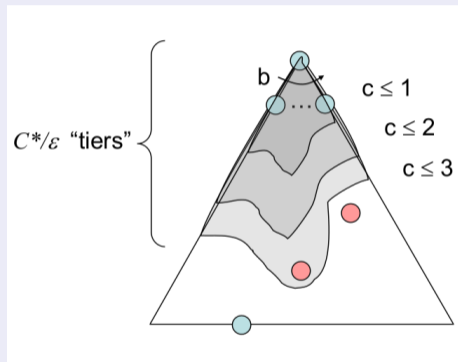
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- if solution exists, finite cost

\Rightarrow Yes

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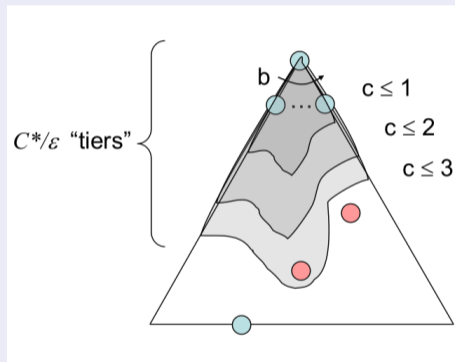
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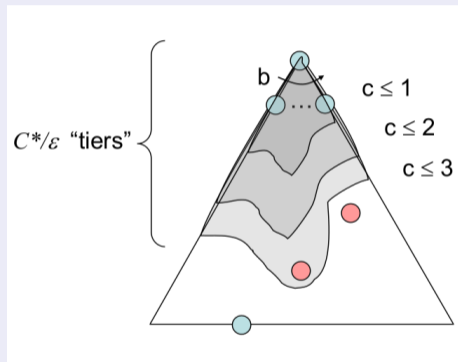
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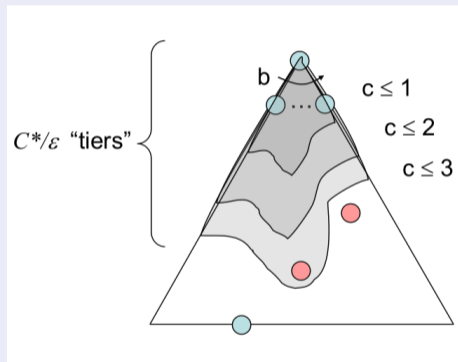
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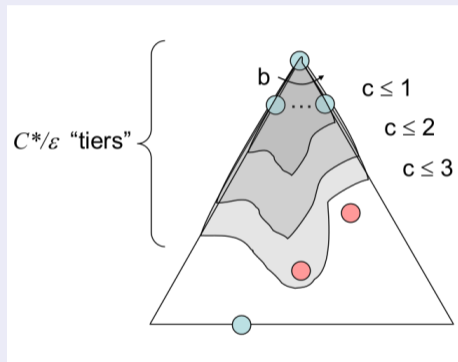
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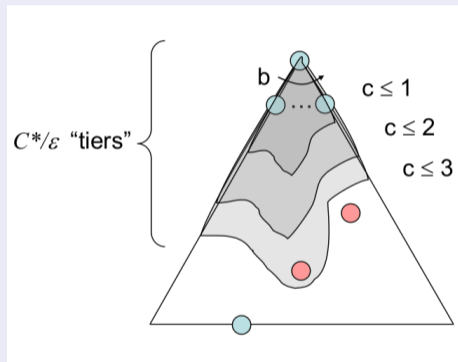
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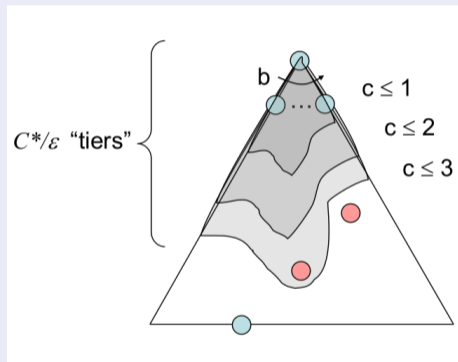
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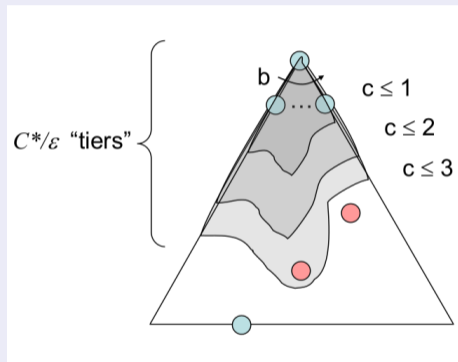
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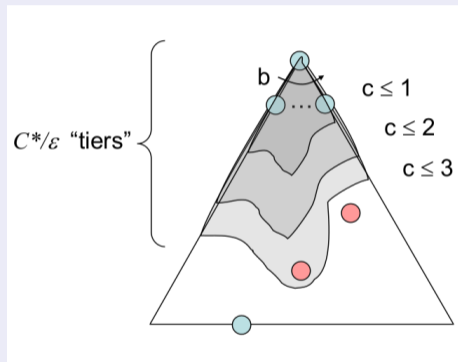
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Depth-First Search Strategy (DFS)

Depth-First Search

- Idea: Expand first the deepest unexpanded nodes
- Implementation: frontier/fringe implemented as a LIFO queue (aka stack)
 - ⇒ novel successors pushed to the top of the stack

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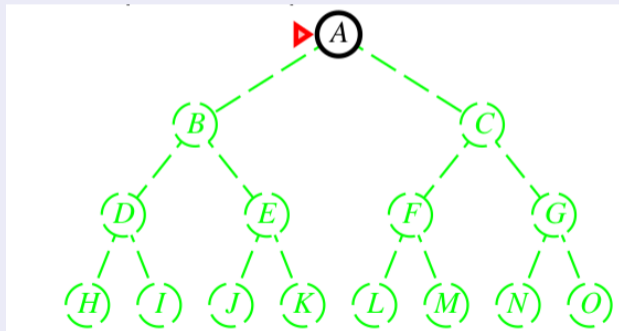
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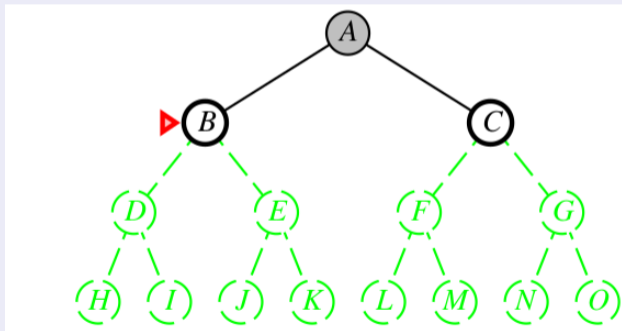
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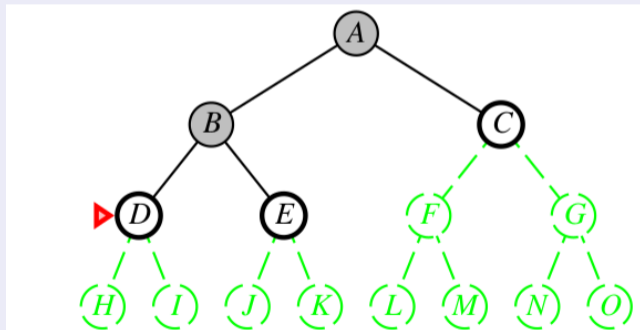
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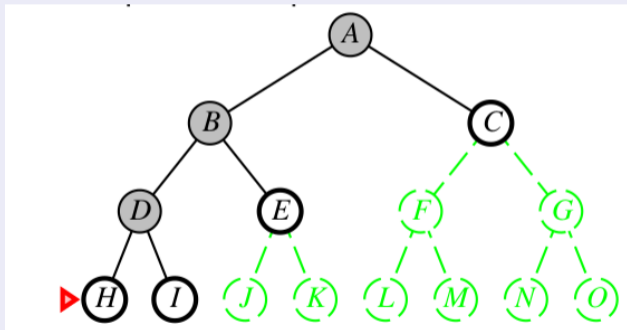
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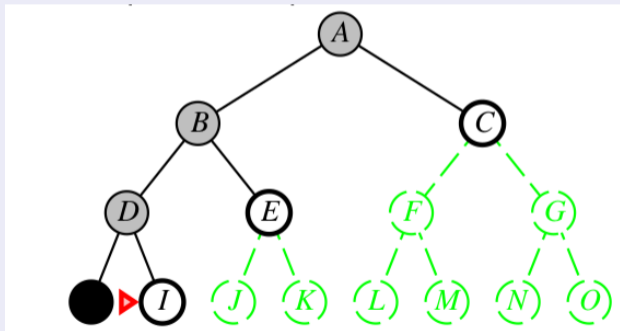
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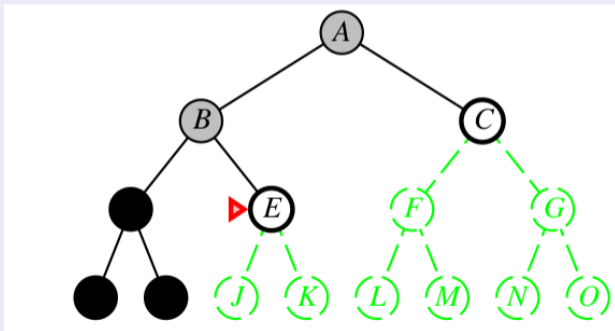
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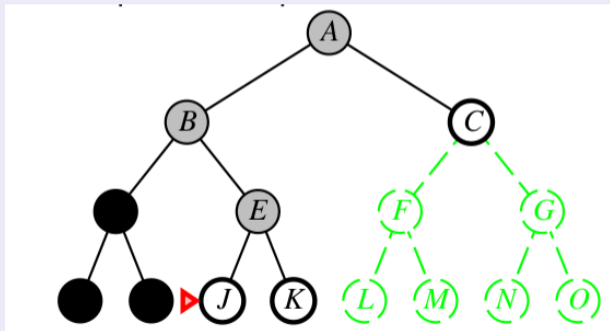
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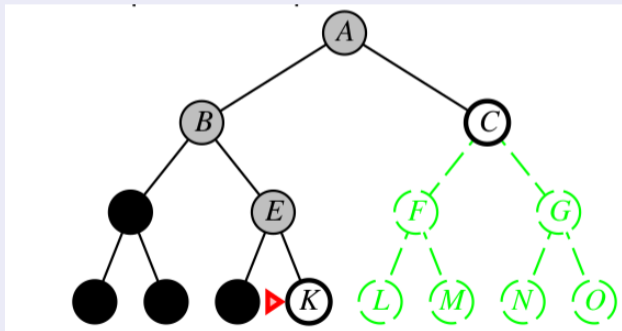
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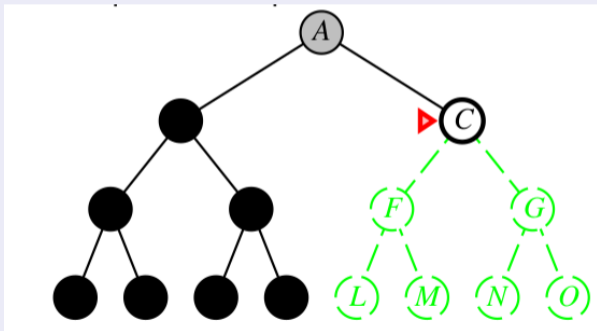
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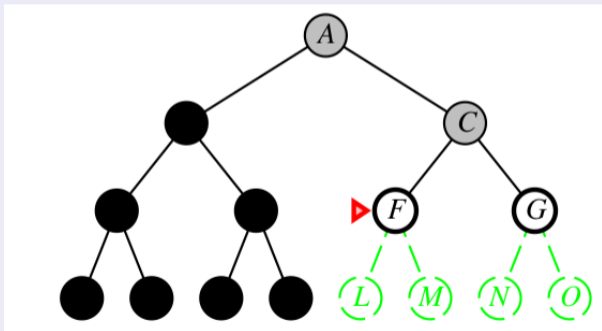
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Depth-First Search

DFS, Graph version (Tree version without “explored”)

Similar to BFS, using a LIFO access for frontier/fringe rather than FIFO.

```
function BREADTHDepth-FIRST-SEARCH(problem) returns a solution, or failure
  node ← a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
  if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
  frontier ← a FIFOLIFO queue with node as the only element
  explored ← an empty set
  loop do
    if EMPTY?(frontier) then return failure
    node ← POP(frontier) /* chooses the shallowestdeepest node in frontier */
    add node.STATE to explored
    for each action in problem.ACTIONS(node.STATE) do
      child ← CHILD-NODE(problem, node, action)
      if child.STATE is not in explored or frontier then
        if problem.GOAL-TEST(child.STATE) then return SOLUTION(child)
        frontier ← INSERT(child, frontier)
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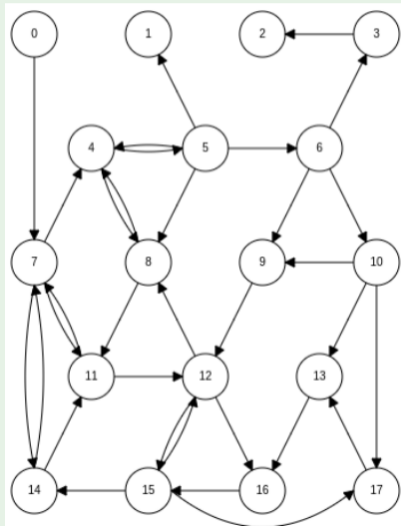
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Exercises

- 1 As with previous example, with DFS **graph** search.
- 2 Consider the following graph, initial state 0, goal state 17:
 - explore it using DFS, tree version
 - explore it using DFS, graph versionchildren should be expanded in numerical order

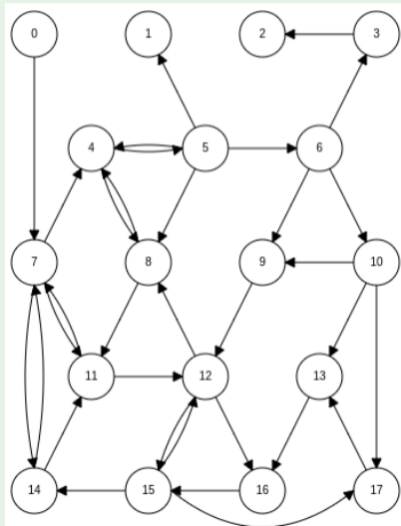
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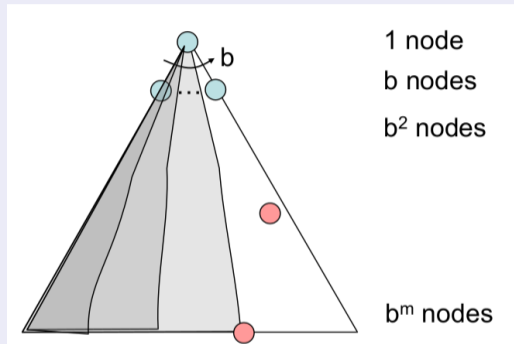
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Depth-First Search (DFS): Properties

- How many steps?
 - could process the whole tree!
⇒ if m finite, takes $O(b^m)$ time
- How much memory?
 - only siblings on path to root
⇒ $O(bm)$ memory size
- Is it complete?
 - if infinite state space: no
 - if finite state space:
 - if depth $> m$: no
 - if depth $< m$: yes
- Is it optimal?
 - No, regardless of depth/cost
⇒ "leftmost" solution

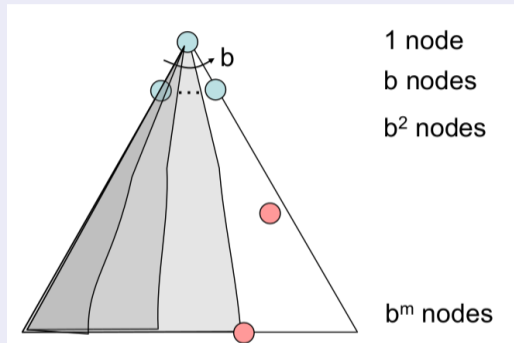


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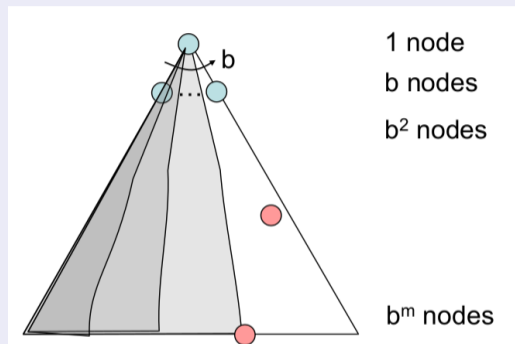


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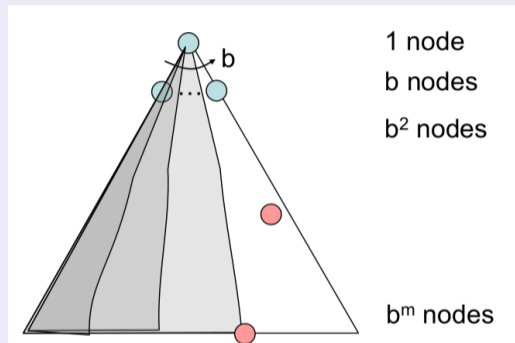


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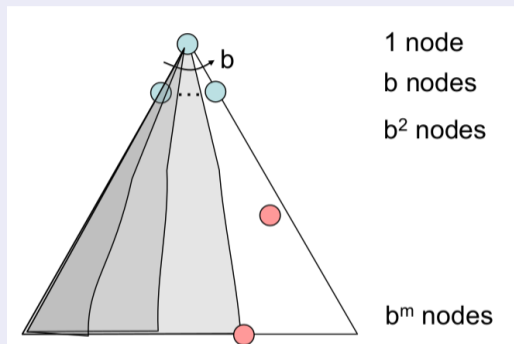


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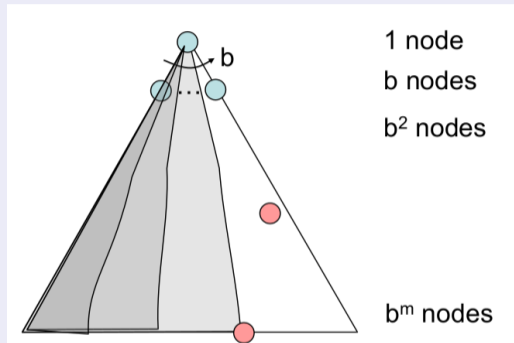


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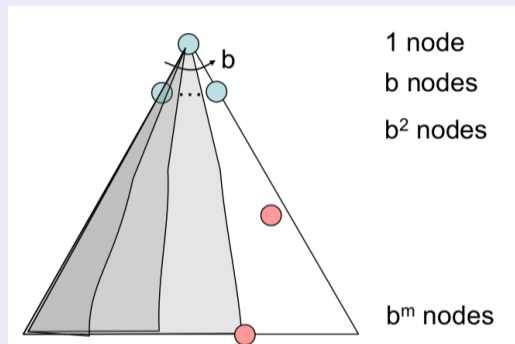


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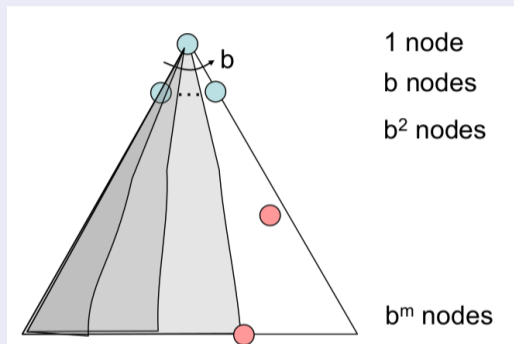
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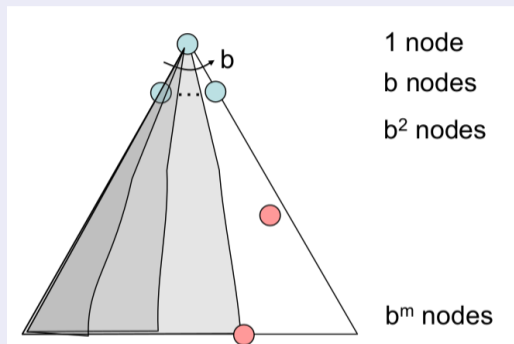


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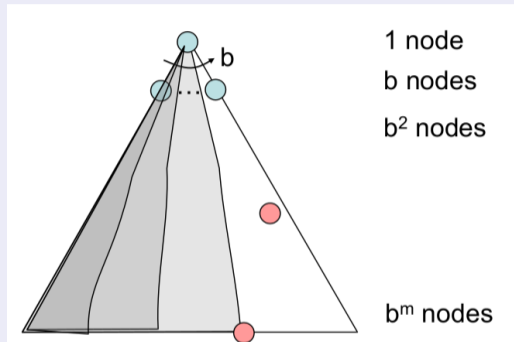


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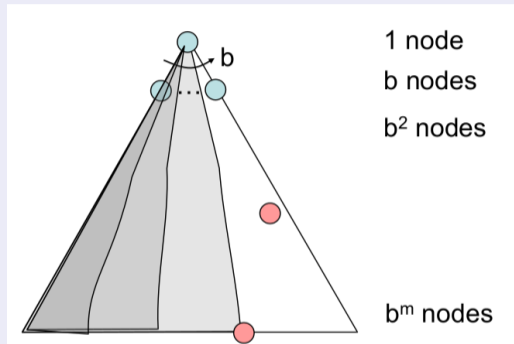


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A Variant of DFS: Backtracking Search

Backtracking Search

- Idea: **only one successor is generated at the time**
 - each partially-expanded node remembers which successor to generate next
 - generate a successor by modifying the current state description, rather than copying it first
 - Applied in CSP, SAT/SMT and Logic Programming

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- 1 Problem-Solving Agents
- 2 Example Problems
- 3 Search Generalities
 - Tree Search
 - Graph Search
 - Implementation Issues & Strategies
- 4 Uninformed Search Strategies**
 - Breadth-First Search
 - Uniform-cost Search
 - Depth-First Search
 - Depth-Limited Search & Iterative Deepening**
- 5 Informed Search Strategies
 - Greedy Best-First Search
 - A* Search
 - Memory-bounded Heuristic Search (hints)
 - Heuristic Functions

Depth-Limited Search (DLS) Strategy

Depth-Limited Search (DLS)

- Idea: **depth-first search with depth limit l**
 - i.e., nodes at depth l treated as having no successors
 - DFS is DLS with $l = +\infty$
- **solves the infinite-path problem of DFS**
⇒ allows DFS deal with infinite-state spaces
- **useful also if maximum-depth is known by domain knowledge**
 - e.g., if maximum node distance in a graph (**diameter**) is known
 - Ex: Romania trip: 9 steps
- Drawbacks (d : depth of the shallowest goal):
 - if $d > l \Rightarrow$ incomplete
 - if $d < l \Rightarrow$ takes $O(b^l)$ instead of $O(b^d)$ steps

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Depth-Limited Search (DLS) Strategy [cont.]

Recursive DLS

function DEPTH-LIMITED-SEARCH(*problem*, *limit*) **returns** a solution, or failure/cutoff
return RECURSIVE-DLS(MAKE-NODE(*problem*.INITIAL-STATE), *problem*, *limit*)

function RECURSIVE-DLS(*node*, *problem*, *limit*) **returns** a solution, or failure/cutoff
if *problem*.GOAL-TEST(*node*.STATE) **then return** SOLUTION(*node*)

else if *limit* = 0 **then return** *cutoff*

else

cutoff_occurred? \leftarrow false

for each *action* **in** *problem*.ACTIONS(*node*.STATE) **do**

child \leftarrow CHILD-NODE(*problem*, *node*, *action*)

result \leftarrow RECURSIVE-DLS(*child*, *problem*, *limit* - 1)

if *result* = *cutoff* **then** *cutoff_occurred?* \leftarrow true

else if *result* \neq *failure* **then return** *result*

if *cutoff_occurred?* **then return** *cutoff* **else return** *failure*

Iterative-Deepening Search Strategy (IDS)

Iterative-Deepening Search

- Idea: call iteratively DLS for increasing depths $l = 0, 1, 2, 3, \dots$
- combines the advantages of breadth- and depth-first strategies
 - complete (like BFS)
 - takes $O(b^l)$ steps (like BFS and DFS)
 - requires $O(bd)$ memory (like DFS)
 - explores a single branch at a time (like DFS)
 - optimal only if step cost = 1
 - optimal variants exist: iterative-lengthening search (see AIMA)
- The favorite search strategy when the search space is very large and depth is not known

```
function ITERATIVE-DEEPENING-SEARCH(problem) returns a solution, or failure
for depth = 0 to  $\infty$  do
    result  $\leftarrow$  DEPTH-LIMITED-SEARCH(problem, depth)
if result  $\neq$  cutoff then return result
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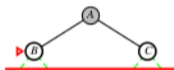
Limit = 0



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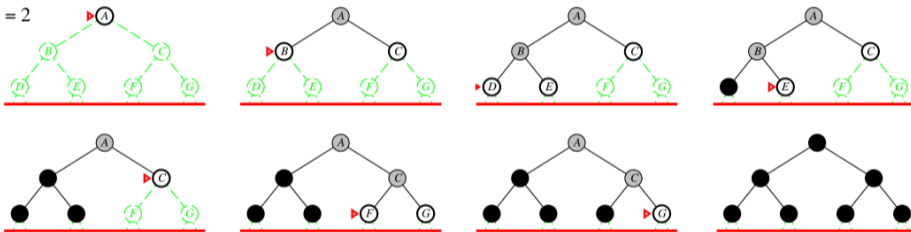
Limit = 1



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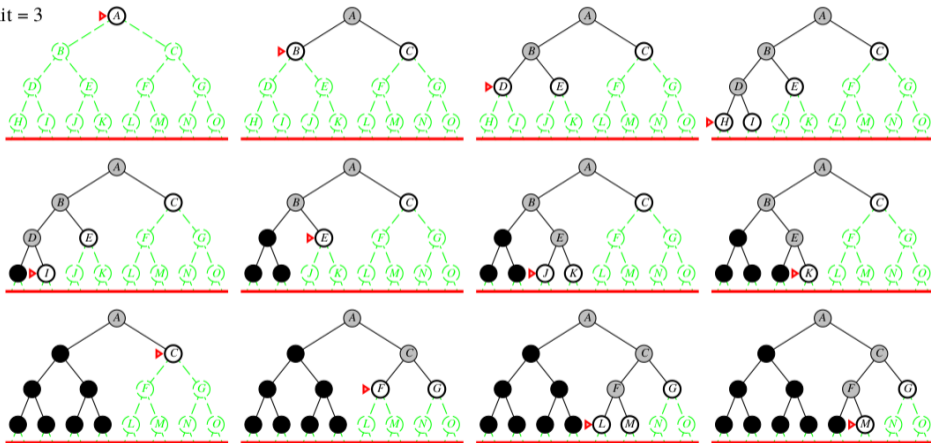
Limit = 2



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Iterative-Deepening Search (IDS) [cont.]

Limit = 3



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Exercises

1 Consider the following graph, initial state 0, goal state 17:

1 explore it using IDS, tree version

2 explore it using IDS, graph version

children should be expanded in numerical order

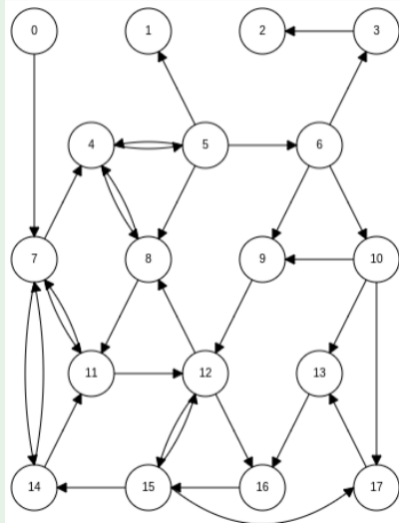
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Remark: Why “only” $O(b^d)$ steps?

- may seem wasteful since states are generated multiple times...
- ... however, only a small fraction of nodes are multiply generated
- number of repeatedly-generated nodes decreases exponentially with number of repetitions
 - depth 1 (b nodes): repeated d times
 - depth 2 (b^2 nodes): repeated $d - 1$ times
 - ...
 - depth d (b^d nodes): repeated 1 time

⇒ The total number of generated nodes is:

$$N(\text{IDS}) = (d)b^1 + (d-1)b^2 + \dots + (1)b^d = O(b^d)$$

$$N(\text{BFS}) = b^1 + b^2 + \dots + b^d = O(b^d)$$

- Ex: with $b = 10$ and $d = 5$:

$$N(\text{IDS}) = 50 + 400 + 3,000 + 20,000 + 100,000 = 123,000$$

$$N(\text{BFS}) = 10 + 100 + 1,000 + 10,000 + 100,000 = 111,110$$

⇒ not significantly worse than BFS

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$$N(\text{BFS}) = 10 + 100 + 1,000 + 10,000 + 100,000 = 111,110$$

⇒ not significantly worse than BFS

Iterative-Deepening Search Strategy (IDS) [cont.]

Remark: Why “only” $O(b^d)$ steps?

- may seem wasteful since states are generated multiple times...
- ... however, only a small fraction of nodes are multiply generated
- number of repeatedly-generated nodes decreases exponentially with number of repetitions
 - depth 1 (b nodes): repeated d times
 - depth 2 (b^2 nodes): repeated $d - 1$ times
 - ...
 - depth d (b^d nodes): repeated 1 time

⇒ The total number of generated nodes is:

$$N(IDS) = (d)b^1 + (d-1)b^2 + \dots + (1)b^d = O(b^d)$$

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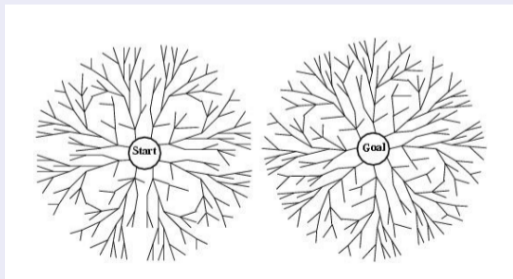
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Bidirectional Search [hints]

- Idea: **Two simultaneous searches:**
 - forward: from start node
 - backward: from goal nodechecking if the node belongs to the other frontier before expansion
- Rationale: $b^{d/2} + b^{d/2} \ll b^d$
⇒ number of steps and memory consumption are $\approx 2b^{d/2}$
- backward search can be tricky in some cases (e.g. **8-queens**)



Uninformed Search Strategies: Comparison

Evaluation of tree-search strategies

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes ^a	Yes ^{a,b}	No	No	Yes ^a	Yes ^{a,d}
Time	$O(b^d)$	$O(b^{1+\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{1+\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(b\ell)$	$O(bd)$	$O(b^{d/2})$
Optimal?	Yes ^c	Yes	No	No	Yes ^c	Yes ^{c,d}

^a: complete if b is finite

^b: complete if step costs $\geq \epsilon$ for some positive ϵ

^c: optimal if step costs are all identical

^d: if both directions use breadth-first search

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For graph searches, the main differences are:

- depth-first search is complete for finite-state spaces
- space & time complexities are bounded by the state space size

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Informed Search Strategies

Some general principles

- The intelligence of a system cannot be measured only in terms of search capacity, but in **the ability to use knowledge about the problem to reduce/mitigate the combinatorial explosion**
- If the system has some control on the order in which candidate solutions are generated, then it is useful to use this order so that actual solutions have a high chance to appear earlier
- For a system with limited processing capacity:
intelligence is the wise choice of what to do next

Heuristic search and heuristic functions

Heuristic search and heuristic functions

- Uninformed UCS strategy ignores the goal when selecting nodes

⇒ Idea: **don't ignore the goal when selecting nodes**

- Intuition: often **extra knowledge can be used to guide the search towards the goal: heuristics**
- A **heuristic** is a function $h(n)$ that estimates how close a state n is to a goal
 - designed for a particular search problem
 - Ex Manhattan distance, Euclidean distance for pathing

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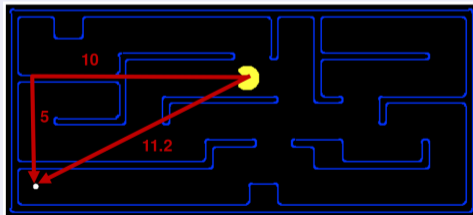
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Best-first Search Strategies

General approach of informed search: Best-first search

- **Best-first search**: node selected for expansion based on an **evaluation function $f(n)$**
 - represent a **cost estimate** \implies choose node which appears best
 - implemented like uniform-cost search, with f instead of g \implies the frontier is a priority queue sorted in decreasing order of $f(n)$
 - both tree-based and graph-based versions
 - most often f includes a **heuristic function $h(n)$**
- **Heuristic function $h(n) \in \mathbb{R}^+$** :
estimated cost of the cheapest path from the state at node n to a goal state
 - $h(n) \geq 0 \forall n$
 - If G is goal, then $h(G) = 0$
 - implements **extra domain knowledge**
 - **depends only on state, not on node** (e.g., independent on paths)
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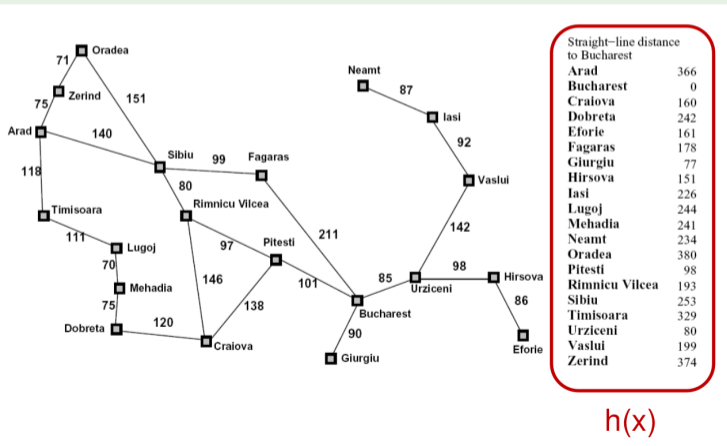
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Example: Straight-Line Distance $h_{SLD}(n)$

- $h(n) \stackrel{\text{def}}{=} h_{SLD}(n)$: straight-line distance heuristic
 - different from actual minimum-path distance
 - cannot be computed from the problem description itself



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Greedy Best-First Search (aka Greedy Search)

- Idea: Expand first the node n with lowest estimate cost to the closest goal, $h(n)$
- Implementation: same as uniform-cost search, with $g(n) \stackrel{\text{def}}{=} h(n)$
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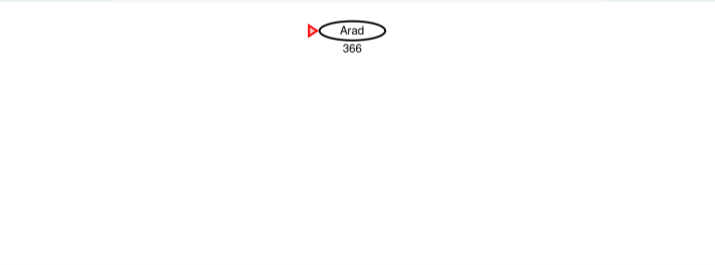
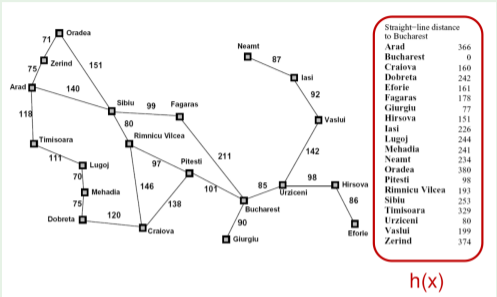
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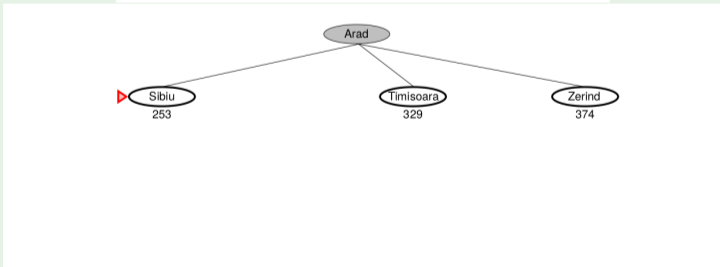
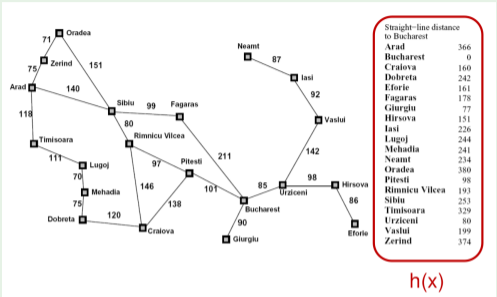
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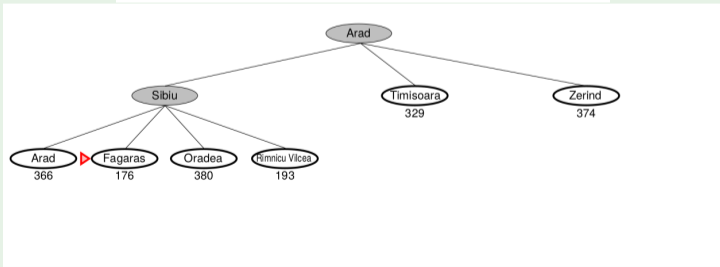
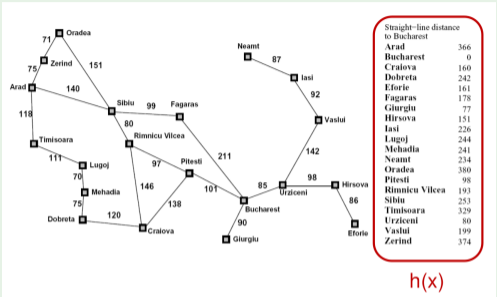
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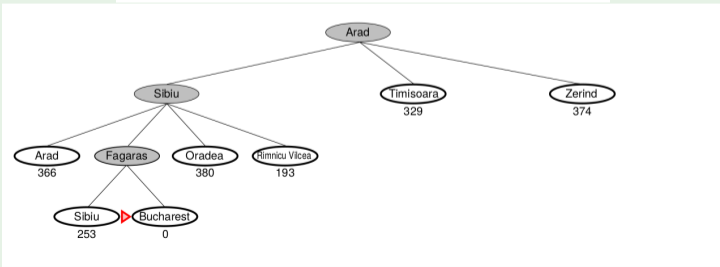
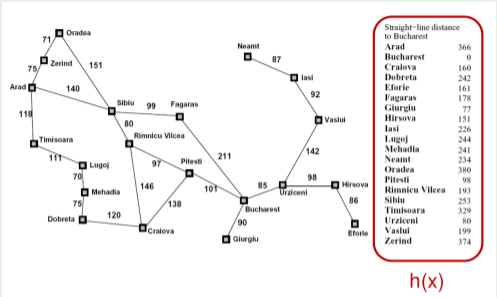
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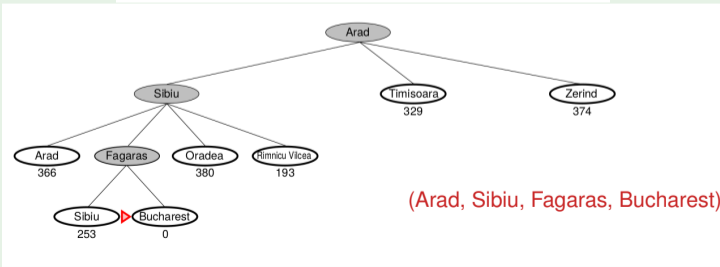
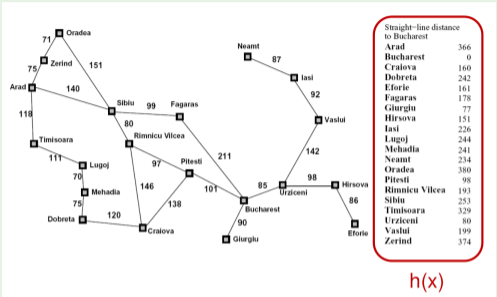
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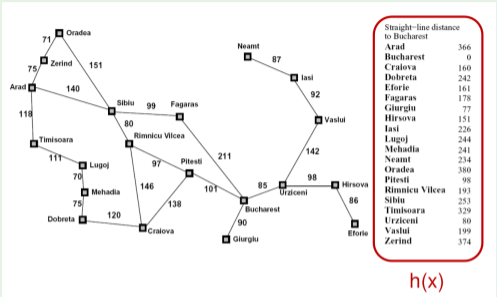
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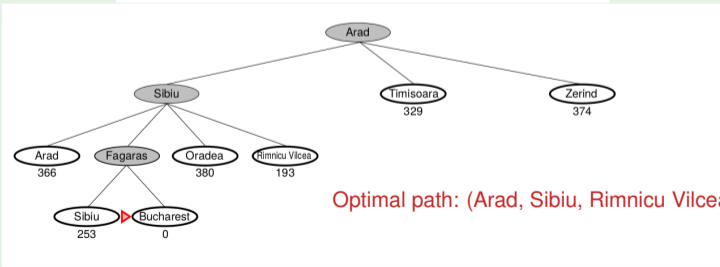
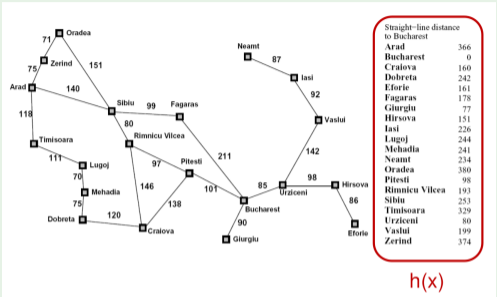
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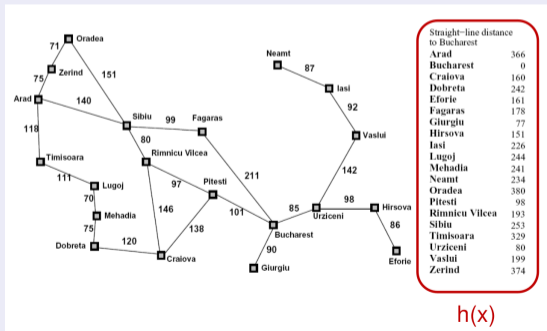


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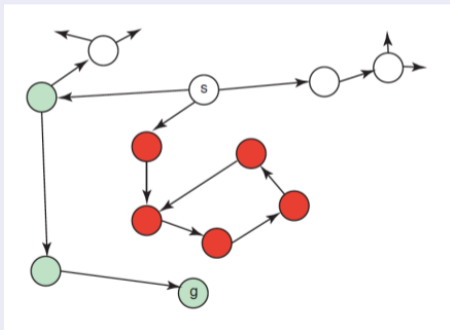
Greedy Best-First Search: (Non-)Optimality

- Greedy best-first search is not optimal
 - it is not guaranteed to find the best solution
 - it is not guaranteed to find the best path toward a solution
- picks the node with minimum (estimated) distance to goal, regardless the cost to reach it
 - Ex: when in Sibiu, it picks Fagaras rather than Rimnicu Vitea



Greedy Best-First Search: (In-)Completeness

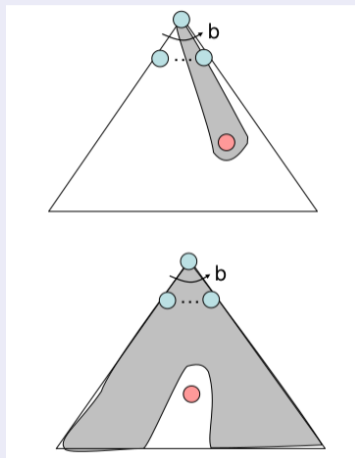
- **Tree-based Greedy best-first search is not complete**
 - may lead to infinite loops
- Graph-based version complete (if state space finite)
- substantially same completeness issues as DFS



(Courtesy of Maria Simi, UniPI)

Greedy Best-First Search (GBFS): Properties

- How many steps?
 - in worst cases may explore all states
⇒ takes $O(b^d)$ time
 - if good heuristics:
⇒ may give good improvements
- How much memory?
 - max frontier size: b^d (as with UCS)
⇒ $O(b^d)$ memory size
- Is it complete?
 - tree: no
 - graph: yes if space finite
- Is it optimal?
 - No

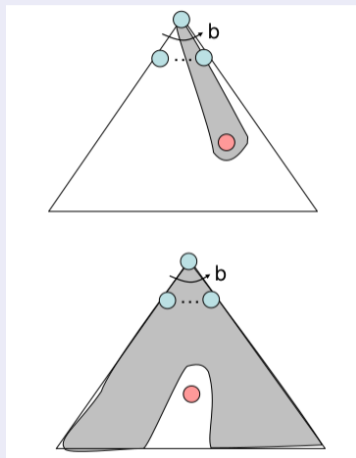


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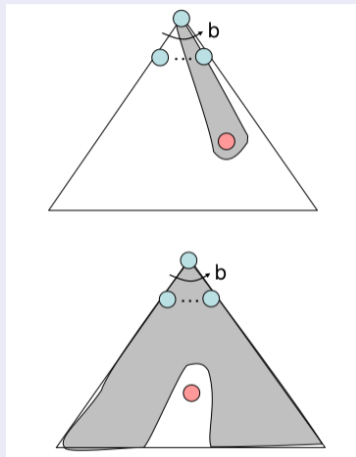


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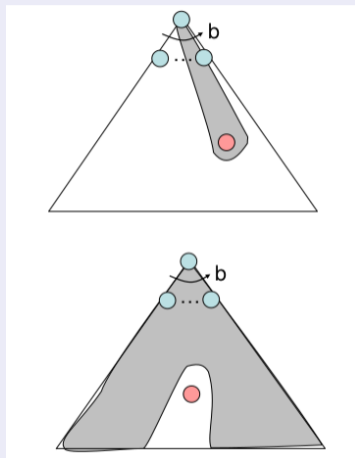


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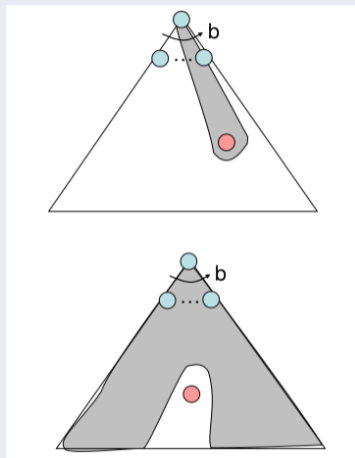


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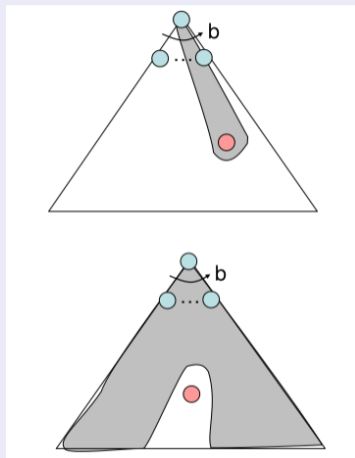


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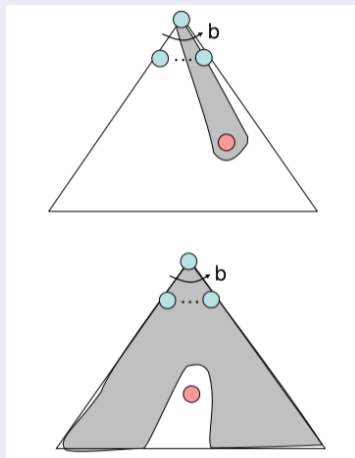


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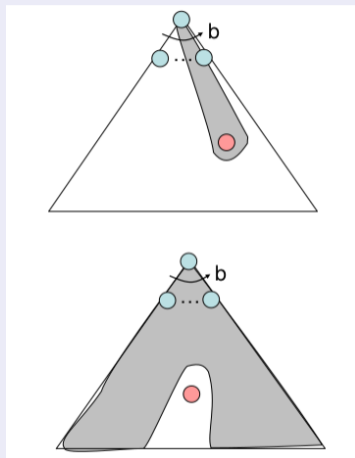


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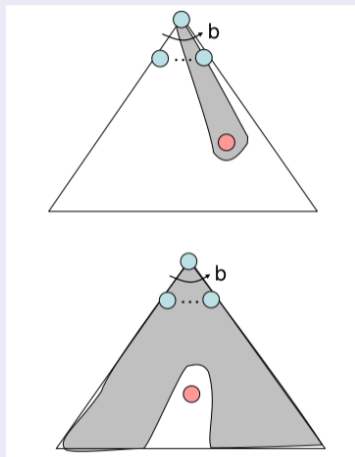


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 - Breadth-First Search
 - Uniform-cost Search
 - Depth-First Search
 - Depth-Limited Search & Iterative Deepening
- 5 **Informed Search Strategies**
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 - **A* Search**
 - Memory-bounded Heuristic Search (hints)
 - Heuristic Functions

A* Search Strategy

A* Search

- Best-known form of best-first search
- Idea: **avoid expanding paths that are already expensive**
- Combine Uniform-Cost and Greedy search: $f(n) = g(n) + h(n)$
 - $g(n)$: cost so far to reach n
 - $h(n)$: estimated cost to goal from n
 - $f(n)$: estimated total cost of path through n to goal

⇒ Expand first the node n with lowest estimated cost of the cheapest solution through n

- Implementation: same as uniform-cost search, with $g(n) + h(n)$ instead of $g(n)$

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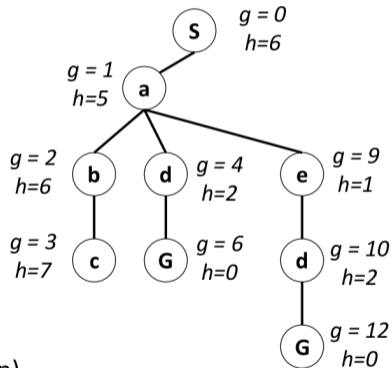
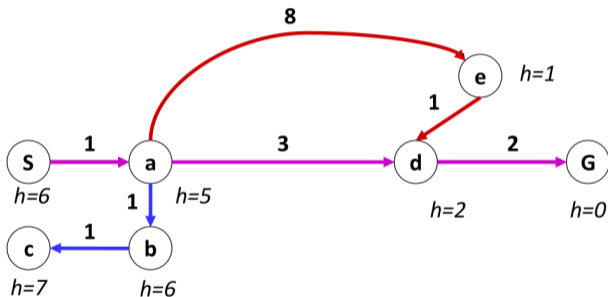
A* Search Strategy [cont.]

A*, Graph version (Tree version: without “explored”)

```
function UNIFORM-COST-SEARCH(problem) returns a solution, or failure
    node ← a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
    frontier ← a priority queue ordered by PATH-COST, with node as the only element
    explored ← an empty set
    loop do
        if EMPTY?(frontier) then return failure
        node ← POP(frontier) /* chooses the lowest-cost node in frontier */
        if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
        add node.STATE to explored
        for each action in problem.ACTIONS(node.STATE) do
            child ← CHILD-NODE(problem, node, action)
            if child.STATE is not in explored or frontier then
                frontier ← INSERT(child, frontier)
            else if child.STATE is in frontier with higher PATH-COST then
                replace that frontier node with child
```

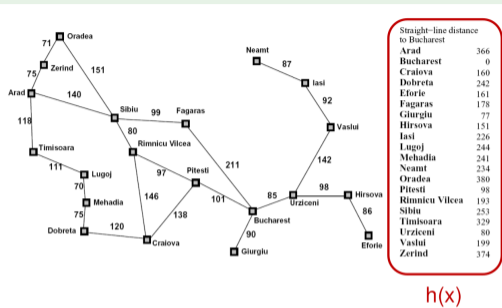
A* Search Strategy: Example

- Uniform-cost orders by path cost, or *backward cost* $g(n)$
- Greedy orders by goal proximity, or *forward cost* $h(n)$



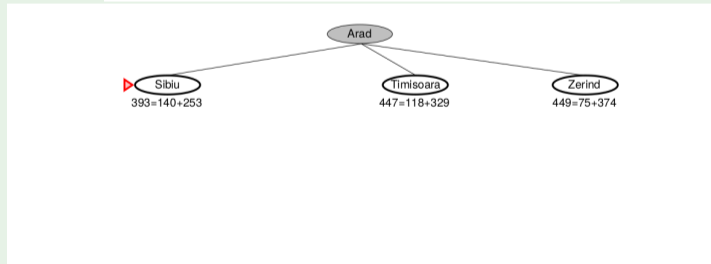
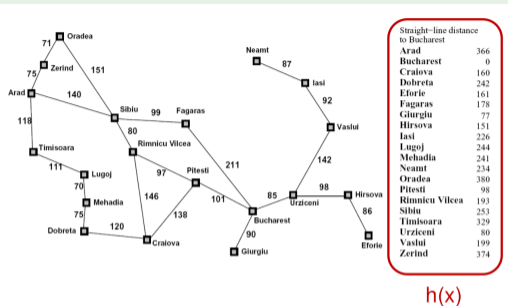
- A* Search orders by the sum: $f(n) = g(n) + h(n)$

A* Search Strategy: Example

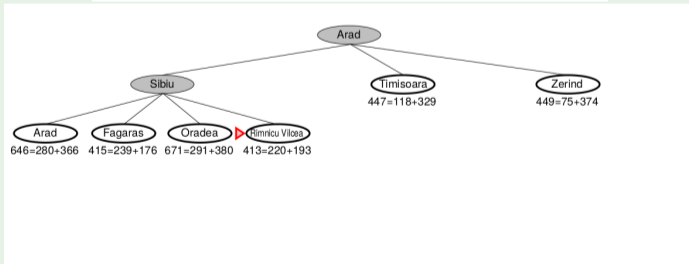
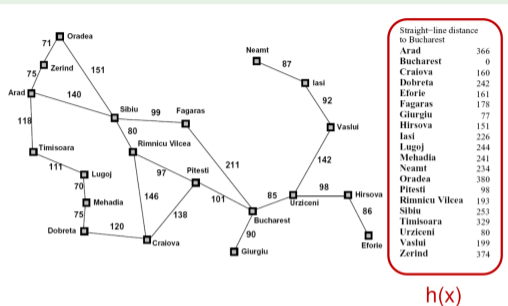


▶ Arad
366=0+366

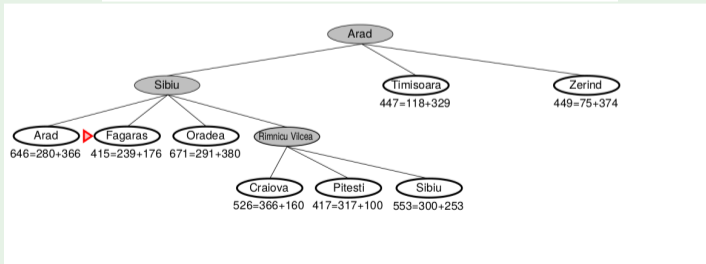
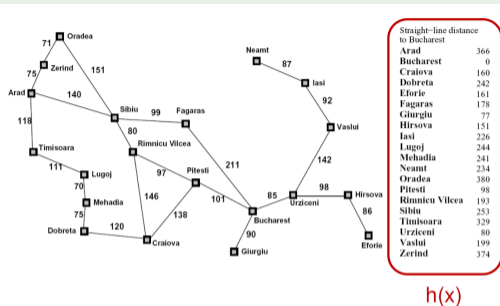
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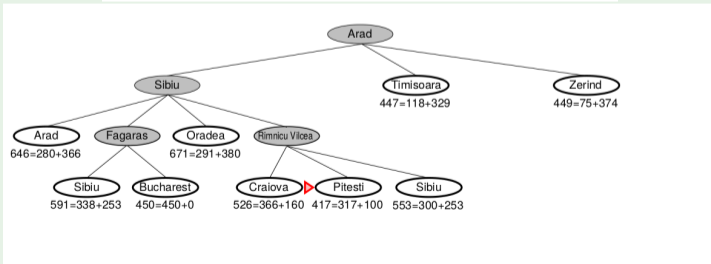
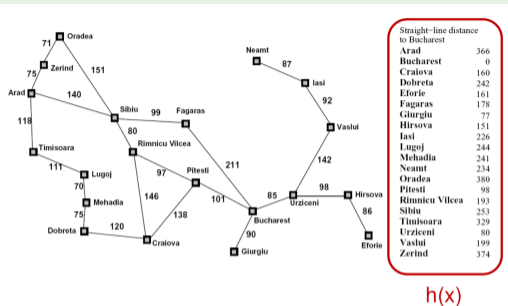
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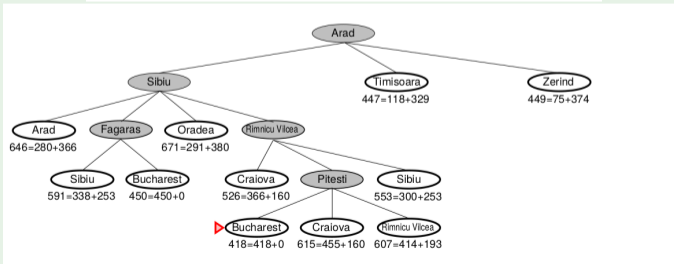
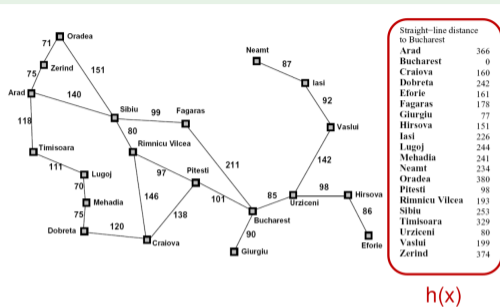
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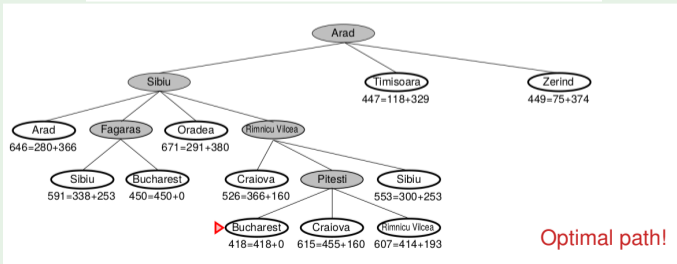
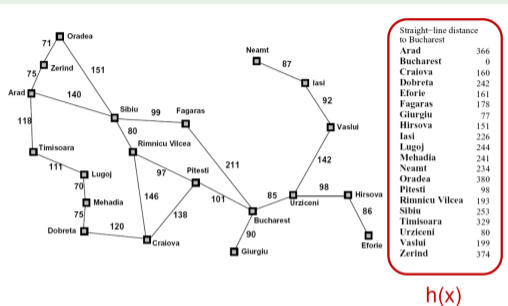
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A* Search Strategy: Example



Exercise

- Modify the Romanian Map example as follows:
 - drop the arcs from Faragas to Bucharest and from Pitesti to Bucharest
 - add one arc from Oradea to Neamt of length 250.
- Execute the A^* algorithm from Arad to Bucharest with such new map

A* Search: Admissible and Consistent Heuristics

Admissible heuristics $h(n)$

- $h(n)$ is **admissible** (aka **optimistic**) iff it never overestimates the cost to reach the goal:
 - $h(n) \leq h^*(n)$ where $h^*(n)$ is the true cost from n
 - ex: the straight-line distance $h_{SDL}()$ to Bucharest

Consistent heuristics $h(n)$

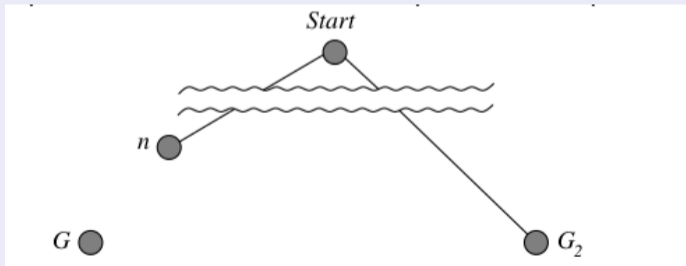
- $h(n)$ is **consistent** (aka **monotonic**) iff, for every successor n' of n generated by any action a with step cost $c(n, a, n')$,
 $h(n) \leq c(n, a, n') + h(n')$
 - verifies the triangular inequality
 - ex: the straight-line distance $h_{SDL}()$ to Bucharest
 - if $h(n)$ is consistent, then $h(n)$ is admissible (straightforward)

A* Tree Search: Optimality

If $h(n)$ is admissible, then A* tree search is optimal

- Suppose some sub-optimal goal G_2 is in the frontier queue.
- Consider any unexpanded node n on a shortest path to an optimal goal G .
- Then:
$$\begin{aligned} f(G_2) &= g(G_2) \quad \text{since } h(G_2) = 0 \\ &> g(G) \quad \text{since } G_2 \text{ sub-optimal} \\ &\geq f(n) \quad \text{since } h \text{ is admissible} \end{aligned}$$

⇒ A* will not pick G_2 from the frontier queue before n



A* Tree Search: Optimality

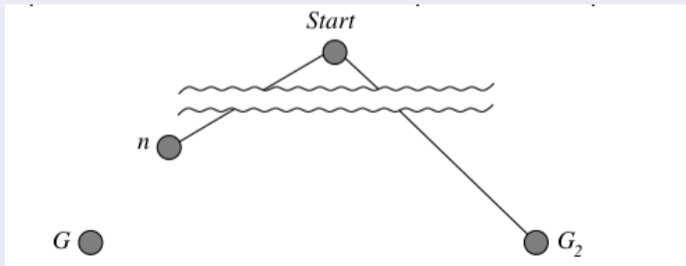
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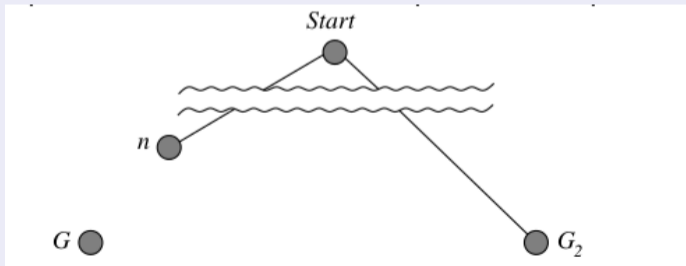
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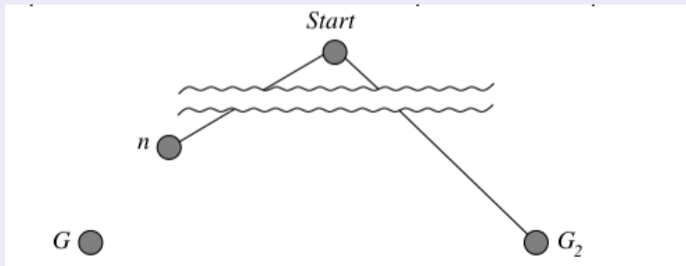
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A* Graph Search: Properties

Properties

1 if $h(n)$ is consistent, then $h(n)$ is admissible (straightforward)

2 If $h(n)$ is consistent, then $f(n)$ is non-decreasing along any path:

- let n' be a successor of n :

$$f(n') = g(n') + h(n') = \overbrace{g(n) + c(n, a, n')}^{g(n')} + h(n') \geq g(n) + h(n) = f(n)$$

3 Graph A* selects node n from the frontier only if the optimal path to n has been found

- if not so, there would be a node n' in the frontier on the optimal path to n (because of the graph separation property)
- since f is non-decreasing along any path, $f(n') \leq f(n)$
- since n' is on the optimal path to n , $f(n') < f(n)$

$\implies n'$ would have been selected before n

\implies A* (graph search) expands nodes in non-decreasing order of f

As soon as we progress along an optimal path, h (and hence f) becomes progressively less optimistic and more realistic.

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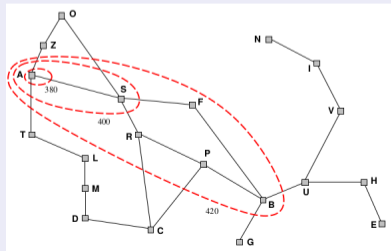
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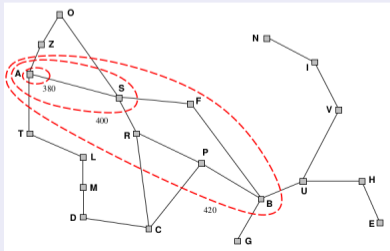
- A* expands nodes in order of non-decreasing f value
- Gradually adds “ f -contours” of nodes (as BFS adds layers)
 - contour i has all nodes with $f = f_i$, s.t. $f_i < f_{i+1}$
 - cannot expand contour f_{i+1} until contour f_i is fully expanded
- If C^* is the cost of the optimal solution path
 - 1 A* expands all nodes s.t. $f(n) < C^*$
 - 2 A* might expand some of the nodes on “goal contour” s.t. $f(n) = C^*$ before selecting a goal node.
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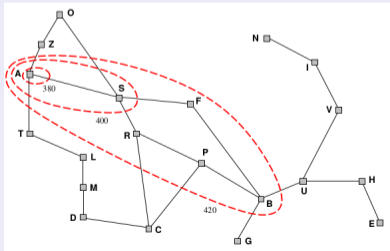
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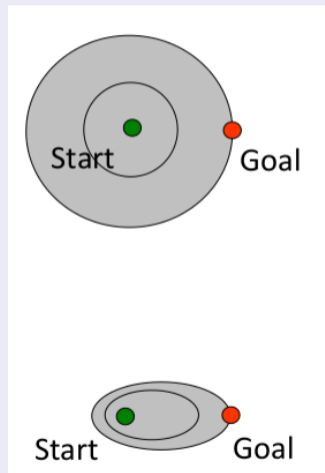
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UCS vs A* Contours

Intuition

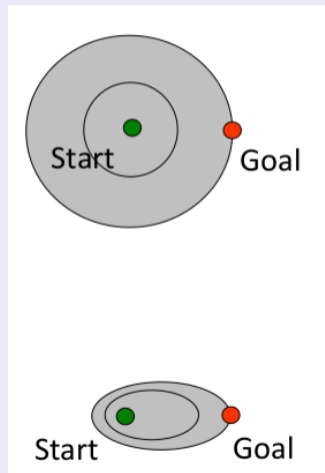
- UCS expands equally in all “directions”
- A* expands mainly toward the goal



UCS vs A* Contours

Intuition

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A* Search: Completeness

If all step costs exceed some finite ϵ and b is finite,
then there are only finitely many nodes n s.t. $f(n) \leq C^*$
 \implies A* is complete.

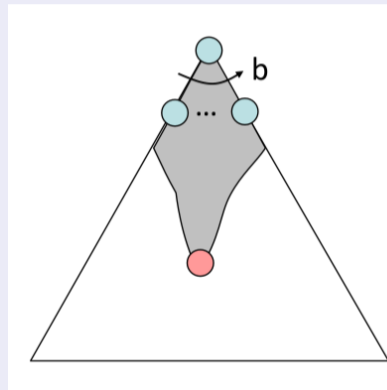
A* Search: Properties

(Under simplified hypotheses) it can be shown the following.

Let $\epsilon \stackrel{\text{def}}{=} (h^* - h)/h^*$ (relative error)

b^ϵ : effective branching factor

- How many steps?
 - takes $O((b^\epsilon)^d)$ time
 - if good heuristics, may give dramatic improvements
- How much memory?
 - Keeps all nodes in memory
 - $\Rightarrow O((b^\epsilon)^d)$ memory size (exponential, like UCS)
- Is it complete?
 - yes
- Is it optimal?
 - yes



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Memory requirement is a major problem also for A*

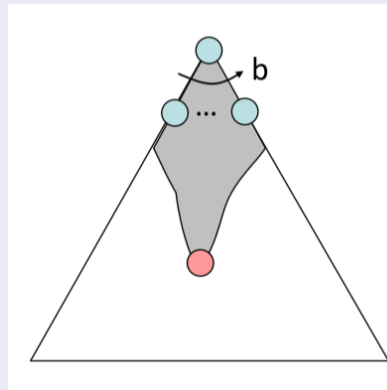
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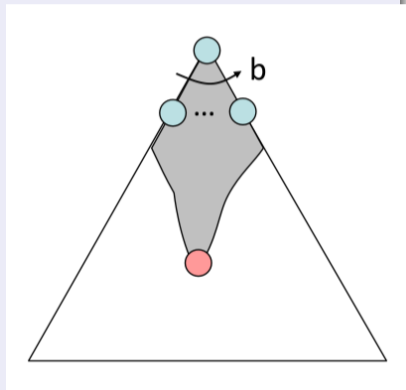
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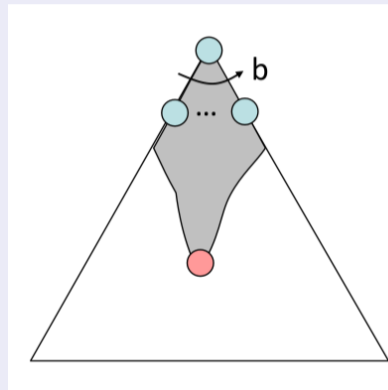
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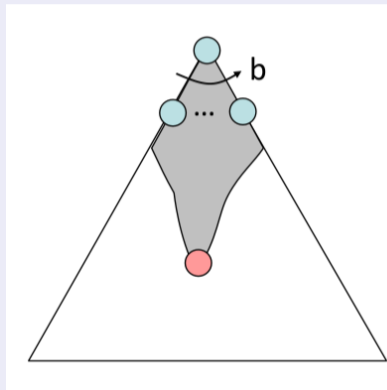
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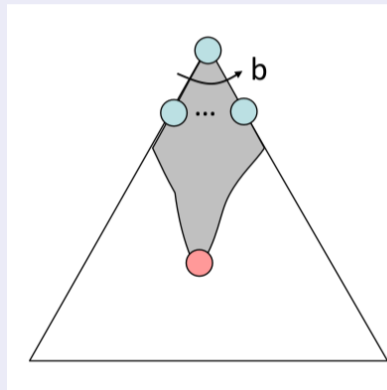
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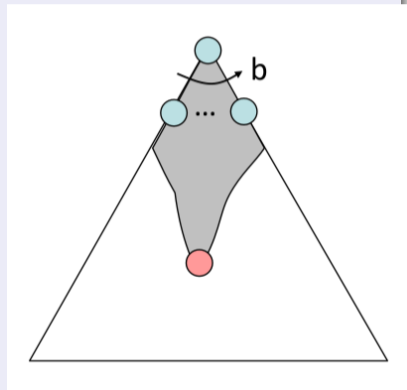
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 - Keeps all nodes in memory
 - $\implies O((b^\epsilon)^d)$ memory size (exponential, like UCS)
- Is it complete?
 - yes
- Is it optimal?
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Memory requirement is a major problem also for A*

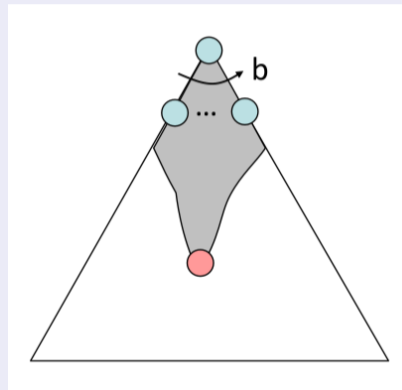
A* Search: Properties

(Under simplified hypotheses) it can be shown the following.

Let $\epsilon \stackrel{\text{def}}{=} (h^* - h)/h^*$ (relative error)

b^ϵ : effective branching factor

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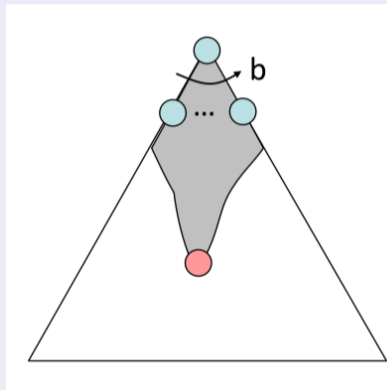
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Outline

- 1 Problem-Solving Agents
- 2 Example Problems
- 3 Search Generalities
 - Tree Search
 - Graph Search
 - Implementation Issues & Strategies
- 4 Uninformed Search Strategies
 - Breadth-First Search
 - Uniform-cost Search
 - Depth-First Search
 - Depth-Limited Search & Iterative Deepening
- 5 **Informed Search Strategies**
 - Greedy Best-First Search
 - A* Search
 - **Memory-bounded Heuristic Search (hints)**
 - Heuristic Functions

Memory-bounded Heuristic Search (hints)

Some solutions to A^* space problems
(maintain completeness and optimality)

- Iterative-deepening A^* (IDA^*)
 - here cutoff information is the f-cost ($g+h$) instead of depth
- Recursive best-first search (RBFS)
 - attempts to mimic standard best-first search with linear space
- (simple) Memory-bounded A^* ((S)MA*)
 - drop the worst-leaf node when memory is full

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Admissible Heuristics

Main problem

What is the best admissible/consistent heuristic?

Dominance of Admissible Heuristics

Dominance

Let $h_1(n)$, $h_2(n)$ admissible heuristics.

- $h_2(n)$ dominates $h_1(n)$ iff $h_2(n) \geq h_1(n)$ for all n .

$\implies h_2(n)$ is better for search

- is nearer to $h^*(n)$

Let $h_1(n)$, $h_2(n)$ admissible heuristics. Let $h_{12} \stackrel{\text{def}}{=} \max(h_1(n), h_2(n))$.

- h_{12} is also admissible
- h_{12} dominates both $h_1(n)$, $h_2(n)$

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Admissible Heuristics: Example

Ex: Heuristics for the 8-puzzle

- $h_1(n)$: number of misplaced tiles
- $h_2(n)$: total Manhattan distance over all tiles
 - (i.e., # of squares from desired location of each tile)
- $h_1(S)?$ 6
- $h_2(S)?$ $4+0+3+3+1+0+2+1 = 14$
- $h^*(S)?$ 26
- both $h_1(n), h_2(n)$ admissible (\leq number of actual steps to solve)
- $h_2(n)$ dominates $h_1(n)$

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

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Quality of Heuristics

Effective branching factor

- **Effective branching factor b^*** : the branching factor that a uniform tree of depth d would have in order to contain $N+1$ nodes

$$N + 1 = 1 + b^* + (b^*)^2 + \dots + (b^*)^d$$

N being the number of nodes generated by the A^* search

- ex: if $d=5$ and $N = 52$, then $b^* = 1.92$
- experimental measure of b^* is fairly constant for hard problems
⇒ can provide a good guide to the heuristic's overall usefulness
- Ideal value of b^* is 1

Admissible Heuristics: Example [cont.]

Average performances on 100 random samples of 8-puzzle

Iterative-deepening search (IDS) vs. A^*

d	Search Cost (nodes generated)			Effective Branching Factor		
	IDS	$A^*(h_1)$	$A^*(h_2)$	IDS	$A^*(h_1)$	$A^*(h_2)$
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	3644035	227	73	2.78	1.42	1.24
14	–	539	113	–	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

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⇒ Dramatic performance improvement

Admissible Heuristics from Relaxed Problems

Idea: Admissible heuristics can be derived from the exact solution cost of a relaxed version of the problem

- Relaxed 8-puzzle: a tile can move from any tile to any other tile
⇒ $h_1(n)$ gives the shortest solution
- Relaxed 8-puzzle: a tile can move to any adjacent square
⇒ $h_2(n)$ gives the shortest solution

- The relaxed problem adds edges to the state space
⇒ any optimal solution in the original problem is also a solution in the relaxed problem
⇒ the cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- the derived heuristic is an exact cost for the relaxed problem
⇒ must obey the triangular inequality
⇒ consistent

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Inferring Automatically Admissible Heuristics

Idea: If a problem definition is written down in a formal language, it is possible to construct relaxed problems automatically

Example

- 8-puzzle actions:

- we can generate three relaxed problems by removing one or both of the conditions

- (a) a tile can move from square A to square B if A is adjacent to B

- (b) a tile can move from square A to square B if B is blank

- (c) a tile can move from square A to square B

⇒ (a) corresponds to $h_2(n)$, (c) corresponds to $h_1(n)$,

The tool ABSolver can generate such heuristics automatically.

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Example

- 8-puzzle actions:

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Learning Admissible Heuristics

- Another way to find an admissible heuristic is through **learning from experience**:
 - Experience = solving lots of 8-puzzles
 - An **inductive learning** algorithm can be used to predict costs for other states that arise during search