Fundamentals of Artificial Intelligence
Chapter 10: **Classical Planning**

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M.S. Course "Artificial Intelligence Systems", academic year 2023-2024

Last update: Thursday 16th November, 2023, 17:35

Outline

1. Basics on Planning
   - The Problem
   - The PDDL Language
2. Search Strategies and Heuristics
   - Forward and Backward Search
   - Heuristics
3. Planning Graphs, Heuristics and Graphplan
   - Planning Graphs
   - Heuristics Driven by Planning Graphs
   - The Graphplan Algorithm
4. Other Approaches (hints)
   - Planning as SAT Solving

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Automated Planning (aka “Planning”)

Automated Planning

Synthesize a sequence of actions (plan) to be performed by an agent leading from an initial state of the world to a set of target states (goal)

- Planning is both:
  - an application per se
  - a common activity in many applications
    (e.g. design & manufacturing, scheduling, robotics,...)
- Similar to problem-solving agents (Ch.03), with factored/structured representation of states
- “Classical” Planning (this chapter):
  fully observable, deterministic, static environments with single agents

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Automated Planning [cont.]

Automated Planning

- Given:
  - an initial state
  - a set of actions you can perform
  - a (set of) state(s) to achieve (goal)
- Find:
  - a plan: a partially- or totally-ordered set of actions needed to achieve the goal from the initial state

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Decidability and Complexity

- PlanSAT: the question of whether there exists any plan that solves a planning problem
  - decidable for classical planning
  - with function symbols, the number of states becomes infinite

    $\Longrightarrow$ undecidable
  - in PSPACE
    - harder than NP, no polynomial-size witness (e.g., Tower of Hanoi)
- Bounded PlanSAT: the question of whether there exists any plan of a given length k or less
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A Language for Planning: PDDL

Planning Domain Definition Language (PDDL)

- A state is a conjunction of fluents: ground, function-less atoms
  - ex: $Poor \wedge Unknown$, $At(Truck_1, Melbourne) \wedge At(Truck_2, Sydney)$
  - ex of non-fluents: $At(x, y)$ (non ground), $\neg Poor$ (negated), $At(Father(Fred), Sydney)$ (not function-less)
  - closed-world assumption: all non-mentioned fluents are false
  - unique-name assumption: distinct names refer to distinct objects
- Actions are described by a set of action schemata
  - concise description: describe which fluent change

  $\Longrightarrow$ the other fluents implicitly maintain their values
- Action Schema: consists in action name, a list of variables in the schema, the precondition, the effect (aka postcondition)
  - precondition and effect are conjunctions of literals (positive or negated atomic sentences)
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PDDL: Example

- Action schema:
  $Action(Fly(p, from, to),$
  $\quad PRECOND : Plane(p) \wedge Airport(from) \wedge Airport(to) \wedge At(p, from)$
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- Action instantiation:
  $Action(Fly(P_1, SFO, JFK),$
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A Language for Planning: PDDL [cont.]

- Precondition: must hold to ensure the action can be executed
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  - action is applicable in state s if the preconditions are satisfied by s
- Effect: represent the effects of the action on the world
  - defines the result of executing the action
- Add list (ADD(a)): (the fluents in) the positive literals in the action's effects
  - ex: $\{At(p, to)\}$
- Delete list (DEL(a)): (the fluents in) the negative literals in the action's effects
  - ex: $\{At(p, from)\}$
- Result of action a in state s: RESULT(s,a)$\stackrel{\text{def}}{=}$(s$\setminus$DEL(a) $\cup$ ADD(a))
  - start from s
  - remove the fluents that appear as negative literals in effect
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  - ex: $Fly(P_1, SFO, JFK) \Longrightarrow$ remove $At(P_1, SFO)$, add $At(P_1, JFK)$

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Sometimes we want to propositionalize a PDDL problem:
replace each action schema with a set of ground actions.

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A Language for Planning: PDDL [cont.]

Time in PDDL

- Fluents do not explicitly refer to time
- Times and states are implicit in the action schemata:
  - the precondition always refers to time t
  - the effect to time t+1.

PDDL Problem

- A set of action schemata defines a planning domain
- PDDL problem: a planning domain, an initial state and a goal
  - the initial state is a conjunction of ground atoms (positive literals)
    - closed-world assumption: any not-mentioned atoms are false
  - the goal is a conjunction of literals (positive or negative)
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    - a goal g may represent a set of states (the set of states entailing g)
- Ex: goal: $At(p, SFO) \land Plane(p)$:
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A Language for Planning: PDDL [cont.]

Planning as a search problem

All components of a search problem

- an initial state
- an ACTIONS function
- a RESULT function
- and a goal test

Example: Air Cargo Transport

$$
\begin{aligned}
&Init(At(C_1, SFO) \land At(C_2, JFK) \land At(P_1, SFO) \land At(P_2, JFK)\\
&\quad \land Cargo(C_1) \land Cargo(C_2) \land Plane(P_1) \land Plane(P_2)\\
&\quad \land Airport(JFK) \land Airport(SFO))\\
&Goal(At(C_1, JFK) \land At(C_2, SFO))\\
&Action(Load(c, p, a),\\
&\quad \text{PRECOND: } At(c, a) \land At(p, a) \land Cargo(c) \land Plane(p) \land Airport(a)\\
&\quad \text{EFFECT: } \neg At(c, a) \land In(c, p))\\
&Action(Unload(c, p, a),\\
&\quad \text{PRECOND: } In(c, p) \land At(p, a) \land Cargo(c) \land Plane(p) \land Airport(a)\\
&\quad \text{EFFECT: } At(c, a) \land \neg In(c, p))\\
&Action(Fly(p, from, to),\\
&\quad \text{PRECOND: } At(p, from) \land Plane(p) \land Airport(from) \land Airport(to)\\
&\quad \text{EFFECT: } \neg At(p, from) \land At(p, to))
\end{aligned}
$$

One solution: [$Load(C_1, P_1, SFO)$, $Fly(P_1, SFO, JFK)$, $Unload(C_1, P_1, JFK)$, $Load(C2, P2, JFK)$, $Fly(P2, JFK, SFO)$, $Unload(C2, P2, SFO)$]

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&\mathit{Init}(\mathit{At}(C_1,\ \mathit{SFO}) \land \mathit{At}(C_2,\ \mathit{JFK}) \land \mathit{At}(P_1,\ \mathit{SFO}) \land \mathit{At}(P_2,\ \mathit{JFK}) \\
&\quad \land \mathit{Cargo}(C_1) \land \mathit{Cargo}(C_2) \land \mathit{Plane}(P_1) \land \mathit{Plane}(P_2) \\
&\quad \land \mathit{Airport}(\mathit{JFK}) \land \mathit{Airport}(\mathit{SFO})) \\
&\mathit{Goal}(\mathit{At}(C_1,\ \mathit{JFK}) \land \mathit{At}(C_2,\ \mathit{SFO})) \\
&\mathit{Action}(\mathit{Load}(c,\ p,\ a), \\
&\quad \text{PRECOND: } \mathit{At}(c,\ a) \land \mathit{At}(p,\ a) \land \mathit{Cargo}(c) \land \mathit{Plane}(p) \land \mathit{Airport}(a) \\
&\quad \text{EFFECT: } \neg\, \mathit{At}(c,\ a) \land \mathit{In}(c,\ p)) \\
&\mathit{Action}(\mathit{Unload}(c,\ p,\ a), \\
&\quad \text{PRECOND: } \mathit{In}(c,\ p) \land \mathit{At}(p,\ a) \land \mathit{Cargo}(c) \land \mathit{Plane}(p) \land \mathit{Airport}(a) \\
&\quad \text{EFFECT: } \mathit{At}(c,\ a) \land \neg\, \mathit{In}(c,\ p)) \\
&\mathit{Action}(\mathit{Fly}(p,\ \mathit{from},\ \mathit{to}), \\
&\quad \text{PRECOND: } \mathit{At}(p,\ \mathit{from}) \land \mathit{Plane}(p) \land \mathit{Airport}(\mathit{from}) \land \mathit{Airport}(\mathit{to}) \\
&\quad \text{EFFECT: } \neg\, \mathit{At}(p,\ \mathit{from}) \land \mathit{At}(p,\ \mathit{to}))
\end{aligned}
$$

One solution: [$\mathit{Load}(C_1, P_1, \mathit{SFO})$, $\mathit{Fly}(P_1, \mathit{SFO}, \mathit{JFK})$, $\mathit{Unload}(C_1, P_1, \mathit{JFK})$, $\mathit{Load}(C2, P2, \mathit{JFK})$, $\mathit{Fly}(P2, \mathit{JFK}, \mathit{SFO})$, $\mathit{Unload}(C2, P2, \mathit{SFO})$]

Example: Spare Tire Problem

$$
\begin{array}{l}
Init(Tire(Flat) \wedge Tire(Spare) \wedge At(Flat, Axle) \wedge At(Spare, Trunk)) \\
Goal(At(Spare, Axle)) \\
Action(Remove(obj, loc), \\
\quad \text{PRECOND: } At(obj, loc) \\
\quad \text{EFFECT: } \neg At(obj, loc) \wedge At(obj, Ground)) \\
Action(PutOn(t, Axle), \\
\quad \text{PRECOND: } Tire(t) \wedge At(t, Ground) \wedge \neg At(Flat, Axle) \\
\quad \text{EFFECT: } \neg At(t, Ground) \wedge At(t, Axle)) \\
Action(LeaveOvernight, \\
\quad \text{PRECOND: } \\
\quad \text{EFFECT: } \neg At(Spare, Ground) \wedge \neg At(Spare, Axle) \wedge \neg At(Spare, Trunk) \\
\qquad\qquad \wedge \neg At(Flat, Ground) \wedge \neg At(Flat, Axle) \wedge \neg At(Flat, Trunk))
\end{array}
$$

(We assume that the car is parked in a particularly bad neighborhood, so that the effect of leaving it overnight is that the tires disappear.)

One solution: [*Remove*(*Flat*, *Axle*), *Remove*(*Spare*, *Trunk*), *PutOn*(*Spare*, *Axle*)]

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\begin{array}{l}
\mathit{Init}(\mathit{Tire}(\mathit{Flat}) \land \mathit{Tire}(\mathit{Spare}) \land \mathit{At}(\mathit{Flat}, \mathit{Axle}) \land \mathit{At}(\mathit{Spare}, \mathit{Trunk})) \\
\mathit{Goal}(\mathit{At}(\mathit{Spare}, \mathit{Axle})) \\
\mathit{Action}(\mathit{Remove}(\mathit{obj}, \mathit{loc}), \\
\quad \text{PRECOND: } \mathit{At}(\mathit{obj}, \mathit{loc}) \\
\quad \text{EFFECT: } \neg \mathit{At}(\mathit{obj}, \mathit{loc}) \land \mathit{At}(\mathit{obj}, \mathit{Ground})) \\
\mathit{Action}(\mathit{PutOn}(t, \mathit{Axle}), \\
\quad \text{PRECOND: } \mathit{Tire}(t) \land \mathit{At}(t, \mathit{Ground}) \land \neg \mathit{At}(\mathit{Flat}, \mathit{Axle}) \\
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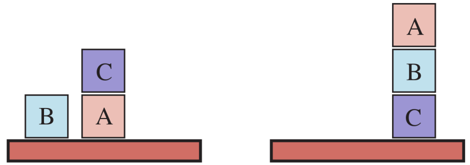
(We assume that the car is parked in a particularly bad neighborhood, so that the effect of leaving it overnight is that the tires disappear.)

One solution: [*Remove*(*Flat*, *Axle*), *Remove*(*Spare*, *Trunk*), *PutOn*(*Spare*, *Axle*)]

Example: Blocks World



Start State Goal State

$$
\begin{aligned}
&Init(On(A, Table) \wedge On(B, Table) \wedge On(C, A) \\
&\quad \wedge Block(A) \wedge Block(B) \wedge Block(C) \wedge Clear(B) \wedge Clear(C)) \\
&Goal(On(A, B) \wedge On(B, C)) \\
&Action(Move(b, x, y), \\
&\quad \text{PRECOND: } On(b, x) \wedge Clear(b) \wedge Clear(y) \wedge Block(b) \wedge Block(y) \wedge \\
&\qquad\qquad (b \neq x) \wedge (b \neq y) \wedge (x \neq y), \\
&\quad \text{EFFECT: } On(b, y) \wedge Clear(x) \wedge \neg On(b, x) \wedge \neg Clear(y)) \\
&Action(MoveToTable(b, x), \\
&\quad \text{PRECOND: } On(b, x) \wedge Clear(b) \wedge Block(b) \wedge (b \neq x), \\
&\quad \text{EFFECT: } On(b, Table) \wedge Clear(x) \wedge \neg On(b, x))
\end{aligned}
$$

One solution: [*MoveToTable*(*C*, *A*), *Move*(*B*, *Table*, *C*), *Move*(*A*, *Table*, *B*)]

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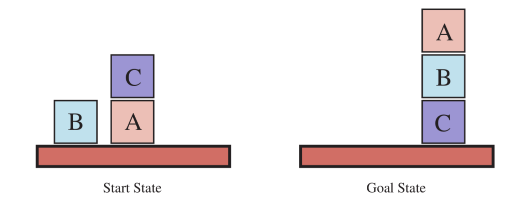

$$
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Outline

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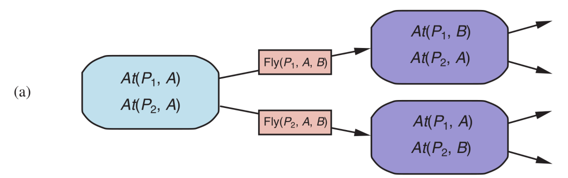
Two Main Approaches

(a) Forward search (aka progression search)

- start in the initial state
- use actions to search forward for a goal state

(b) Backward search (aka regression search)

- start from goals
- use reverse actions to search forward for the initial state



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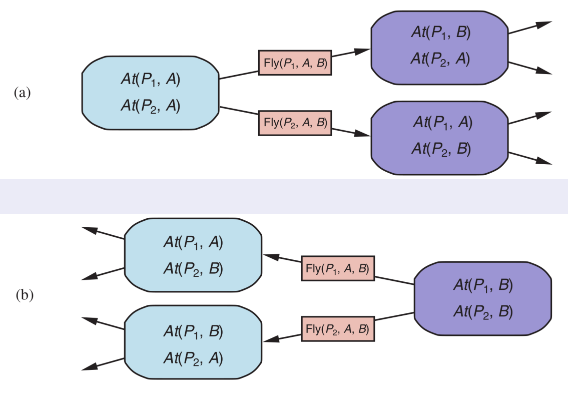

(© S. Russell & P. Norwig, AIMA)

Forward Search

- Forward search (aka progression search)
  - choose actions whose preconditions are satisfied
  - add positive effects, delete negative
- Goal test: does the state satisfy the goal?
- Step cost: each action costs 1

$\Longrightarrow$ We can use any of the search algorithms from Ch. 03, 04
  - need keeping track of the actions used to reach the goal
- Breadth-first and best-first
  - Sound: if they return a plan, then the plan is a solution
  - Complete: if a problem has a solution, then they will return one
  - Require exponential memory wrt. solution length! $\Longrightarrow$ unpractical
- Depth-first search and greedy search
  - Sound
  - Not complete
    - may enter in infinite loops
    - (classical planning only): made complete by loop-checking
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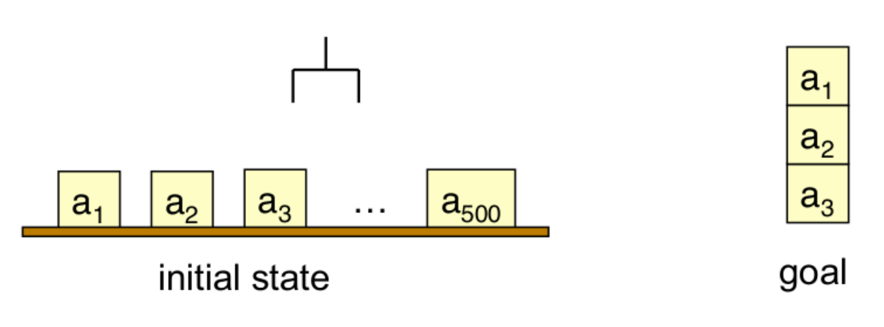
Branching Factor of Forward Search

- Planning problems can have huge state spaces
- Forward search can have a very large branching factor
  - ex: $pickup(a_1), pickup(a_2), \ldots, pickup(a_{500})$

$\Longrightarrow$ Forward-search can waste time trying lots of irrelevant actions

$\Longrightarrow$ Need a good heuristic to guide the search



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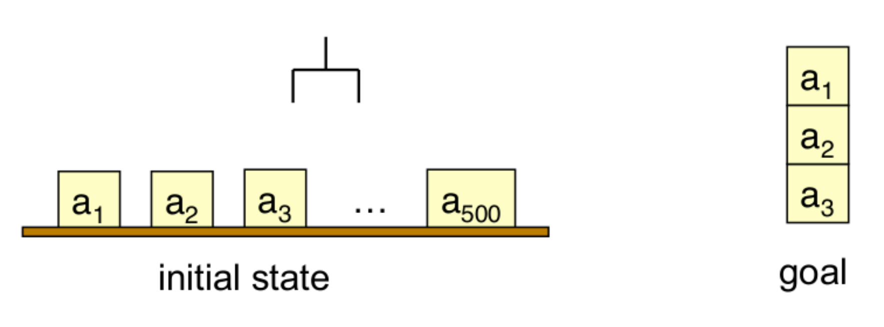

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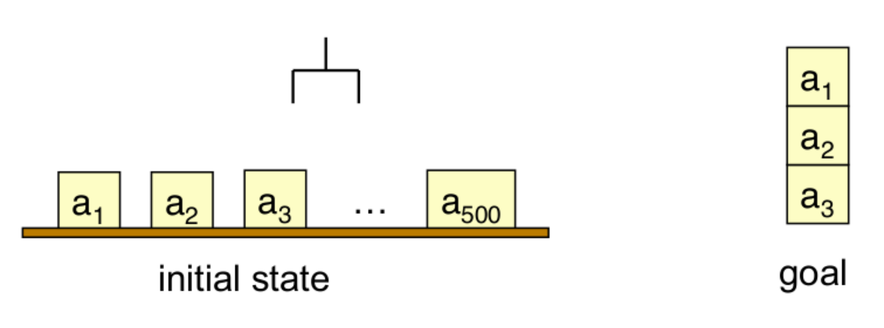

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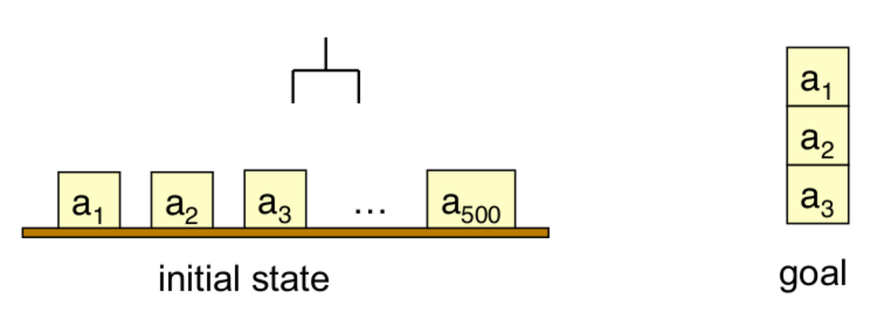

Backward Search (aka Regression or Relevant-States)

- Predecessor (sub)goal g' of ground goal g via ground action a:
  $Pos(g') \stackrel{\text{def}}{=} (Pos(g) \setminus Add(a)) \cup Pos(Precond(a))$
  $Neg(g') \stackrel{\text{def}}{=} (Neg(g) \setminus Del(a)) \cup Neg(Precond(a))$
- Note: Both g and g' represent many states
  - irrelevant ground atoms unassigned

- Consider the goal $At(C_1, SFO) \wedge At(C_2, JFK)$
- Consider the ground action:
  $Action(Unload(C_1, P_1, SFO),$
  $PRECOND : In(C_1, P_1) \wedge At(P_1, SFO) \wedge Cargo(C_1) \wedge Plane(P_1) \wedge Airport(SFO)$
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Backward Search [cont.]

- Idea: deal with partially un-instantiated actions and states
  - avoid unnecessary instantiations

  $\Longrightarrow$ no need to produce a goal for every possible instantiation
- use the most general unifier $\Longrightarrow$ compute weakest precondition
- standardize action schemata first (rename vars into fresh ones)

- Consider the goal $At(C_1, SFO) \land At(C_2, JFK)$
- Consider the partially-instantiated action:

  $Action(Unload(C_1, p', SFO),$
  $PRECOND : In(C_1, p') \land At(p', SFO) \land Cargo(C_1) \land Plane(p') \land Airport(SFO)$
  $EFFECT : At(C_1, SFO) \land \neg In(C_1, p'))$
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  $In(C_1, p') \land At(p', SFO) \land Cargo(C_1) \land Plane(p') \land Airport(SFO) \land At(C_2, JFK)$
- Represents states with all possible planes

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Backward Search [cont.]

- Idea: deal with partially un-instantiated actions and states
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- use the most general unifier $\Longrightarrow$ compute weakest precondition
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Which action to choose?

- Relevant action: could be the last step in a plan for goal g
  - at least one of the action's effects (positive or negative) must unify with an element of the goal

  (see AIMA book for formal definition)
- Consistent action: must not undo desired literals of the goal
  - inconsistent actions are also non-relevant

- Ex: consider the goal $At(C_1, SFO) \wedge At(C_2, JFK)$

  is relevant (previous example)

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Outline

Heuristics for (Forward-Search) Planning

A^* for Planning

- Recall: A^* is a best-first algorithm which
  - uses an evaluation function f(s) = g(s) + h(s),
  - g(s): (exact) cost to reach s
  - h(s): admissible (optimistic) heuristics
    (never overestimates the distance to the goal)
- A technique for admissible heuristics: problem relaxation

  $\Longrightarrow$ h(s): the exact cost of a solution to the relaxed problem
- Forms of problem relaxation exploiting problem structure
  - Add arcs to the search graph $\Longrightarrow$ make it easier to search
    - ignore-preconditions heuristics
    - ignore-delete-lists heuristics
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Ignore (some) Preconditions Heuristics

- Ignore all preconditions drops all preconditions from actions
  - every action is applicable in any state
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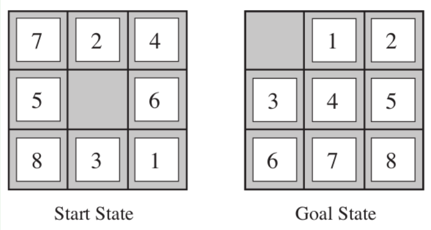
Ignore-Preconditions Heuristics: Example

Sliding tiles

$Action(Slide(t, s_1, s_2),$
$\quad PRECOND : Tile(t) \wedge Blank(s_2) \wedge On(t, s_1) \wedge Adjacent(s_1, s_2)$
$\quad EFFECT : On(t, s_2) \wedge Blank(s_1) \wedge \neg On(t, s_1) \wedge \neg Blank(s_2))$

- Remove the preconditions $Blank(s_2) \wedge Adjacent(s_1, s_2)$
  $\Longrightarrow$ we get the number-of-misplaced-tiles heuristics
- Remove the precondition $Blank(s_2)$
  $\Longrightarrow$ we get the Manhattan-distance heuristics



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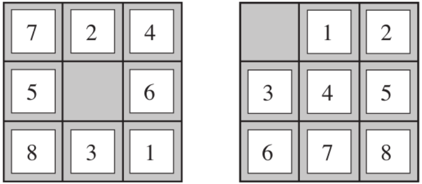

Start State

Goal State

(Courtesy of Maria Simi, UniPI)

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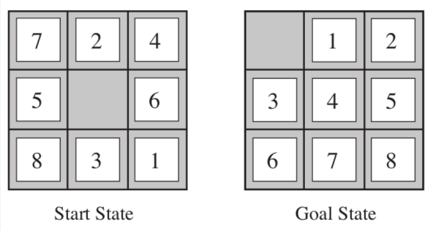

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Ignore Delete-list Heuristics

- Assumption: goals & preconditions contain only positive literals
  - reasonable in many domains
- Idea: Remove the delete lists from all actions
  - No action will ever undo the effect of actions,

  $\Longrightarrow$ there is a monotonic progress towards the goal
- Still NP-hard to find the optimal solution of the relaxed problem
  - can be approximated in polynomial time, with hill-climbing
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Ignore Delete-list Heuristics: Example (Hoffmann'05)

- Planning state spaces with ignore-delete-lists heuristic
  - height above the bottom plane is the heuristic score of a state
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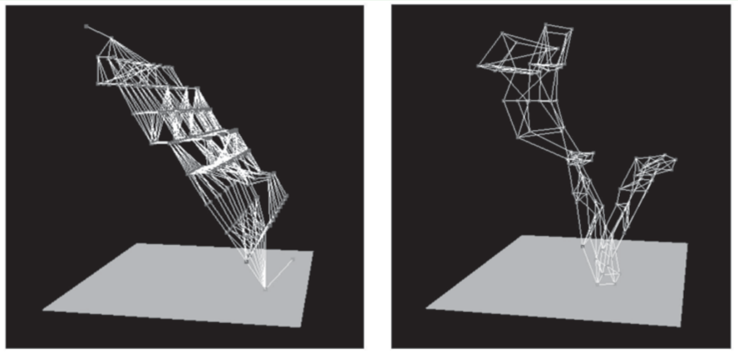
$\Longrightarrow$ No local minima, non dead-ends, non backtracking

$\Longrightarrow$ Search for the goal is straightforward for hill-climbing



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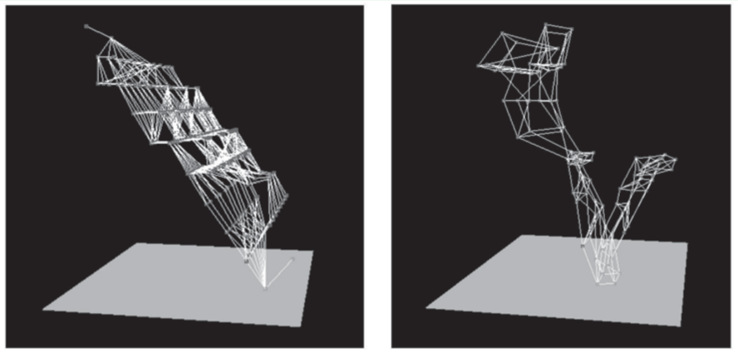

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State Abstractions

- Many-to-one mapping from states in the ground/original representation of the problem to a more abstract representation
  - drastically reduces the number of states
- Common strategy: ignore some (less-relevant) fluents
  - drop k fluents $\Longrightarrow$ reduce search space by 2^k factors
  - relevance based on (heuristic) evaluation or domain knowledge

- Air cargo problem: 10 airports, 50 planes, 200 pieces of cargo
  $\Longrightarrow 10^{50} \cdot (50+10)^{200} \approx 10^{405}$ states (*)
- Consider particular problem in that domain
  - all packages are at 5 airports
  - all packages at a given airport have the same destination
- Abstraction: drop all “At” fluents except for these involving one plane and one package at each of the the 5 airports
  $\Longrightarrow 10^{5} \cdot (5+10)^{5} \approx 10^{11}$ states (*)
  - abstract solution shorter than ground solutions $\Longrightarrow$ admissible
  - abstract solution easy to extend: add Load and Unload actions

(*) wrong in AIMA III Ed, corrected in later editions

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(*) wrong in AIMA III Ed, corrected in later editions

State Abstractions

- Many-to-one mapping from states in the ground/original representation of the problem to a more abstract representation
  - drastically reduces the number of states
- Common strategy: ignore some (less-relevant) fluents
  - drop k fluents $\Longrightarrow$ reduce search space by 2^k factors
  - relevance based on (heuristic) evaluation or domain knowledge

- Air cargo problem: 10 airports, 50 planes, 200 pieces of cargo
  $\Longrightarrow$ $10^{50} \cdot (50+10)^{200} \approx 10^{405}$ states (*)
- Consider particular problem in that domain
  - all packages are at 5 airports
  - all packages at a given airport have the same destination
- Abstraction: drop all "At" fluents except for these involving one plane and one package at each of the the 5 airports
  $\Longrightarrow$ $10^{5} \cdot (5+10)^{5} \approx 10^{11}$ states (*)
  - abstract solution shorter than ground solutions $\Longrightarrow$ admissible
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Other Strategies for Planning

Other strategies to define heuristics

- Problem decomposition
  - “divide & conquer” problem into subproblem
  - solve subproblems independently
- Using a data structure called “planning graph” (next section)

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Outline

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Generalities

Planning Graph

- A data structure which is a rich source of information:
  - can be used to give better heuristic estimates h(s)
  - can drive an algorithm called Graphplan
- A polynomial-size over-approximation to the (exponential) search tree
  - can be constructed very quickly

\- cannot answer definitively if goal g is reachable from initial state

\+ may discover that the goal is not reachable

\+ can estimate the most-optimistic step # to reach g

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Planning Graph: Definition

- A directed graph, built forward and organized into levels
  - level S_0: contain each ground fluent that holds in the initial state
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  - level A_i: contains all ground actions with preconditions in S_i
  - level S_{i+1}: all the effects of all the actions in A_i
    - each S_i may contain both P_j and $\neg P_j$

  until $S_N = S_{N-1}$ ("leveled off").
- Contains persistence actions (aka maintenance actions, no-ops)
  - say that a literal l persists if no action negates it
- Mutual exclusion links (mutex) connect
  - incompatible pairs of actions
  - incompatible pairs of literals

Deals with ground states and actions only

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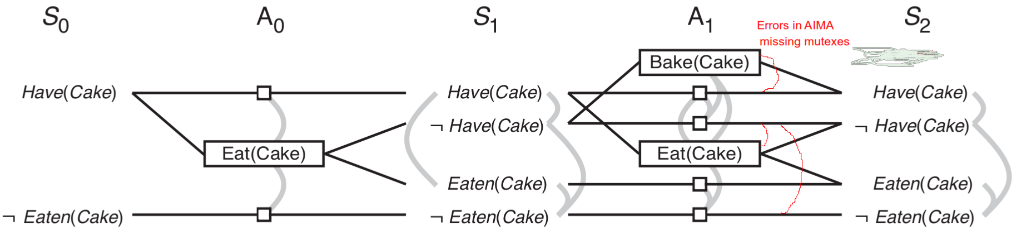
Planning Graph: Example

Init(*Have*(*Cake*))
Goal(*Have*(*Cake*) ∧ *Eaten*(*Cake*))
Action(*Eat*(*Cake*)
  PRECOND: *Have*(*Cake*)
  EFFECT: ¬ *Have*(*Cake*) ∧ *Eaten*(*Cake*))
Action(*Bake*(*Cake*)
  PRECOND: ¬ *Have*(*Cake*)
  EFFECT: *Have*(*Cake*))

You would like to eat your cake and still have a cake. Fortunately, you can bake a new one.

Rectangles indicate actions
Small squares persistence actions (**no-ops**)
Straight lines indicate preconditions
and effects
Mutex links are shown as curved gray lines



Mutex Computation

- Two actions at the same action-level have a mutex relation if
  - Inconsistent effects: an effect of one negates an effect of the other
  - Interference: one deletes a precondition of the other
  - Inconsistent preconditions (aka competing needs): they have mutually exclusive preconditions
- Otherwise they don't interfere with each other
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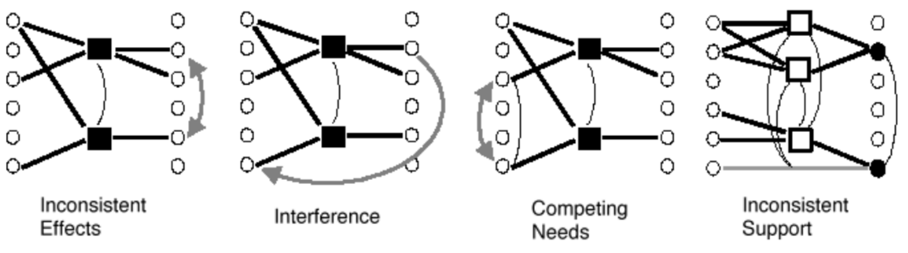

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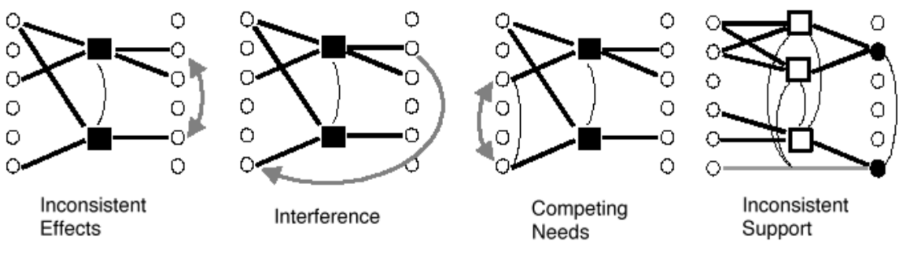

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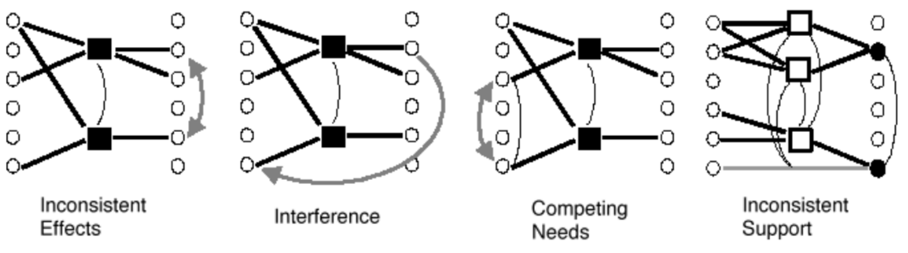

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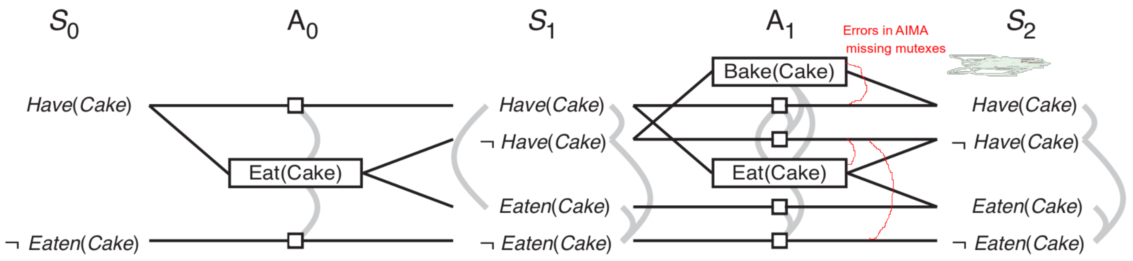

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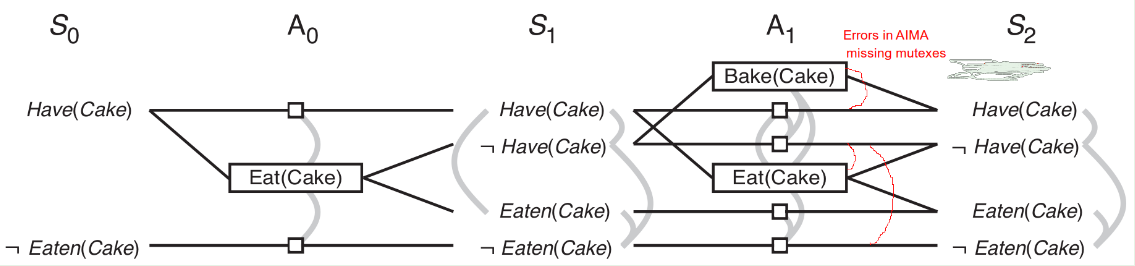

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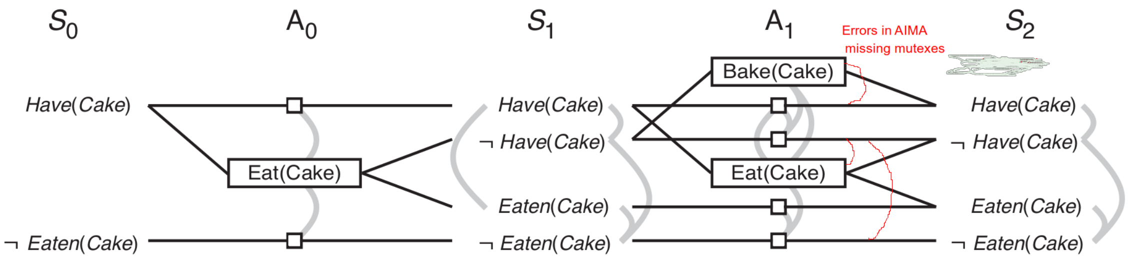

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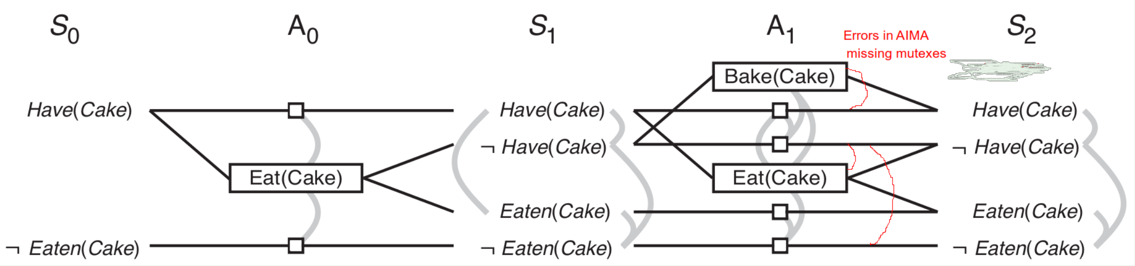

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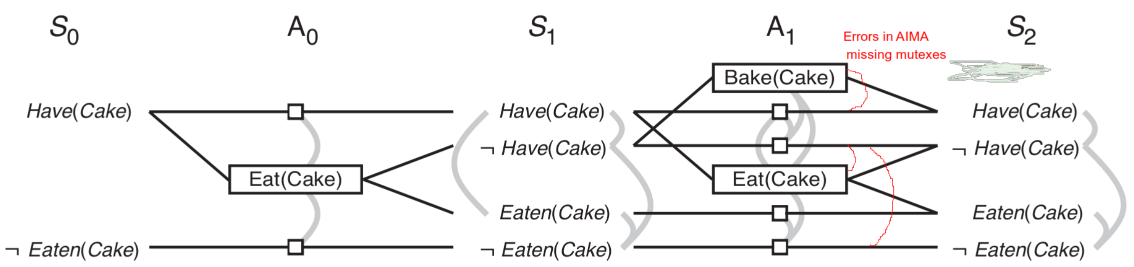
Mutex Computation: Example [cont.]

- Two literals at the same state-level have a mutex relation if
  - inconsistent support: one is the negation of the other
    ex.: *Have*(*Cake*), ¬*Have*(*Cake*)
  - all ways of achieving them are pairwise mutex
    ex.: (S_1): *Have*(*Cake*) in mutex with *Eaten*(*Cake*)
    because persistence of *Have*(*Cake*), *Eat*(*Cake*) are mutex



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Building of the Planning Graph

Create initial layer S_0:

1. insert into S_0 all literals in the initial state

Repeat for increasing values of $i = 0, 1, 2, ...$:

Create action layer A_i:

1. for each action schema, for each way to unify its preconditions to non-mutually exclusive literals in S_i, enter an action node into A_i
2. for every literal in S_i, enter a no-op action node into A_i
3. add mutexes between the newly-constructed action nodes

Create state layer S_{i+1}:

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Planning Graphs: Properties

- Literals and actions increase monotonically and are finite
  $\Longrightarrow$ we eventually reach a level where they stabilize
- Mutexes decrease monotonically (and cannot become less than zero)
  $\Longrightarrow$ they too eventually must level off

$\Longrightarrow$ When we reach this stable state, if one of the goal literals is missing or is mutex with another goal literal, then it will remain so
$\Longrightarrow$ we can stop

Planning Graphs: Complexity

- A planning graph is polynomial in the size of the problem:
  - a graph with n levels, a actions, l literals, has size $O(n(a+l)^2)$
  - time complexity is also $O(n(a+l)^2)$

$\Longrightarrow$ The process of constructing the planning graph is very fast

  - does not require choosing among actions

Outline

Planning Graphs for Heuristic Estimation

Information provided by Planning Graphs

- Each level S_i represents a set of possible belief states
  - two literals connected by a mutex belong to different belief state
- A literal not appearing in the final level of the graph cannot be achieved by any plan

  $\Longrightarrow$ if a goal literal is not in the final level, the problem is unsolvable
- The level S_j a literal l appears first is never greater than the level it can be achieved in a plan
  - j is called the level cost of literal l
- the level cost of a literal g_i in the graph constructed starting from state s, is an estimate of the cost to achieve it from s (i.e. h(g))
  - this estimate is admissible
  - ex: from s_0 Have(cake) has cost 0 and Eaten(cake) has cost 1
- Planning graph admits several actions per level

  $\Longrightarrow$ inaccurate estimate
- Serialization: enforcing only one action per level (adding mutex)

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Planning Graphs for Heuristic Estimation [cont.]

Estimating the heuristic cost of a conjunction of goal literals

- Max-level heuristic: the maximum level cost of the sub-goals
  - admissible
- Level-sum heuristic: the sum of the level costs of the goals
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The Graphplan Algorithm

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function GRAPHPLAN($problem$) **returns** solution or failure

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typo in AIMA book
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      if $solution \neq failure$ **then return** $solution$
    if $graph$ and $nogoods$ have both leveled off **then return** $failure$
    $graph \leftarrow$ EXPAND-GRAPH($graph$, $problem$)

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[Recall] Example: Spare Tire Problem

$$
\begin{aligned}
&Init(Tire(Flat) \wedge Tire(Spare) \wedge At(Flat, Axle) \wedge At(Spare, Trunk))\\
&Goal(At(Spare, Axle))\\
&Action(Remove(obj, loc),\\
&\quad \text{PRECOND: } At(obj, loc)\\
&\quad \text{EFFECT: } \neg At(obj, loc) \wedge At(obj, Ground))\\
&Action(PutOn(t,\ Axle),\\
&\quad \text{PRECOND: } Tire(t) \wedge At(t, Ground) \wedge \neg At(Flat, Axle)\\
&\quad \text{EFFECT: } \neg At(t, Ground) \wedge At(t, Axle))\\
&Action(LeaveOvernight,\\
&\quad \text{PRECOND:}\\
&\quad \text{EFFECT: } \neg At(Spare, Ground) \wedge \neg At(Spare, Axle) \wedge \neg At(Spare, Trunk)\\
&\qquad\qquad \wedge \neg At(Flat, Ground) \wedge \neg At(Flat, Axle) \wedge \neg At(Flat,\ Trunk))
\end{aligned}
$$

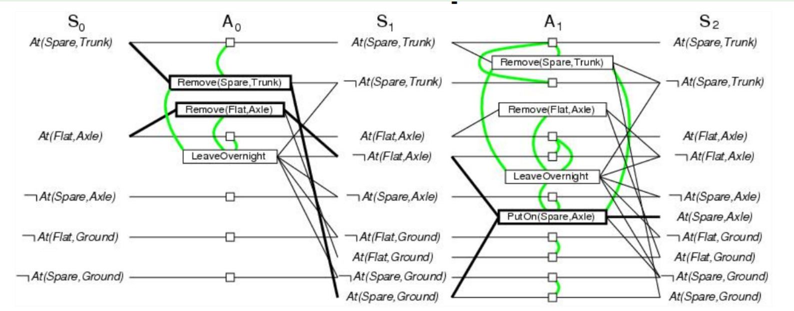
(We assume that the car is parked in a particularly bad neighborhood, so that the effect of leaving it overnight is that the tires disappear.)

One solution: [*Remove*(*Flat*, *Axle*), *Remove*(*Spare*, *Trunk*), *PutOn*(*Spare*, *Axle*)]

Graphplan: Example

Spare Tire problem

- Initial plan 5 literals from initial state and the Closed-World-Assumption literals (S_0).
  - fixed literals (e.g. *Tire*(*Flat*)) ignored here
  - irrelevant literals ignored here
- Goal *At*(*Spare*, *Axle*) not present in S_0
  $\Longrightarrow$ no need to call EXTRACT-SOLUTION
- Graph and nogoods not leveled off $\Longrightarrow$ invoke EXPAND-GRAPH



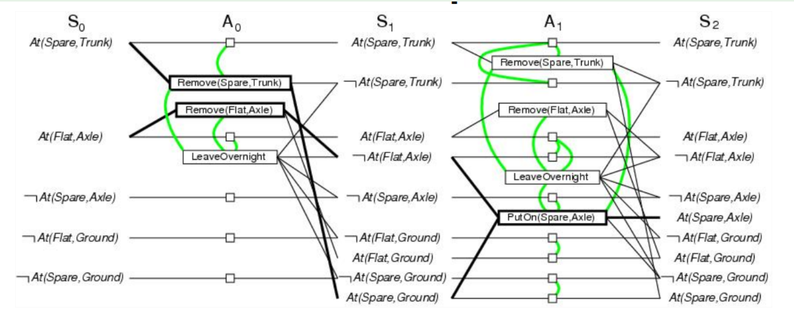
(© S. Russell & P. Norwig, AIMA) (inter-fluent mutexes omitted for readability)

Graphplan: Example [cont.]

Spare Tire problem

- Invoke EXPAND-GRAPH
  - add actions A_0, persistence actions and mutexes
  - add fluents S_1 and mutexes
- Goal $At(Spare, Axle)$ not present in S_1

  $\Longrightarrow$ no need to call EXTRACT-SOLUTION
- Graph and nogoods not leveled off $\Longrightarrow$ invoke EXPAND-GRAPH



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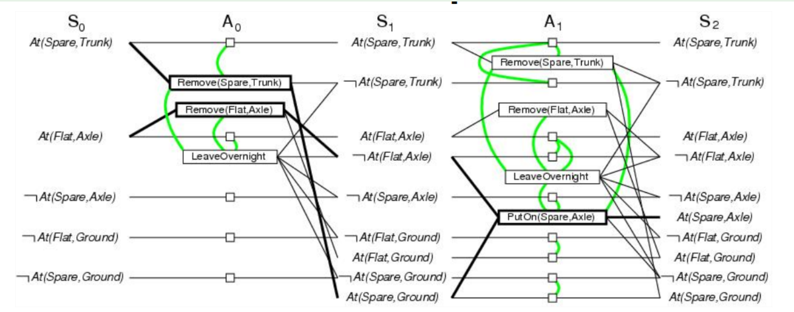
Graphplan: Example [cont.]

Spare Tire problem

- Invoke EXPAND-GRAPH
  - add actions A_1, persistence actions and mutexes
  - add fluents S_2 and mutexes
- Goal $At(Spare, Axle)$ present in S_2
  - call EXTRACT-SOLUTION
- Solution found!



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Exercise

- Consider the following variant of the Spare Tire problem: add $At(Flat, Trunk)$ to the goal
- Write the (non-serialized) planning graph
- Extract a plan from the graph
- Do the same with the serialized planning graph

The Graphplan Algorithm [cont.]

Graphplan "family" of algorithms, depending on approach used in EXTRACT-SOLUTION(...)

About EXTRACT-SOLUTION(...)

- Can be formulated as an (incremental) SAT problem
  - one proposition for each ground action and fluent
  - clauses represent preconditions, effects, no-ops and mutexes
- Can be formulated as a backward search problem
- Planning problem restricted to planning graph
  - mutexes found by EXPAND-GRAPH prune paths in the search tree

  $\Longrightarrow$ much faster than unrestricted planning
- (if P.G. not serialized) may produce partial order plans

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Partial-Order Plans

Partial-Order vs. Total-Order Plans

- Total-order plans: strictly linear sequences of actions
  - disregards the fact that some action are mutually independent
- Partial-order plans: set of precedence constraints between action pairs
  - form a directed acyclic graph
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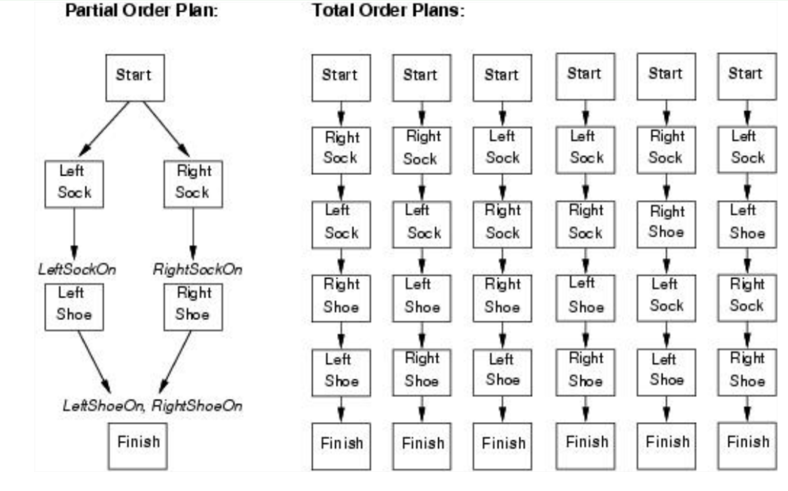
Partial-Order Plans: Example

Socks & Shoes Examples



(Courtesy of Michela Milano, UniBO)

Termination of Graphplan

- Theorem: If the graph and the no-goods have both leveled off, and no solution is found we can safely terminate with failure
- Intuition (proof sketch):
  - Literals and actions increase monotonically and are finite
    $\Longrightarrow$ we eventually reach a level where they stabilize
  - Mutexes and no-goods decrease monotonically (and cannot become less than zero)
    $\Longrightarrow$ they too eventually must level off

  $\Longrightarrow$ When we reach this stable state, if one of the goal literals is missing or is mutex with another goal literal, then it will remain so
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Exercise

- Socks & Shoes example:
  1. Formalize the Socks & Shoes example in PDDL
  2. Write the non-serialized planning graph
  3. Compute the level cost for every fluent
  4. Choose some states, compute h(s) using the three heuristics
  5. Extract a plan from the graph in (2)
  6. Compare h(s) with the level they occur in the plan
  7. Write the serialized planning graph
  8. Repeat steps (3)-(6) with the serialized graph
- Do same steps (1)-(8) for the Air Cargo Transport example

Outline

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Planning as SAT Solving

- Encode bounded planning problem into a propositional formula

$\Longrightarrow$ Solve it by (incremental) calls to a SAT solver

- A model for the formula (if any) is a plan of length t
- Many variants in the encoding
- Extremely efficient with many problems of interest

function SATPLAN($init$, $transition$, $goal$, T_{max}) **returns** solution or failure
  inputs: $init$, $transition$, $goal$, constitute a description of the problem
    T_{max}, an upper limit for plan length

  for $t = 0$ **to** T_{max} **do**
    $cnf \leftarrow$ TRANSLATE-TO-SAT($init$, $transition$, $goal$, t)
    $model \leftarrow$ SAT-SOLVER(cnf)
    if $model$ is not null **then**
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    - ex: $\langle At(P_1, SFO), 3\rangle \Longrightarrow At\_P_1\_SFO\_3$
    - ex: $\langle Fly(P_1, SFO, JFK), 3\rangle \Longrightarrow Fly\_P_1\_SFO\_JFK\_3$
  - returns propositional formula: $Init^0 \land (\bigwedge_{i=1}^{t-1} Transition^{i,i+1}) \land Goal^t$
- $Init^0$ and $Goal^t$: conjunctions of literals at step 0 and t resp.
  - ex: $Init^0$: $At\_P_1\_SFO\_0 \land At\_P_2\_JFK\_0$
  - ex: $Goal^3$: $At\_P_1\_JFK\_3 \land At\_P_2\_SFO\_3$
- $Transition^{i,i+1}$: encodes transition from steps i to $i+1$
  - Actions: $Action^i \to (Precond^i \land Effects^{i+1})$
    ex: $Fly\_P_1\_SFO\_JFK\_2 \to (At\_P_1\_SFO\_2 \land At\_P_1\_JFK\_3)$
  - No-Ops: for each fluent F and step i:

$$F^{i+1} \leftrightarrow \bigvee_k ActionCausingF_k^i \lor (F^i \land \bigwedge_j \neg ActionCausingNotF_j^i)$$

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Consider the socks & shoes example

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  - no need to CNF-ize here (human beings don't like CNFs)
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