Fundamentals of Artificial Intelligence

Laboratory

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Exercise 8.1 - Which of the following are correct?

a. False |= True. This is true because False has no models and hence entails every sentence and because True is true in all models and hence is entailed by every sentence.

b. True |= False. This is false.

c. (A ∧ B) |= (A ⇔ B). This is true because the left-hand side has exactly one model that is one of the two models of the right-hand side.

d. A ⇔ B |= A ∨ B. This is false because one of the models of A ⇔ B has both A and B false, which does not satisfy A ∨ B.

e. A ⇔ B |= ¬A ∨ B. This is true because the RHS is A ⇒ B, one of the conjuncts in the definition of A ⇔ B.

f. (A ∧ B) ⇒ C |= (A ⇒ C) ∨ (B ⇒ C). This is true because the RHS is false only when both disjuncts are false, i.e., when A and B are true and C is false, in which case the LHS is also false. This may seem counterintuitive, and would not hold if ⇒ is interpreted as “causes.”

g. (A ∨ B) ∧ (¬C ∨ ¬D ∨ E) |= (A ∨ B). This is true; removing a conjunct only allows more models.

h. (A ∨ B) ∧ (¬C ∨ ¬D ∨ E) |= (A ∨ B) ∧ (¬D ∨ E). This is false; removing a disjunct allows fewer models.

i. (A ∨ B) ∧ ¬(A ⇒ B) is satisfiable. This is satisfiable; model has A and ¬B.

j. (A ⇔ B) ∧ (¬A ∨ B) is satisfiable. This is satisfiable; RHS is entailed by LHS so models are those of A ⇔ B.

Exercise 8.2 - Homework

Using truth tables, verify the following equivalences:

a. $P \land Q \Leftrightarrow Q \land P$

b. $P \lor Q \Leftrightarrow Q \lor P$

c. $P \land (Q \land R) \Leftrightarrow (P \land Q) \land R$

d. $P \lor (Q \lor R) \Leftrightarrow (P \lor Q) \lor R$

Exercise 8.3

Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following sentences?

a. $B \lor C$.
Sentence is false only if B and C are false, which occurs in 4 cases for A and D, leaving 12.

b. $\neg A \lor \neg B \lor \neg C \lor \neg D$.
Sentence is false only if A, B, C, and D are true, which occurs in 1 case, leaving 15.

c. $(A \Rightarrow B) \land A \land \neg B \land C \land D$.
The last four conjuncts specify a model in which the first conjunct is false, so 0.

Exercise 8.4

According to some political pundits, a person who is radical (R) is electable (E) if he/she is conservative (C), but otherwise is not electable.

a. Which of the following are correct representations of this assertion?

(i) $(R \wedge E) \Leftrightarrow C$

(ii) $R \Rightarrow (E \Leftrightarrow C)$

(iii) $R \Rightarrow ((C \Rightarrow E) \vee \neg E)$

Exercise 8.4

According to some political pundits, a person who is radical (R) is electable (E) if he/she is conservative (C), but otherwise is not electable.

a. Which of the following are correct representations of this assertion?

(i) $(R \land E) \Leftrightarrow C$

No.
This sentence asserts, among other things, that all conservatives are radical, which is not what was stated.

Exercise 8.4

According to some political pundits, a person who is radical (R) is electable (E) if he/she is conservative (C), but otherwise is not electable.

a. Which of the following are correct representations of this assertion?

(ii) $R \Rightarrow (E \Leftrightarrow C)$

Yes.
This says that if a person is a radical then they are electable if and only if they are conservative.

Exercise 8.4

According to some political pundits, a person who is radical (R) is electable (E) if he/she is conservative (C), but otherwise is not electable.

a. Which of the following are correct representations of this assertion?

(iii) $R \Rightarrow ((C \Rightarrow E) \vee \neg E)$

No.
This is equivalent to $\neg R \vee \neg C \vee E \vee \neg E$ which is a tautology, true under any assignment.

Exercise 8.5

Show using the propositional resolution calculus that:

(1) $P \lor \neg Q$

(2) $\neg P \lor R \lor S$

(3) Q

(4) $\neg P \lor \neg R$

(5) $Q \lor P \lor R$

(6) $S \lor \neg Q$

(7) R

is not satisfiable;

Exercise 8.5 - Solution

Show using the propositional resolution calculus that:

(1) P ∨ ¬Q

(2) ¬P ∨ R ∨ S

(3) Q

(4) ¬P ∨ ¬R

(5) Q ∨ P ∨ R

(6) S ∨ ¬Q

(7) R

(8) P

Exercise 8.5 - Solution

Show using the propositional resolution calculus that:

(1) P ∨ ¬Q

(2) ¬P ∨ R ∨ S

(3) Q

(4) ¬P ∨ ¬R

(5) Q ∨ P ∨ R

(6) S ∨ ¬Q

(7) R

(8) P

(9) ¬R

Exercise 8.5 - Solution

Show using the propositional resolution calculus that:

(1) P ∨ ¬Q

(2) ¬P ∨ R ∨ S

(3) Q

(4) ¬P ∨ ¬R

(5) Q ∨ P ∨ R

(6) S ∨ ¬Q

(7) R

(8) P

(9) ¬R

(10) {}

Exercise 8.6

Show using the propositional resolution calculus that:

(1) $P \vee Q$

(2) $\neg Q \vee P \vee \neg R$

(3) $\neg P \vee \neg R$

(4) R

is not satisfiable.

Exercise 8.6

Show using the propositional resolution calculus that:

(1) P ∨ Q

(2) ¬Q ∨ P ∨ ¬R

(3) ¬P ∨ ¬R

(4) R

(5) P ∨ P ∨ ¬R

Exercise 8.6

Show using the propositional resolution calculus that:

(1) $P \lor Q$

(2) $\neg Q \lor P \lor \neg R$

(3) $\neg P \lor \neg R$

(4) R

(5) $P \lor P \lor \neg R$

(6) $P \lor \neg R$

Exercise 8.6

Show using the propositional resolution calculus that:

(1) $P \vee Q$

(2) $\neg Q \vee P \vee \neg R$

(3) $\neg P \vee \neg R$

(4) R

(5) $P \vee P \vee \neg R$

(6) $P \vee \neg R$

(7) $\neg R$

Exercise 8.6

Show using the propositional resolution calculus that:

(1) $P \vee Q$

(2) $\neg Q \vee P \vee \neg R$

(3) $\neg P \vee \neg R$

(4) R

(5) $P \vee P \vee \neg R$

(6) $P \vee \neg R$

(7) $\neg R$

(8) {}

Exercise 8.7

Show using the propositional resolution calculus that:

(1) $G \vee D \vee \neg B$

(2) $\neg E \vee C$

(3) G

(4) $F \vee C \vee \neg A$

(5) $\neg A$

(6) $\neg C \vee B$

(7) $\neg G \vee \neg F \vee E$

(8) F

(9) $\neg F \vee \neg E \vee D$

(10) $\neg F \vee \neg B \vee A$

is not satisfiable;

Exercise 8.7

(1) G ∨ D ∨ ¬B

(2) ¬E ∨ C

(3) G

(4) F ∨ C ∨ ¬A

(5) ¬A

(6) ¬C ∨ B

(7) ¬G ∨ ¬F ∨ E

(8) F

(9) ¬F ∨ ¬E ∨ D

(10) ¬F ∨ ¬B ∨ A

Resolution: deduction of a new clause from a pair of clauses with EXACTLY ONE incompatible variable (resolvent)

(11) [(7), (3)] ⇒ (¬F ∨ E)

(12) [(8), (11)] ⇒ (E)

(13) [(12), (2)] ⇒ (C)

(14) [(13), (6)] ⇒ (B)

(15) [(8), (10)] ⇒ (¬B ∨ A)

(16) [(14), (15)] ⇒ (A)

(17) [(5), (16)] ⇒ {}

Exercise 8.7

(1) G ∨ D ∨ ¬B

(2) ¬E ∨ C

(3) G

(4) F ∨ C ∨ ¬A

(5) ¬A

(6) ¬C ∨ B

(7) ¬G ∨ ¬F ∨ E

(8) F

(9) ¬F ∨ ¬E ∨ D

(10) ¬F ∨ ¬B ∨ A

Resolution: deduction of a new clause from a pair of clauses with EXACTLY ONE incompatible variable (resolvent)

(11) [(7), (3)] ⇒ (¬F ∨ E)

(12) [(8), (11)] ⇒ (E)

(13) [(12), (2)] ⇒ (C)

(14) [(13), (6)] ⇒ (B)

(15) [(8), (10)] ⇒ (¬B ∨ A)

(16) [(14), (15)] ⇒ (A)

(17) [(5), (16)] ⇒ {}

Exercise 8.8

Using DPLL algorithm to find a model for the following proposition:

- $(A \vee B) \wedge (C \vee D) \wedge \neg B$

Exercise 8.8

Using DPLL algorithm to find a model for the following proposition:

- $(A \lor B) \land (C \lor D) \land \neg B$
- $(A \lor \mathbf{B}) \land (C \lor D) \land \mathbf{\neg B}$

Propagate: B = False

Exercise 8.8

Using DPLL algorithm to find a model for the following proposition:

- (A ∨ B) ∧ (C ∨ D) ∧ ¬B
- (A ∨ **B**) ∧ (C ∨ D) ∧ **¬B**
- (**A** ∨ **B**) ∧ (C ∨ D) ∧ **¬B**

Propagate: B = False
Propagate: A = True

Exercise 8.8

Using DPLL algorithm to find a model for the following proposition:

- $(A \lor B) \land (C \lor D) \land \neg B$
- $(A \lor B) \land (C \lor D) \land \neg B$
- $(A \lor B) \land (C \lor D) \land \neg B$
- $(A \lor B) \land (C \lor D) \land \neg B$

Propagate: B = False
Propagate: A = True
Decide: C = True

Interpretation: A=True; B=False; C=True

Exercise 8.9

Using DPLL algorithm to find a model for the following proposition:

- $(\neg A \lor B) \land (\neg C \lor \neg B) \land (C \lor \neg B) \land (A \lor D)$

Exercise 8.9

Using DPLL algorithm to find a model for the following proposition:

- $(\neg A \vee B) \wedge (\neg C \vee \neg B) \wedge (C \vee \neg B) \wedge (A \vee D)$
- $(\neg \mathbf{A} \vee B) \wedge (\neg C \vee \neg B) \wedge (C \vee \neg B) \wedge (\mathbf{A} \vee D)$

Decide: A = True

Exercise 8.9

Using DPLL algorithm to find a model for the following proposition:

- $(\neg A \lor B) \land (\neg C \lor \neg B) \land (C \lor \neg B) \land (A \lor D)$
- $(\neg \mathbf{A} \lor \mathbf{B}) \land (\neg C \lor \neg \mathbf{B}) \land (C \lor \neg \mathbf{B}) \land (\mathbf{A} \lor D)$

Decide: A = True
Propagate: B = True

Exercise 8.9

Using DPLL algorithm to find a model for the following proposition:

- $(\neg A \vee B) \wedge (\neg C \vee \neg B) \wedge (C \vee \neg B) \wedge (A \vee D)$
- $(\neg A \vee B) \wedge (\neg C \vee \neg B) \wedge (C \vee \neg B) \wedge (A \vee D)$
- $(\neg A \vee B) \wedge (\neg C \vee \neg B) \wedge (C \vee \neg B) \wedge (A \vee D)$

Decide: A = True
Propagate: B = True
Propagate: C = False

Exercise 8.9

Using DPLL algorithm to find a model for the following proposition:

- (¬A ∨ B) ∧ (¬C ∨ ¬B) ∧ (C ∨ ¬B) ∧ (A ∨ D)
- (**¬A** ∨ **B**) ∧ (¬C ∨ **¬B**) ∧ (C ∨ **¬B**) ∧ (**A** ∨ D)
- (**¬A** ∨ **B**) ∧ (**¬C** ∨ **¬B**) ∧ (**C** ∨ **¬B**) ∧ (**A** ∨ D)

Decide: A = True
Propagate: B = True
Propagate: C = False

Exercise 8.9

Using DPLL algorithm to find a model for the following proposition:

- $(\neg A \lor B) \land (\neg C \lor \neg B) \land (C \lor \neg B) \land (A \lor D)$
- $(\neg \mathbf{A} \lor B) \land (\neg C \lor \neg B) \land (C \lor \neg B) \land (\mathbf{A} \lor D)$

Decide: A = False

Exercise 8.9

Using DPLL algorithm to find a model for the following proposition:

- $(\neg A \vee B) \wedge (\neg C \vee \neg B) \wedge (C \vee \neg B) \wedge (A \vee D)$
- $(\neg \mathbf{A} \vee B) \wedge (\neg C \vee \neg B) \wedge (C \vee \neg B) \wedge (\mathbf{A} \vee \mathbf{D})$

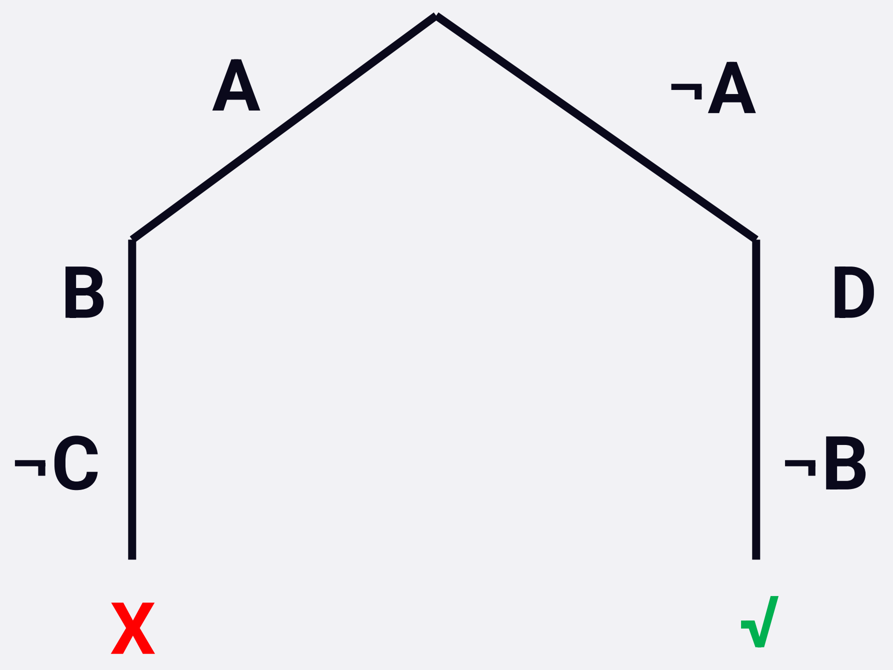
Decide: A = False
Propagate: D = True

Exercise 8.9

Using DPLL algorithm to find a model for the following proposition:

- (¬A ∨ B) ∧ (¬C ∨ ¬B) ∧ (C ∨ ¬B) ∧ (A ∨ D)
- (**¬A** ∨ **B**) ∧ (¬C ∨ **¬B**) ∧ (C ∨ **¬B**) ∧ (**A** ∨ **D**)

Decide: A = False
Propagate: D = True
Decide: B = False



Interpretation: A=False; B=False; D=True

Exercise 8.10

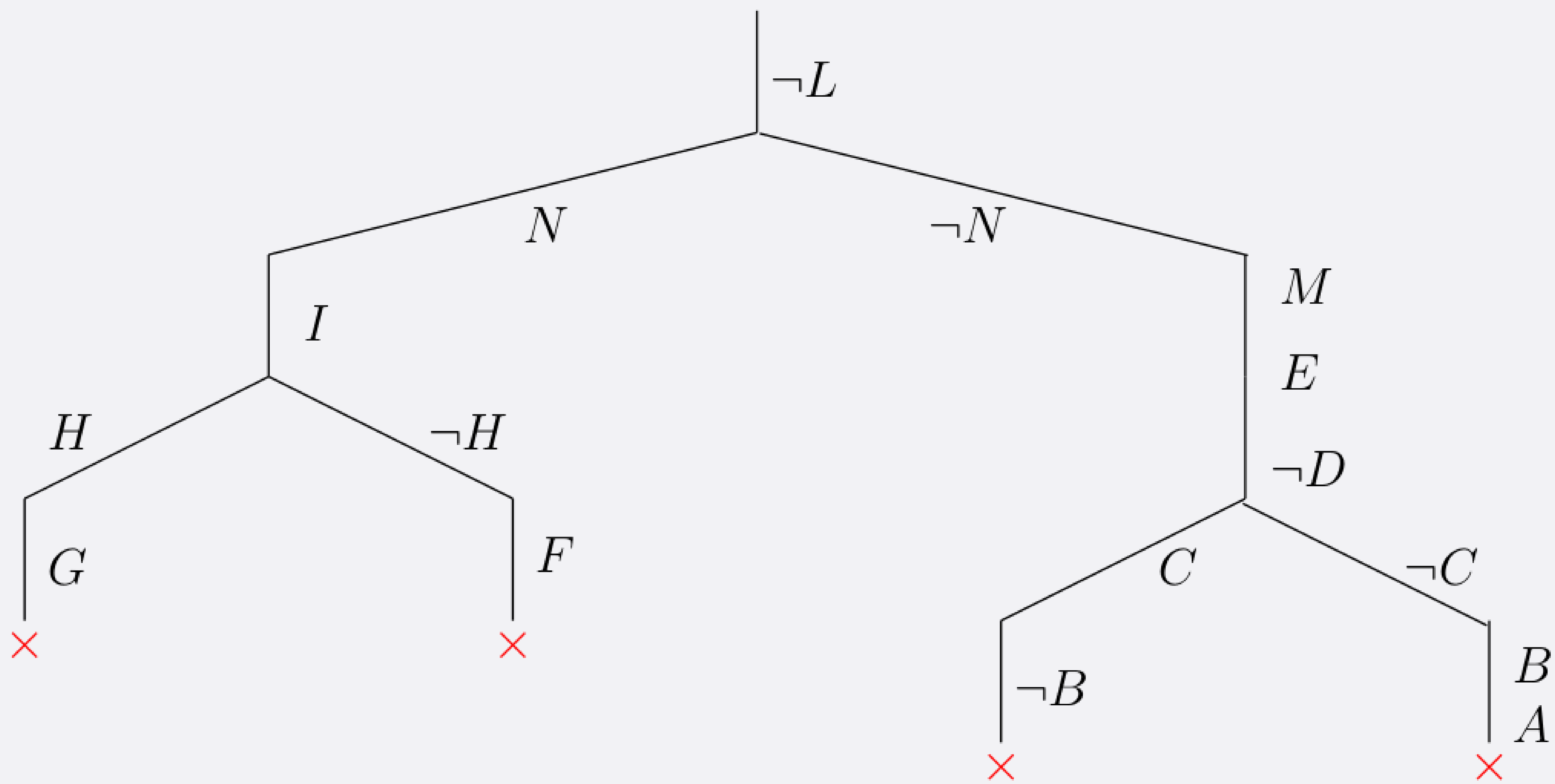
Consider the following CNF formula in PL:

$$
\begin{array}{llll}
(\neg L &) & & \wedge \\
(M & \vee N & \vee L) & \wedge \\
(\neg N & \vee I &) \wedge & \\
(\neg H & \vee \neg N & \vee G) & \wedge \\
(\neg H & \vee \neg G & \vee \neg N) & \wedge \\
(F & \vee \neg N & \vee H) & \wedge \\
(H & \vee \neg F & \vee \neg N) & \wedge \\
(N & \vee E & \vee L) & \wedge \\
(\neg E & \vee \neg D & \vee N) & \wedge \\
(D & \vee C & \vee B) & \wedge \\
(\neg C & \vee L & \vee \neg B) & \wedge \\
(N & \vee \neg C & \vee B) & \wedge \\
(C & \vee A & \vee \neg E) & \wedge \\
(D & \vee C & \vee \neg A) &
\end{array}
$$

Draw the search tree obtained by applying to the above formula the DPLL algorithm without the pure-symbol rule. Variables should be chosen according to **reverse alphabetic order**, and assigned **true** first.

Exercise 8.10

$$
\begin{aligned}
&(\neg L) \wedge \\
&(M \vee N \vee L) \wedge \\
&(\neg N \vee I) \wedge \\
&(\neg H \vee \neg N \vee G) \wedge \\
&(\neg H \vee \neg G \vee \neg N) \wedge \\
&(F \vee \neg N \vee H) \wedge \\
&(H \vee \neg F \vee \neg N) \wedge \\
&(N \vee E \vee L) \wedge \\
&(\neg E \vee \neg D \vee N) \wedge \\
&(D \vee C \vee B) \wedge \\
&(\neg C \vee L \vee \neg B) \wedge \\
&(N \vee \neg C \vee B) \wedge \\
&(C \vee A \vee \neg E) \wedge \\
&(D \vee C \vee \neg A)
\end{aligned}
$$



Exercise 8.11 - Homework

Prove each of the following assertions:

a. α is valid if and only if True |= α. Forward: If alpha is valid it is true in all models, hence it is true in all models of True. Backward: if True |= α then α must be true in all models of True, i.e., in all models, hence α must be valid.

b. For any α, False |= α. False doesn't hold in any model, so α trivially holds in every model of False.

c. α |= β if and only if the sentence (α ⇒ β) is valid. Both sides are equivalent to the assertion that there is no model in which α is true and β is false, i.e., no model in which α ⇒ β is false.

d. α ≡ β if and only if the sentence (α ⇔ β) is valid. Both sides are equivalent to the assertion that α and β have the same truth value in every model.

a. α |= β if and only if the sentence (α ∧ ¬β) is unsatisfiable. As in c, both sides are equivalent to the assertion that there is no model in which α is true and β is false.