

# Fundamentals of Artificial Intelligence

## Laboratory

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# Algorithms source code

- <https://github.com/aimacode/aima-java>
- Simplified and self-contained version of minimax on the laboratory website.

# Arc-consistency

- Given binary-constraint  $C_{X,Y}$ :  $D_X, D_Y$  are arc consistent if  $\forall x \in D_X \exists y \in D_Y$  s.t.  $\langle x,y \rangle \in C_{X,Y}$
- E.g.:  $D_A = \{1, 2, 3, 4\}$ ,  $D_B = \{1, 2, 3, 4\}$ , and  $C_{A,B} = B < A$  is **NOT** arc consistent as  $A = 1$  is not consistent with  $C_{A,B}$   
 $\Rightarrow D'_A = \{-, 2, 3, 4\}$  and  $D'_B = \{1, 2, 3, -\}$

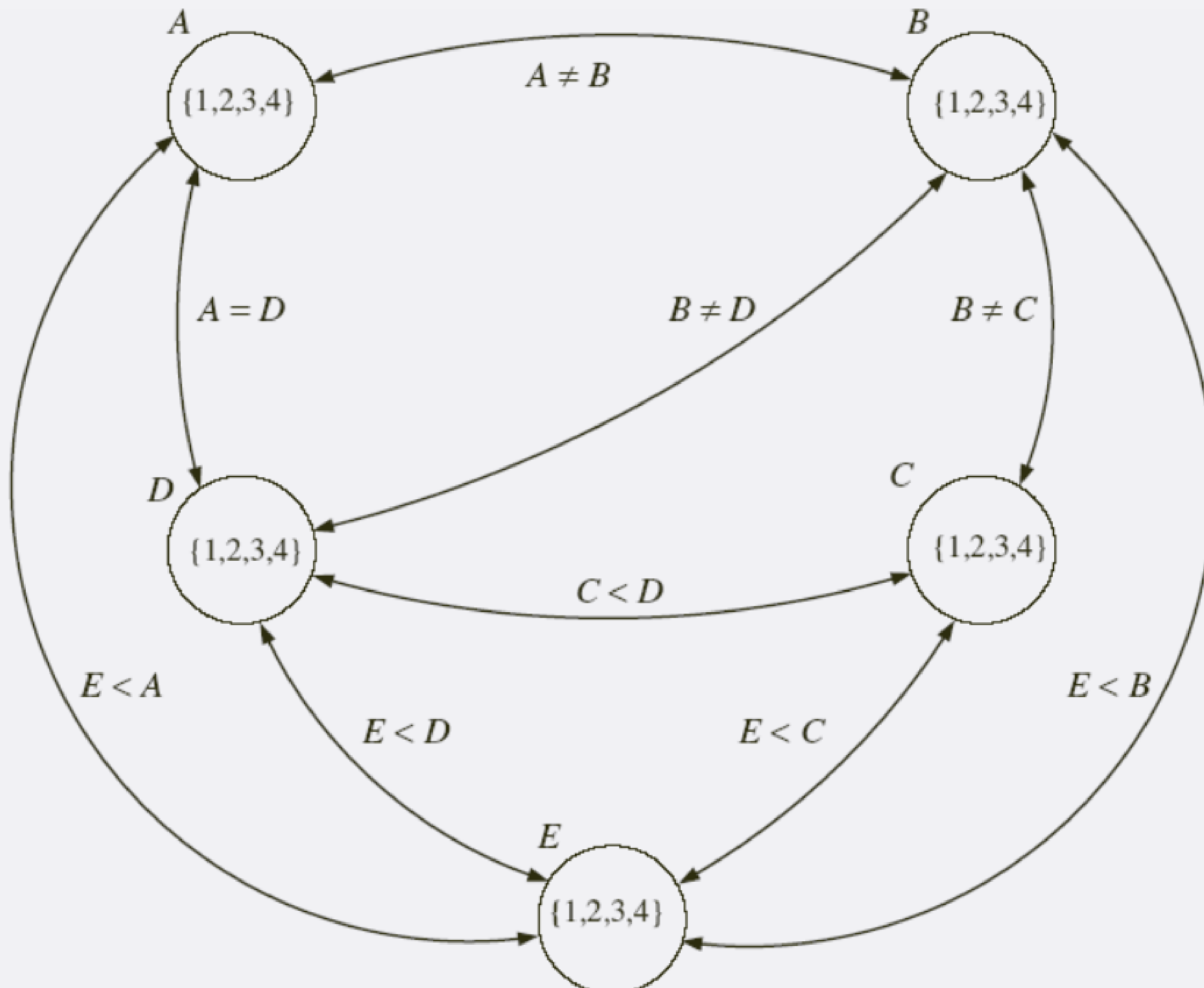
## Exercise 7.1

- Scheduling activities
  - Variables: A, B, C, D, E (starting time of activity)
  - Domains:  $D_i = \{1, 2, 3, 4\}$ , for  $i = \mathbf{A, B, \dots, E}$
  - Constraints:
    - $(B \neq 3)$
    - $(C \neq 2)$
    - $(A \neq B)$
    - $(B \neq C)$
    - $(C < D)$
    - $(A = D)$
    - $(E < A)$
    - $(E < B)$
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    - $(B \neq D)$
  
- Draw the constraint network and find a solution.

# Exercise 7.1

- Constraints network

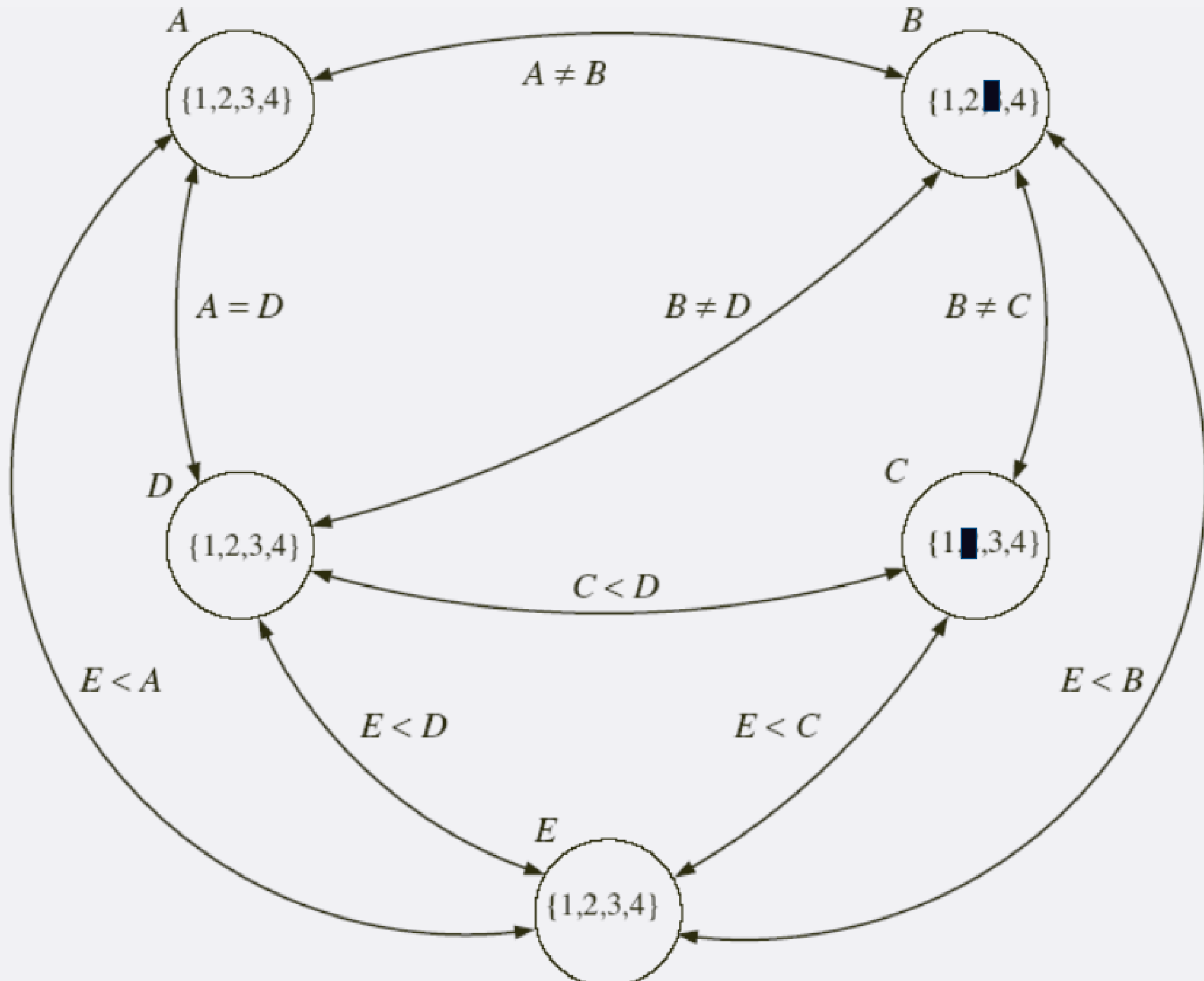
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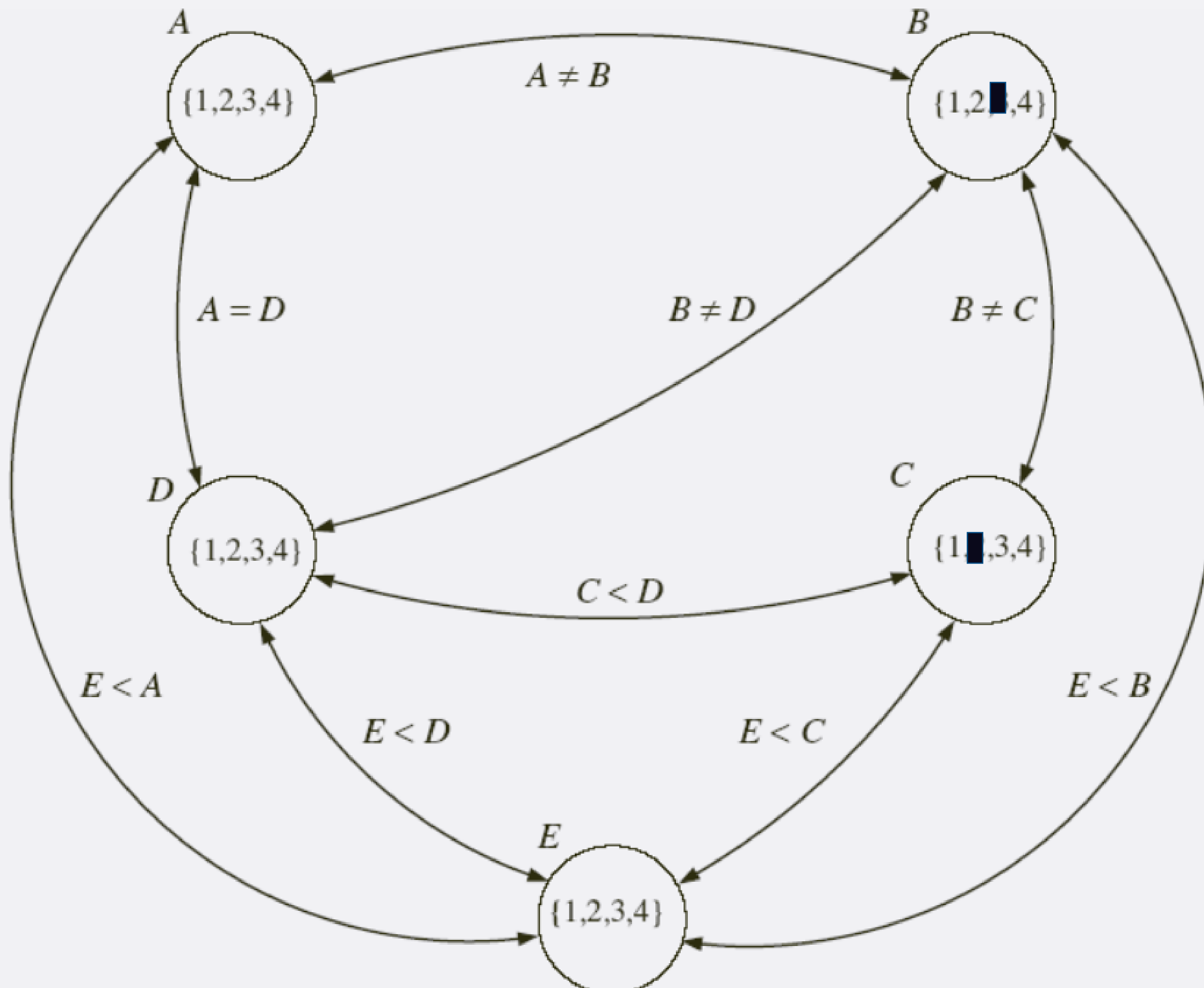
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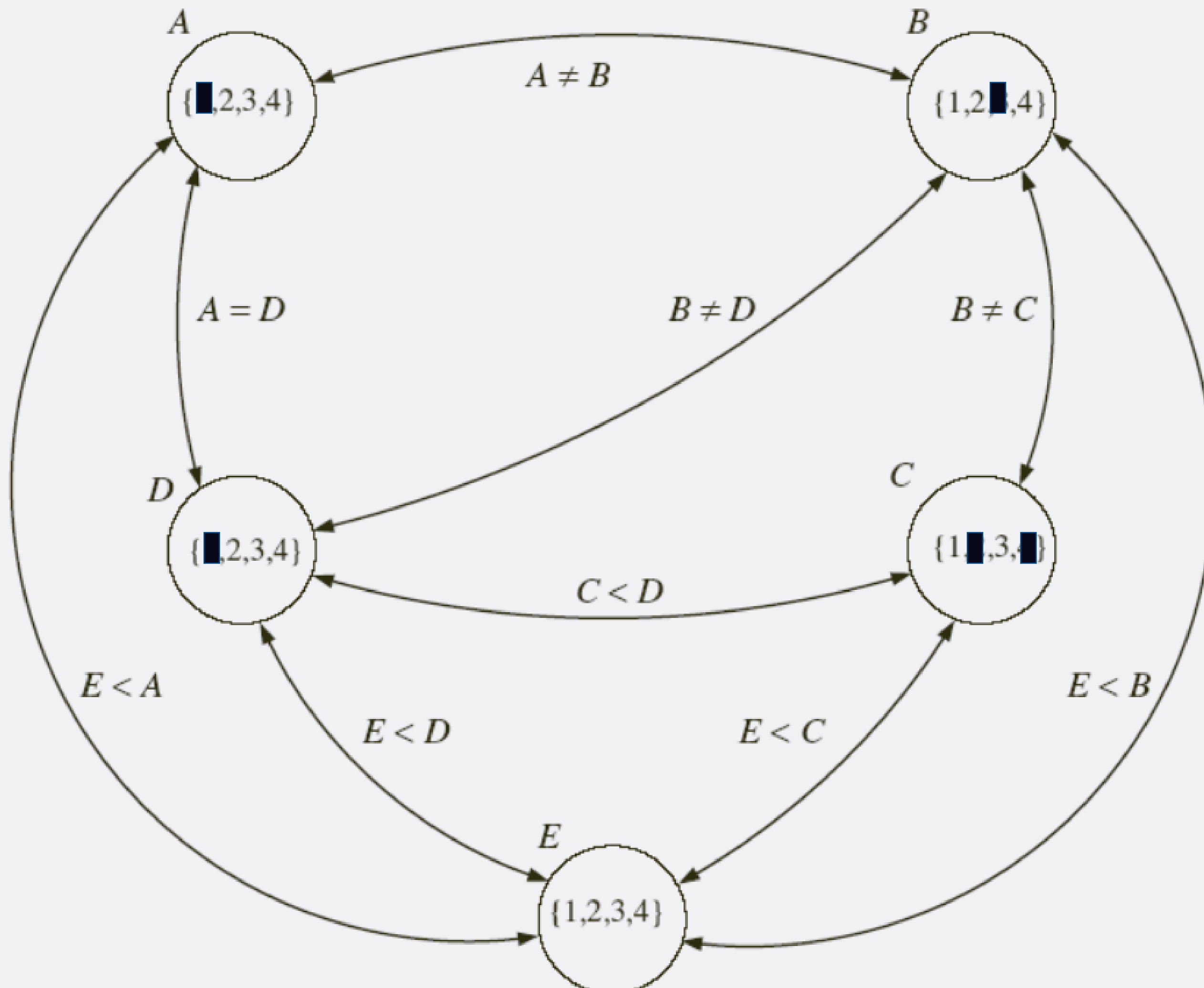
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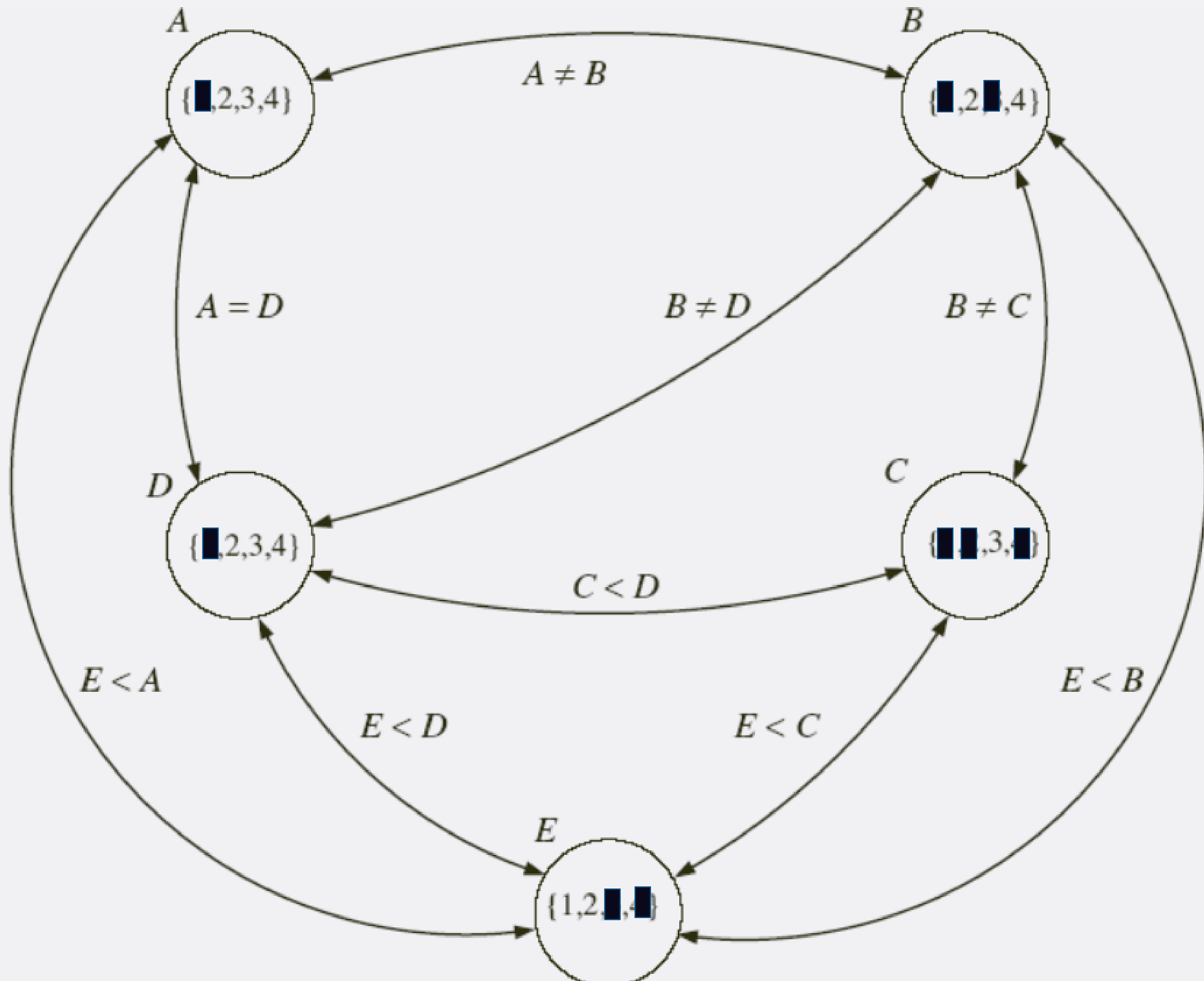




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- Constraints network

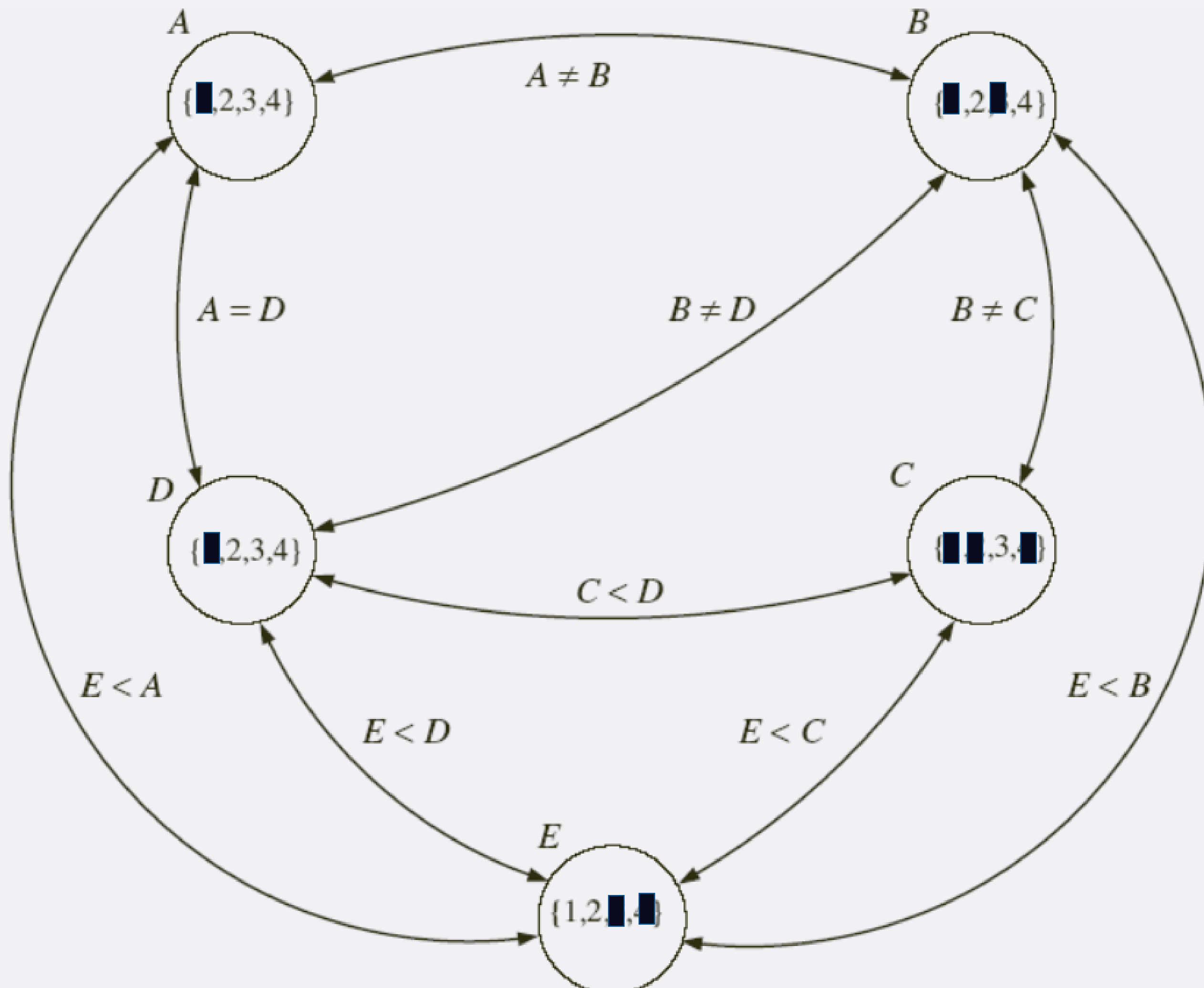
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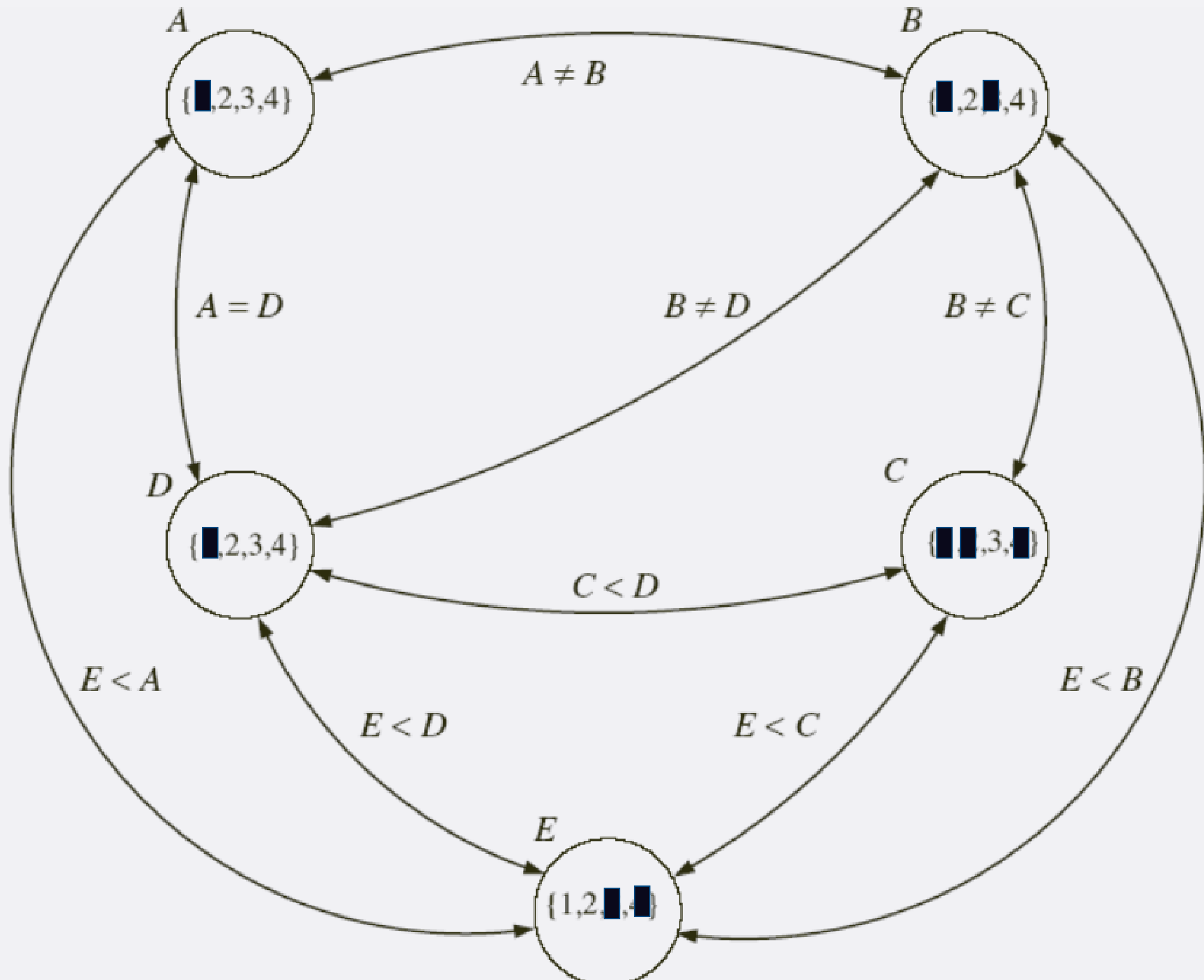
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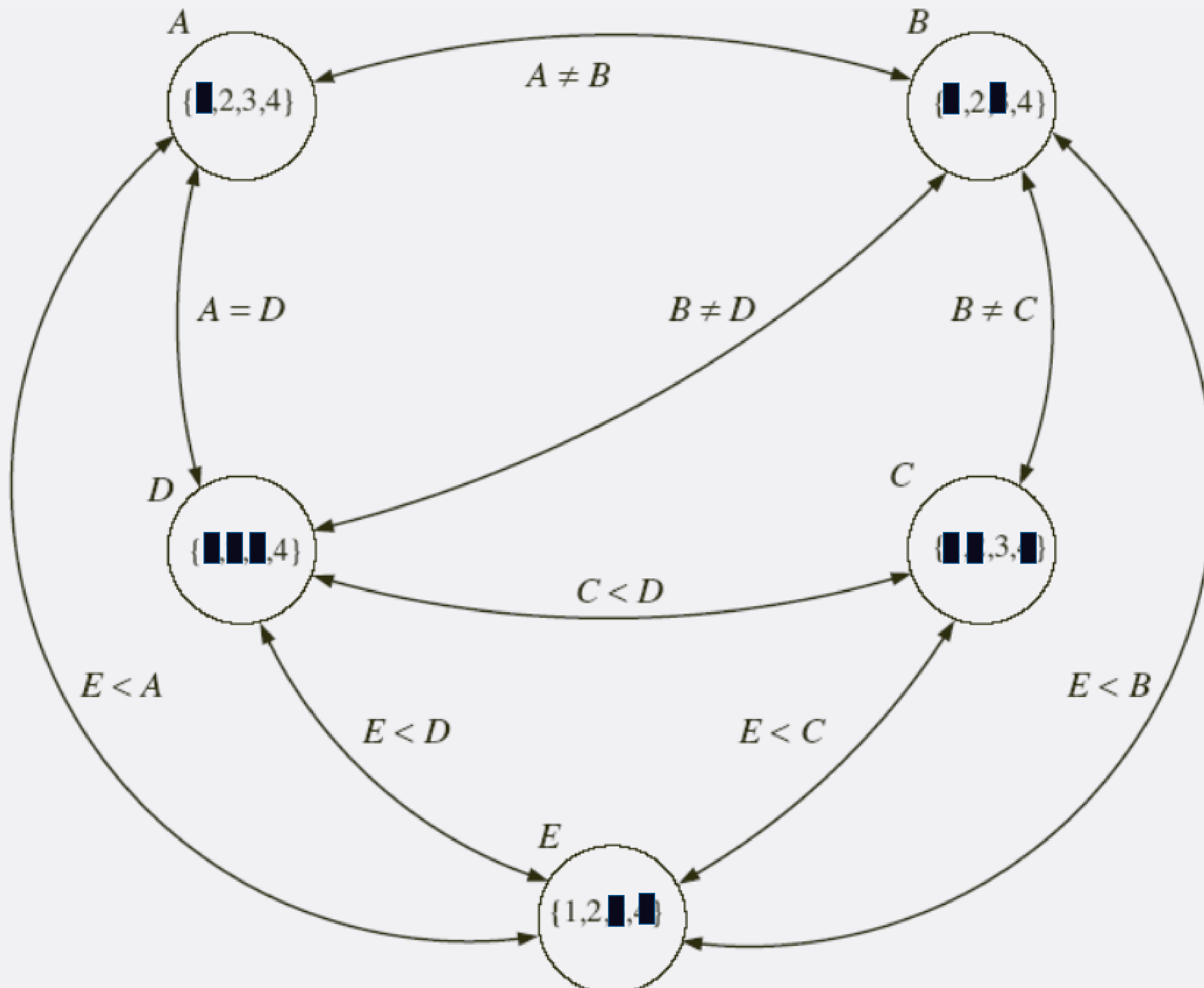
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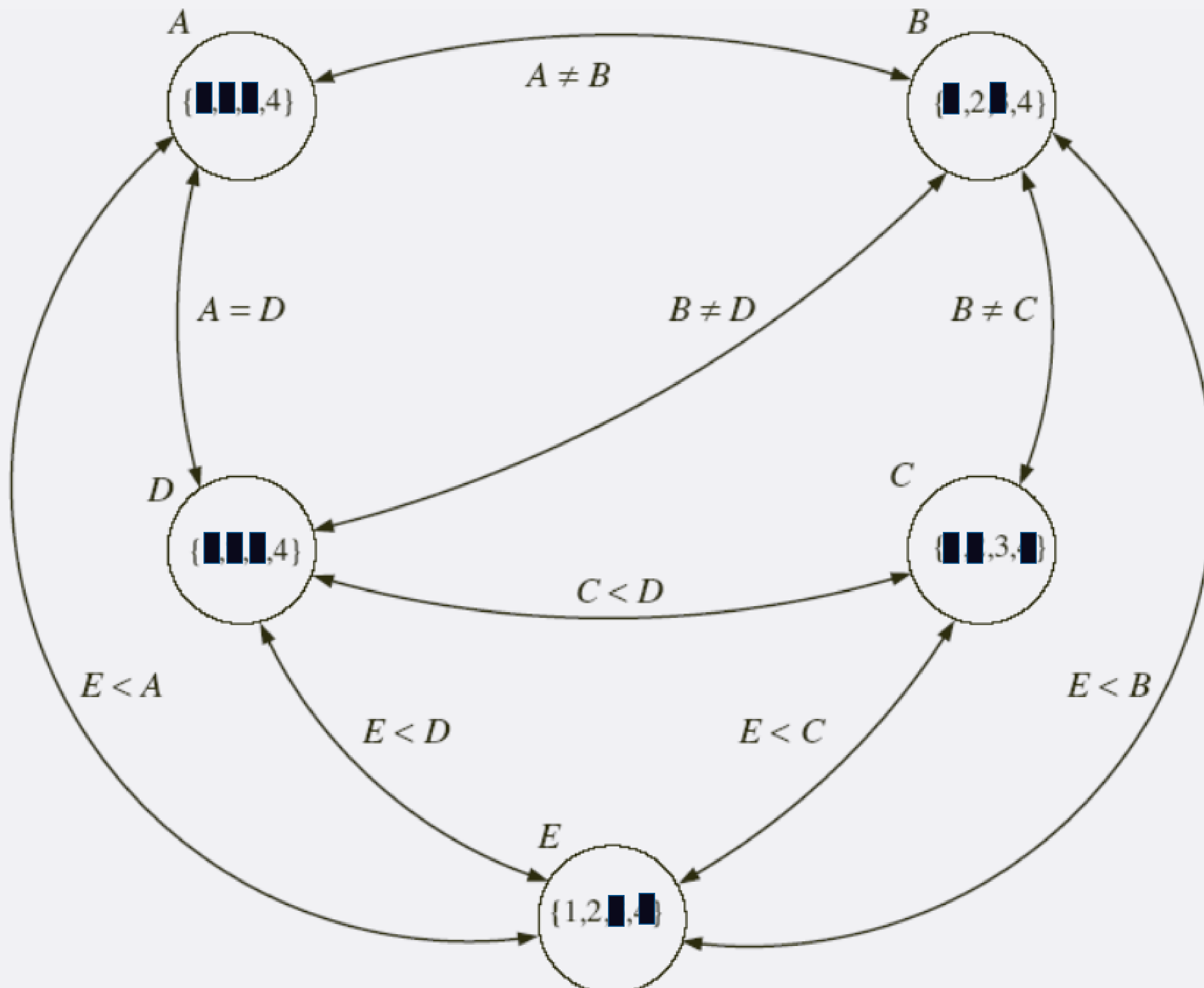
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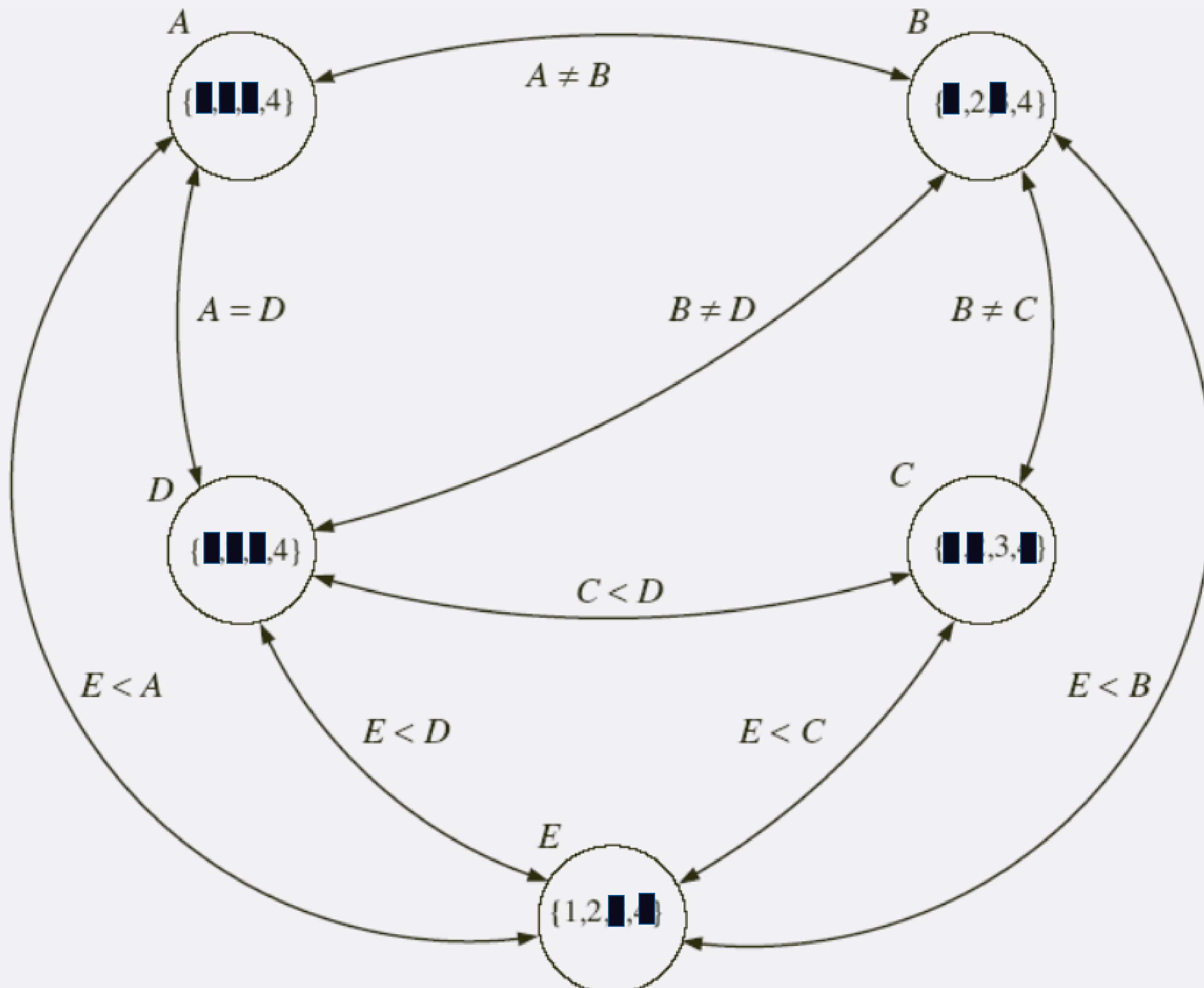
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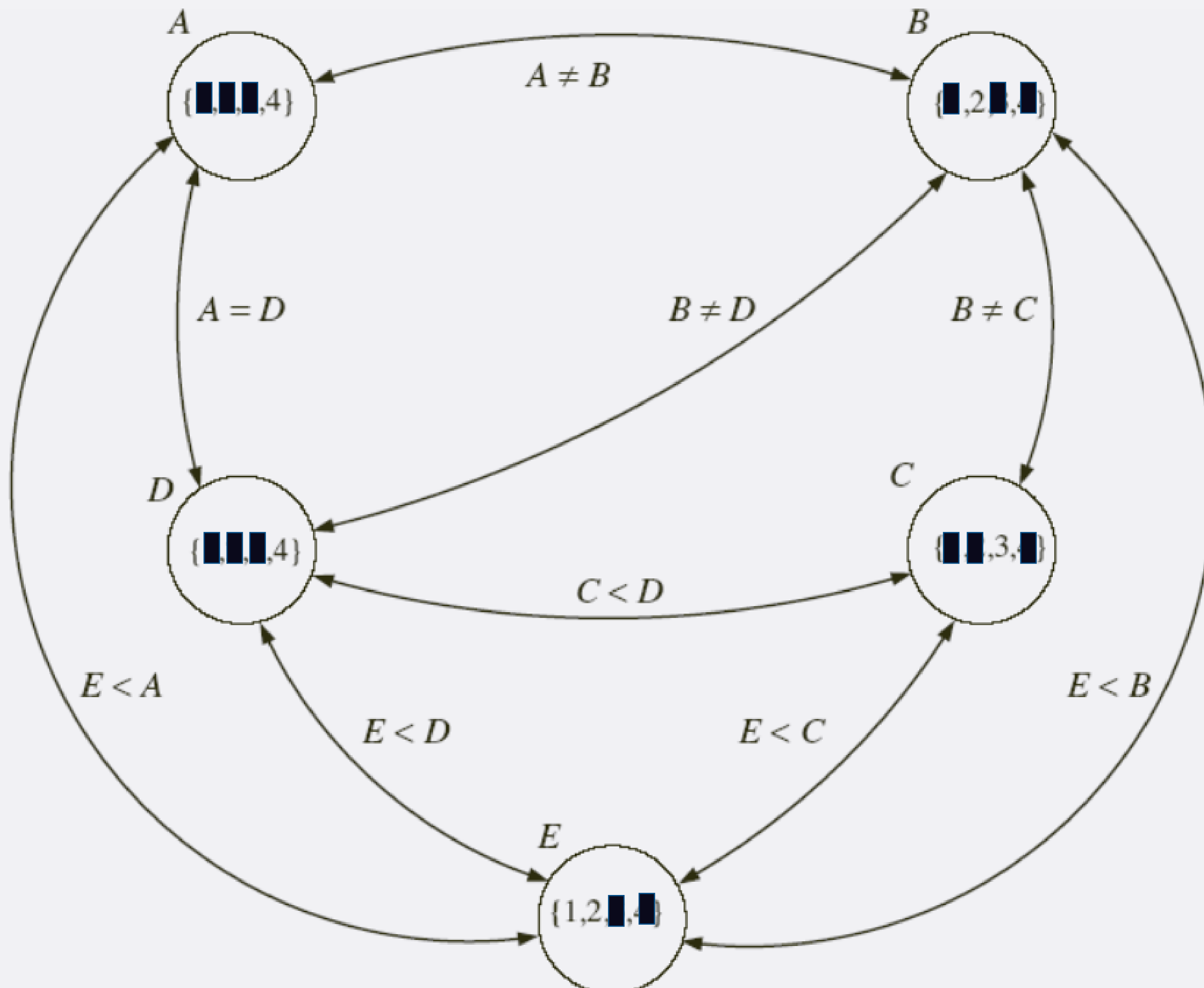
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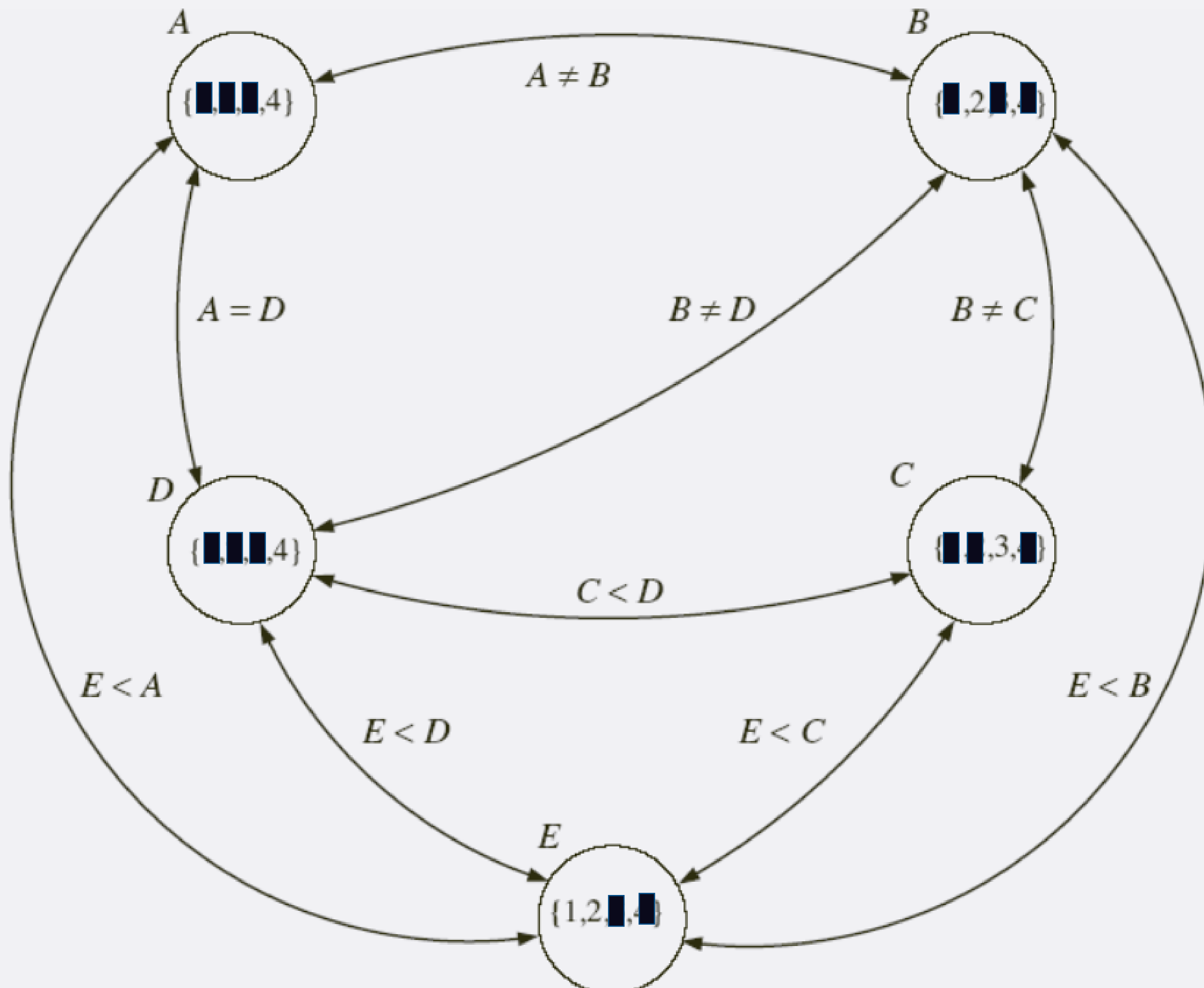
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## Exercise 7.2

- Consider the following binary constraint network:
  - There are 4 variables:  $X_1, X_2, X_3, X_4$
  - Domains:  $D_1=\{1,2,3,4\}$ ,  $D_2=\{3,4,5,8,9\}$ ,  $D_3=\{2,3,5,6,7,9\}$ ,  $D_4=\{3,5,7,8,9\}$
  - The constraints are
    - $X_1 \geq X_2$
    - $X_2 > X_3$  or  $X_3 - X_2 = 2$
    - $X_3 \neq X_4$ .
  
- Tasks:
  - a. Is the network arc-consistent? If not, compute the arc-consistent network. (show the whole process of enforcing arc-consistency and not just the final network)
  
  - b. Is the network consistent? If yes, give a solution.

## Exercise 7.2 - Solution

- Task (a)

No, it is not arc-consistent.

Enforce arc-consistency between  $X_1$  and  $X_2$ :  $D_1 = \{3, 4\}$      $D_2 = \{3, 4\}$

$X_2$  and  $X_3$ :  $D_2 = \{3, 4\}$      $D_3 = \{2, 3, 5, 6\}$

$X_3$  and  $X_4$ :  $D_3 = \{2, 3, 5, 6\}$      $D_4 = \{3, 5, 7, 8, 9\}$

So the arc-consistent domains are

$D_1 = \{3, 4\}$      $D_2 = \{3, 4\}$      $D_3 = \{2, 3, 5, 6\}$      $D_4 = \{3, 5, 7, 8, 9\}$

- Task (b)

$X_1 = 4, X_2 = 4, X_3 = 3, X_4 = 9$

## Exercise 7.3

- Consider the following binary constraint network:
  - There are 4 variables:  $X_1, X_2, X_3, X_4$
  - Domains:  $D_1 = \{1, 2, 5, 7\}$ ,  $D_2 = \{1, 2, 6, 8, 9\}$ ,  $D_3 = \{2, 4, 6, 7, 8, 9\}$ ,  $D_4 = \{1, 2, 3, 8, 9\}$
  - The constraints are
    - $X_1 = X_2$
    - $X_2 < X_3$  or  $X_2 - X_3 = 3$
    - $X_3 > X_4$ .
  
- Tasks:
  - a. Is the network arc-consistent? If not, compute the arc-consistent network. (show the whole process of enforcing arc-consistency and not just the final network)
  
  - b. Is the network consistent? If yes, give a solution.

## Exercise 7.4

- Consider the following binary constraint network:
  - There are 5 variables:  $X_1, X_2, X_3, X_4, X_5$
  - Domains:  $D_1 = \{3, 4, 5, 7, 9\}$ ,  $D_2 = \{2, 4, 6, 8, 9\}$ ,  $D_3 = \{1, 2, 6, 8, 9\}$ ,  
 $D_4 = \{1, 2, 3, 8, 9\}$ ,  $D_5 = \{2, 5, 6, 7, 8\}$
  - The constraints are
    - $X_1 > X_2$  or  $X_2 - X_1 = 2$
    - $X_2 < X_3$
    - $X_2 > X_4$  or  $X_2 - X_4 = 1$
    - $X_3 > X_5$ .
  
- Tasks:
  - a. Is the network arc-consistent? If not, compute the arc-consistent network. (show the whole process of enforcing arc-consistency and not just the final network)
  - b. Is the network consistent? If yes, give a solution.

## Exercise 7.5

- Download “Problem 7.5 Text” from the laboratory website.

## Exercise 7.5

- Question 1

5 variables: AR-1, AR-2, MLR, CR, IWR

4 constraints:

1. IAR says  $\leq 1$  of 15-381, 15-681, and 19-601 can be assigned to the 5 variables.
2. BAR says  $\leq 1$  of 15-211 and 70-122 can be assigned to the 5 variables
3. OR says  $\leq 1$  of 21-484 and 70-311 can be assigned to the 5 variables
4. No double counting says if a variable is assigned to one variable it can't be assigned to another variable

Initial domains:

AR-1: 15-211, 15-212, 15-381, 15-681, 21-484

AR-2: 15-211, 15-212, 15-381, 15-681, 21-484

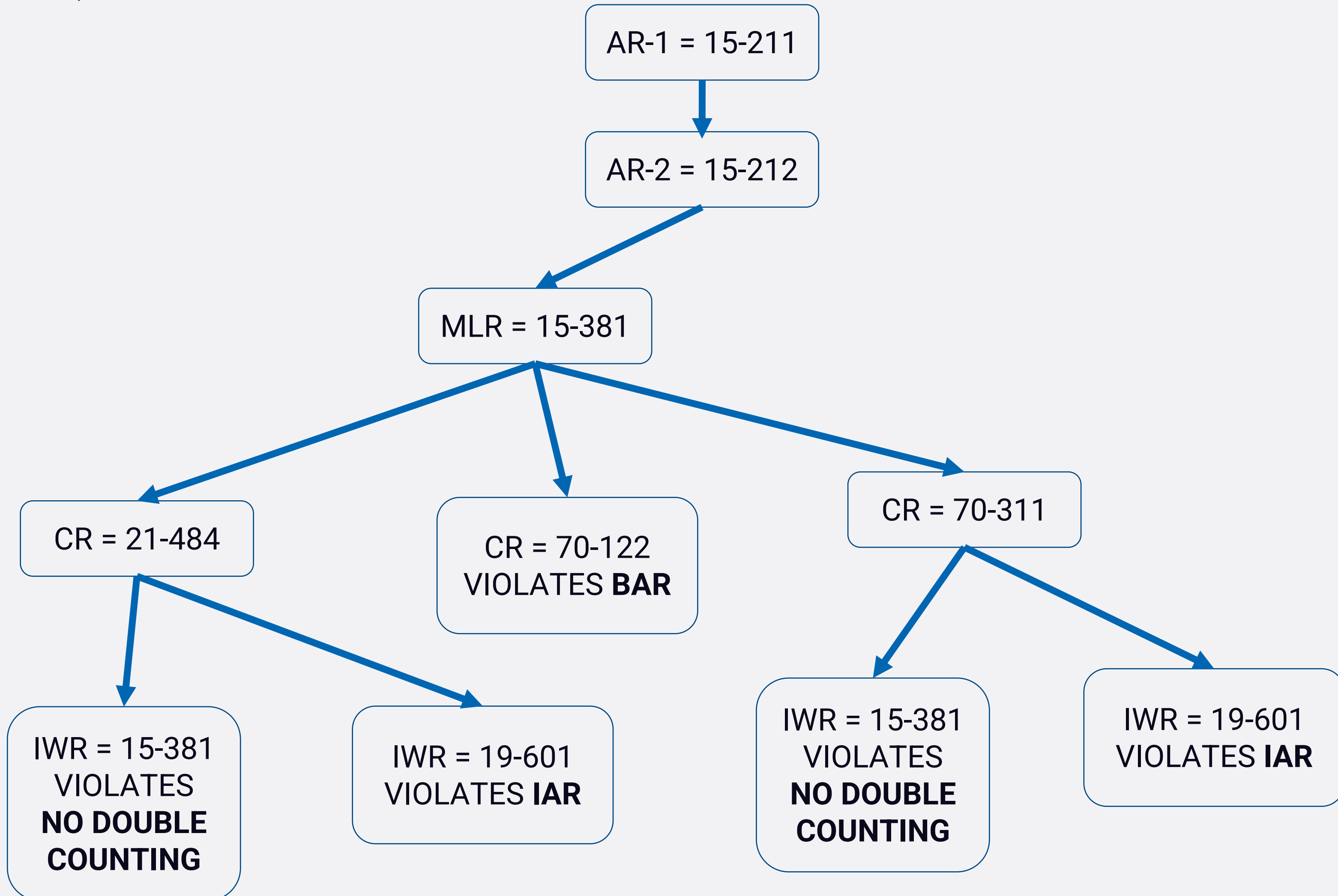
MLR: 15-381, 15-681, 80-310

CR: 21-484, 70-122, 70-311

IWR: 15-381, 19-601

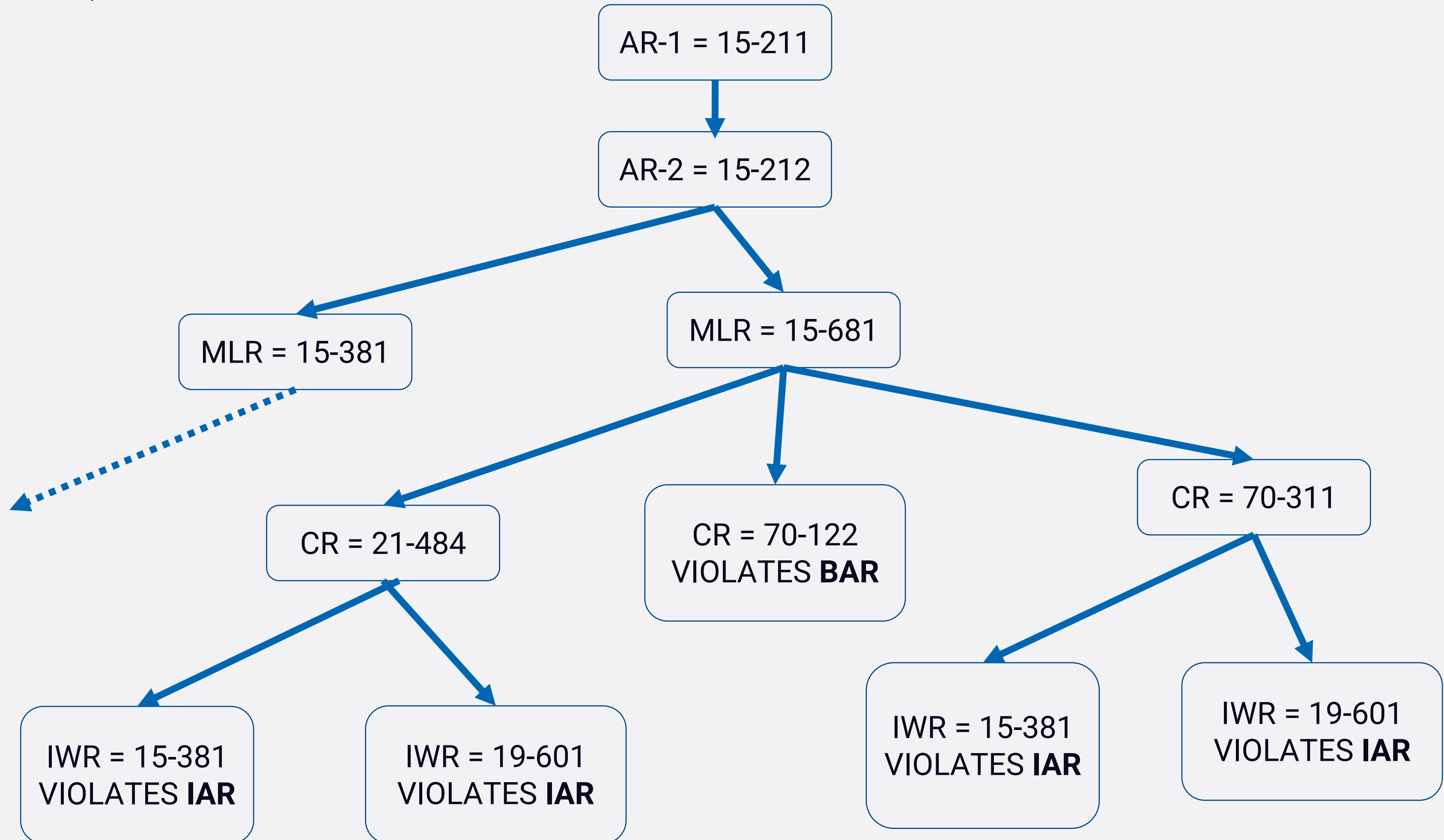
# Exercise 7.5

- Question 2



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