

# Fundamentals of Artificial Intelligence

## Chapter 07: Logical Agents

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# Outline

- 1 Propositional Logic
- 2 Propositional Reasoning
  - Resolution
  - DPLL
  - Reasoning with Horn Formulas
  - Local Search
- 3 Agents Based on Knowledge Representation & Reasoning
  - Knowledge-Based Agents
  - Example: the Wumpus World
- 4 Agents Based on Propositional Reasoning
  - Propositional Logic Agents
  - Example: the Wumpus World

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# Propositional Logic (aka Boolean Logic)



# Basic Definitions and Notation

- **Propositional formula** (aka **Boolean formula** or **sentence**)
  - $\top, \perp$  are formulas
  - a **propositional atom**  $A_1, A_2, A_3, \dots$  is a formula;
  - if  $\varphi_1$  and  $\varphi_2$  are formulas, then  
 $\neg\varphi_1, \varphi_1 \wedge \varphi_2, \varphi_1 \vee \varphi_2, \varphi_1 \rightarrow \varphi_2, \varphi_1 \leftarrow \varphi_2, \varphi_1 \leftrightarrow \varphi_2, \varphi_1 \oplus \varphi_2$   
are formulas.
- Ex:  $\varphi \stackrel{\text{def}}{=} (\neg(A_1 \rightarrow A_2)) \wedge (A_3 \leftrightarrow (\neg A_1 \oplus (A_2 \vee \neg A_4)))$
- **Atoms**( $\varphi$ ): the set  $\{A_1, \dots, A_N\}$  of atoms occurring in  $\varphi$ .
- **Literal**: a propositional atom  $A_i$  (**positive literal**) or its negation  $\neg A_i$  (**negative literal**)
  - Notation: if  $l := \neg A_i$ , then  $\neg l := A_i$
- **Clause**: a disjunction of literals  $\bigvee_j l_j$  (e.g.,  $(A_1 \vee \neg A_2 \vee A_3 \vee \dots)$ )
- **Cube**: a conjunction of literals  $\bigwedge_j l_j$  (e.g.,  $(A_1 \wedge \neg A_2 \wedge A_3 \wedge \dots)$ )

# Semantics of Boolean operators

Truth Table

$\alpha$	$\beta$	$\neg\alpha$	$\alpha\wedge\beta$	$\alpha\vee\beta$	$\alpha\rightarrow\beta$	$\alpha\leftarrow\beta$	$\alpha\leftrightarrow\beta$	$\alpha\oplus\beta$
$\perp$	$\perp$	$\top$	$\perp$	$\perp$	$\top$	$\top$	$\top$	$\perp$
$\perp$	$\top$	$\top$	$\perp$	$\top$	$\top$	$\perp$	$\perp$	$\top$
$\top$	$\perp$	$\perp$	$\perp$	$\top$	$\perp$	$\top$	$\perp$	$\top$
$\top$	$\top$	$\perp$	$\top$	$\top$	$\top$	$\top$	$\top$	$\perp$

- $\wedge$ ,  $\vee$ ,  $\leftrightarrow$  and  $\oplus$  are commutative:

$$(\alpha \wedge \beta) \iff (\beta \wedge \alpha)$$

$$(\alpha \vee \beta) \iff (\beta \vee \alpha)$$

$$(\alpha \leftrightarrow \beta) \iff (\beta \leftrightarrow \alpha)$$

$$(\alpha \oplus \beta) \iff (\beta \oplus \alpha)$$

- $\wedge$ ,  $\vee$ ,  $\leftrightarrow$  and  $\oplus$  are associative:

$$((\alpha \wedge \beta) \wedge \gamma) \iff (\alpha \wedge (\beta \wedge \gamma)) \iff (\alpha \wedge \beta \wedge \gamma)$$

$$((\alpha \vee \beta) \vee \gamma) \iff (\alpha \vee (\beta \vee \gamma)) \iff (\alpha \vee \beta \vee \gamma)$$

$$((\alpha \leftrightarrow \beta) \leftrightarrow \gamma) \iff (\alpha \leftrightarrow (\beta \leftrightarrow \gamma)) \iff (\alpha \leftrightarrow \beta \leftrightarrow \gamma)$$

$$((\alpha \oplus \beta) \oplus \gamma) \iff (\alpha \oplus (\beta \oplus \gamma)) \iff (\alpha \oplus \beta \oplus \gamma)$$

## Remark: Semantics of Implication “ $\rightarrow$ ” (aka “ $\Rightarrow$ ”, “ $\supset$ ”)

The semantics of Implication “ $\alpha \rightarrow \beta$ ” may be counter-intuitive

$\alpha \rightarrow \beta$ : “the antecedent (aka premise)  $\alpha$  implies the consequent (aka conclusion)  $\beta$ ” (aka “if  $\alpha$  holds, then  $\beta$  holds”), but not vice versa

- does not require causation or relevance between  $\alpha$  and  $\beta$ 
  - ex: “5 is odd implies Tokyo is the capital of Japan” is true in p.l. (under the standard interpretation of “5”, “odd”, “Tokyo”, “Japan”)
  - relation between antecedent & consequent: they are both true
- is true whenever its antecedent is false
  - ex: “5 is even implies Sam is smart” is true (regardless the smartness of Sam)
  - ex: “5 is even implies Tokyo is in Italy” is true (!)
  - relation between antecedent & consequent: the former is false
- does not require temporal precedence of  $\alpha$  wrt.  $\beta$ 
  - ex: “the grass is wet implies it must have rained” is true (the consequent precedes temporally the antecedent)

# Syntactic Properties of Boolean Operators

$$\begin{aligned} \neg\neg\alpha &\iff \alpha \\ (\alpha \vee \beta) &\iff \neg(\neg\alpha \wedge \neg\beta) \\ \neg(\alpha \vee \beta) &\iff (\neg\alpha \wedge \neg\beta) \\ (\alpha \wedge \beta) &\iff \neg(\neg\alpha \vee \neg\beta) \\ \neg(\alpha \wedge \beta) &\iff (\neg\alpha \vee \neg\beta) \\ (\alpha \rightarrow \beta) &\iff (\neg\alpha \vee \beta) \\ \neg(\alpha \rightarrow \beta) &\iff (\alpha \wedge \neg\beta) \\ (\alpha \leftarrow \beta) &\iff (\alpha \vee \neg\beta) \\ \neg(\alpha \leftarrow \beta) &\iff (\neg\alpha \wedge \beta) \\ (\alpha \leftrightarrow \beta) &\iff ((\alpha \rightarrow \beta) \wedge (\alpha \leftarrow \beta)) \\ &\iff ((\neg\alpha \vee \beta) \wedge (\alpha \vee \neg\beta)) \\ \neg(\alpha \leftrightarrow \beta) &\iff (\neg\alpha \leftrightarrow \beta) \\ &\iff (\alpha \leftrightarrow \neg\beta) \\ &\iff ((\alpha \vee \beta) \wedge (\neg\alpha \vee \neg\beta)) \\ (\alpha \oplus \beta) &\iff \neg(\alpha \leftrightarrow \beta) \end{aligned}$$

Boolean logic can be expressed in terms of  $\{\neg, \wedge\}$  (or  $\{\neg, \vee\}$ ) only!



# Exercises

1 For every pair of formulas  $\alpha \iff \beta$  below, show that  $\alpha$  and  $\beta$  can be rewritten into each other by applying the syntactic properties of the previous slide

- $(A_1 \wedge A_2) \vee A_3 \iff (A_1 \vee A_3) \wedge (A_2 \vee A_3)$
- $(A_1 \vee A_2) \wedge A_3 \iff (A_1 \wedge A_3) \vee (A_2 \wedge A_3)$
- $A_1 \rightarrow (A_2 \rightarrow (A_3 \rightarrow A_4)) \iff (A_1 \wedge A_2 \wedge A_3) \rightarrow A_4$
- $A_1 \rightarrow (A_2 \wedge A_3) \iff (A_1 \rightarrow A_2) \wedge (A_1 \rightarrow A_3)$
- $(A_1 \vee A_2) \rightarrow A_3 \iff (A_1 \rightarrow A_3) \wedge (A_2 \rightarrow A_3)$
- $A_1 \oplus A_2 \iff (A_1 \vee A_2) \wedge (\neg A_1 \vee \neg A_2)$
- $\neg A_1 \leftrightarrow \neg A_2 \iff A_1 \leftrightarrow A_2$
- $A_1 \leftrightarrow A_2 \leftrightarrow A_3 \iff A_1 \oplus A_2 \oplus A_3$

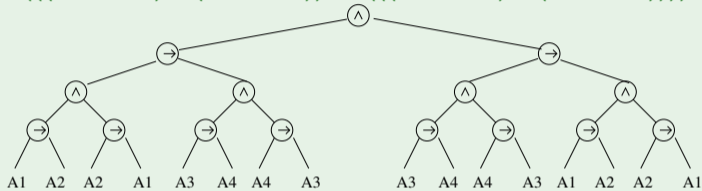
# Tree & DAG Representations of Formulas

- Formulas can be represented either as **trees** or as **DAGS** (**Directed Acyclic Graphs**)
- **DAG representation can be up to exponentially smaller**
  - in particular, when  $\leftrightarrow$ 's are involved

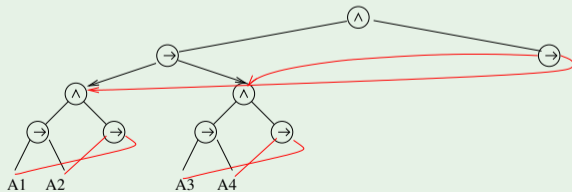
$$\begin{aligned} & (A_1 \leftrightarrow A_2) \leftrightarrow (A_3 \leftrightarrow A_4) \\ & \quad \downarrow \\ & (((A_1 \leftrightarrow A_2) \rightarrow (A_3 \leftrightarrow A_4)) \wedge \\ & ((A_3 \leftrightarrow A_4) \rightarrow (A_1 \leftrightarrow A_2))) \\ & \quad \downarrow \\ & (((A_1 \rightarrow A_2) \wedge (A_2 \rightarrow A_1)) \rightarrow ((A_3 \rightarrow A_4) \wedge (A_4 \rightarrow A_3))) \wedge \\ & (((A_3 \rightarrow A_4) \wedge (A_4 \rightarrow A_3)) \rightarrow (((A_1 \rightarrow A_2) \wedge (A_2 \rightarrow A_1)))) \end{aligned}$$

# Tree & DAG Representations of Formulas: Example

$$(((A_1 \rightarrow A_2) \wedge (A_2 \rightarrow A_1)) \rightarrow ((A_3 \rightarrow A_4) \wedge (A_4 \rightarrow A_3))) \wedge$$
$$(((A_3 \rightarrow A_4) \wedge (A_4 \rightarrow A_3)) \rightarrow (((A_1 \rightarrow A_2) \wedge (A_2 \rightarrow A_1))))$$



*Tree Representation*



*DAG Representation*

## Basic Definitions and Notation [cont.]

- **Total truth assignment**  $\mu$  for  $\varphi$ :  
 $\mu : \mathit{Atoms}(\varphi) \mapsto \{\top, \perp\}$ .
  - represents a **possible world** or a **possible state of the world**
- **Partial Truth assignment**  $\mu$  for  $\varphi$ :  
 $\mu : \mathcal{A} \mapsto \{\top, \perp\}, \mathcal{A} \subset \mathit{Atoms}(\varphi)$ .
  - represents  $2^k$  total assignments,  $k$  is # unassigned variables
- **Notation: set and formula representations of an assignment**
  - $\mu$  can be represented **as a set of literals**:  
EX:  $\{\mu(A_1) := \top, \mu(A_2) := \perp\} \implies \{A_1, \neg A_2\}$
  - $\mu$  can be represented **as a formula (cube)**:  
EX:  $\{\mu(A_1) := \top, \mu(A_2) := \perp\} \implies (A_1 \wedge \neg A_2)$

## Basic Definitions and Notation [cont.]

- A **total** truth assignment  $\mu$  **satisfies**  $\varphi$  ( $\mu$  is a model of  $\varphi$ ,  $\mu \models \varphi$ ):

$$\mu \models A_i \iff \mu(A_i) = \top$$

$$\mu \models \neg\varphi \iff \text{not } \mu \models \varphi$$

$$\mu \models \alpha \wedge \beta \iff \mu \models \alpha \text{ and } \mu \models \beta$$

$$\mu \models \alpha \vee \beta \iff \mu \models \alpha \text{ or } \mu \models \beta$$

$$\mu \models \alpha \rightarrow \beta \iff \text{if } \mu \models \alpha, \text{ then } \mu \models \beta$$

$$\mu \models \alpha \leftrightarrow \beta \iff \mu \models \alpha \text{ iff } \mu \models \beta$$

$$\mu \models \alpha \oplus \beta \iff \mu \models \alpha \text{ iff not } \mu \models \beta$$

- $M(\varphi) \stackrel{\text{def}}{=} \{\mu \mid \mu \models \varphi\}$  (the set of models of  $\varphi$ )
- A **partial** truth assignment  $\mu$  **satisfies**  $\varphi$  iff all its total extensions satisfy  $\varphi$ 
  - (Ex:  $\{A_1\} \models (A_1 \vee A_2)$ ) because  $\{A_1, A_2\} \models (A_1 \vee A_2)$  and  $\{A_1, \neg A_2\} \models (A_1 \vee A_2)$ )
- $\varphi$  is **satisfiable** iff  $\mu \models \varphi$  for some  $\mu$  (i.e.  $M(\varphi) \neq \emptyset$ )
- $\alpha$  **entails**  $\beta$  ( $\alpha \models \beta$ ) iff, for all  $\mu$ s,  $\mu \models \alpha \implies \mu \models \beta$  (i.e.,  $M(\alpha) \subseteq M(\beta)$ )
- $\varphi$  is **valid** ( $\models \varphi$ ) iff  $\mu \models \varphi$  for all  $\mu$ s (i.e.,  $\mu \in M(\varphi)$  for all  $\mu$ s)

# Properties & Results

## Property

$\varphi$  is valid iff  $\neg\varphi$  is unsatisfiable

## Deduction Theorem

$\alpha \models \beta$  iff  $\alpha \rightarrow \beta$  is valid ( $\models \alpha \rightarrow \beta$ )

## Corollary

$\alpha \models \beta$  iff  $\alpha \wedge \neg\beta$  is unsatisfiable

Validity and entailment checking can be straightforwardly reduced to (un)satisfiability checking!

# Equivalence and Equi-Satisfiability

- $\alpha$  and  $\beta$  are **equivalent** iff, for every  $\mu$ ,  $\mu \models \alpha$  iff  $\mu \models \beta$   
(i.e., if  $M(\alpha) = M(\beta)$ )
- $\alpha$  and  $\beta$  are **equi-satisfiable** iff exists  $\mu_1$  s.t.  $\mu_1 \models \alpha$  iff exists  $\mu_2$  s.t.  $\mu_2 \models \beta$   
(i.e., if  $M(\alpha) \neq \emptyset$  iff  $M(\beta) \neq \emptyset$ )
- $\alpha, \beta$  equivalent  
     $\Downarrow \Updownarrow$   
     $\alpha, \beta$  equi-satisfiable
- EX:  $A_1 \vee A_2$  and  $(A_1 \vee \neg A_3) \wedge (A_3 \vee A_2)$  are equi-satisfiable, not equivalent.  
     $\{\neg A_1, A_2, A_3\} \models (A_1 \vee A_2)$ , but  $\{\neg A_1, A_2, A_3\} \not\models (A_1 \vee \neg A_3) \wedge (A_3 \vee A_2)$
- Typically used when  $\beta$  is the result of applying some transformation  $T$  to  $\alpha$ :  $\beta \stackrel{\text{def}}{=} T(\alpha)$ :
  - $T$  is **validity-preserving** [resp. **satisfiability-preserving**] iff  
     $T(\alpha)$  and  $\alpha$  are equivalent [resp. equi-satisfiable]

# Complexity

- For  $N$  variables, there are up to  $2^N$  truth assignments to be checked.
- The problem of deciding the satisfiability of a propositional formula is **NP-complete**

⇒ The most important logical problems (**validity**, **inference**, **entailment**, **equivalence**, ...) can be straightforwardly reduced to **(un)satisfiability**, and are thus **(co)NP-complete**.



**No existing worst-case-polynomial algorithm.**



# Conjunctive Normal Form (CNF)

- $\varphi$  is in **Conjunctive normal form** iff it is a conjunction of disjunctions of literals:

$$\bigwedge_{i=1}^L \bigvee_{j_i=1}^{K_i} l_{j_i}$$

- the disjunctions of literals  $\bigvee_{j_i=1}^{K_i} l_{j_i}$  are called **clauses**
- Easier to handle: list of lists of literals.  
 $\implies$  no reasoning on the recursive structure of the formula

# Classic CNF Conversion $CNF(\varphi)$

- Every  $\varphi$  can be reduced into CNF by, e.g.,

(i) expanding implications and equivalences:

$$\alpha \rightarrow \beta \implies \neg\alpha \vee \beta$$

$$\alpha \leftrightarrow \beta \implies (\neg\alpha \vee \beta) \wedge (\alpha \vee \neg\beta)$$

(ii) pushing down negations recursively:

$$\neg(\alpha \wedge \beta) \implies \neg\alpha \vee \neg\beta$$

$$\neg(\alpha \vee \beta) \implies \neg\alpha \wedge \neg\beta$$

$$\neg\neg\alpha \implies \alpha$$

(iii) applying recursively the DeMorgan's Rule:  $(\alpha \wedge \beta) \vee \gamma \implies (\alpha \vee \gamma) \wedge (\beta \vee \gamma)$

- Resulting formula worst-case **exponential**:

- ex:  $\|CNF(\bigvee_{i=1}^N (l_{i1} \wedge l_{i2}))\| = \| (l_{11} \vee l_{21} \vee \dots \vee l_{N1}) \wedge (l_{12} \vee l_{22} \vee \dots \vee l_{N2}) \wedge \dots \wedge (l_{1N} \vee l_{2N} \vee \dots \vee l_{NN}) \| = 2^N$

- $Atoms(CNF(\varphi)) = Atoms(\varphi)$
- $CNF(\varphi)$  is **equivalent** to  $\varphi$ :  $M(CNF(\varphi)) = M(\varphi)$
- Rarely used in practice.

# Labeling CNF conversion $CNF_{label}(\varphi)$

## Labeling CNF conversion $CNF_{label}(\varphi)$ (aka Tseitin's conversion)

- Every  $\varphi$  can be reduced into CNF by, e.g., applying recursively bottom-up the rules:

$$\varphi \implies \varphi[(l_i \vee l_j)|B] \wedge CNF(B \leftrightarrow (l_i \vee l_j))$$

$$\varphi \implies \varphi[(l_i \wedge l_j)|B] \wedge CNF(B \leftrightarrow (l_i \wedge l_j))$$

$$\varphi \implies \varphi[(l_i \leftrightarrow l_j)|B] \wedge CNF(B \leftrightarrow (l_i \leftrightarrow l_j))$$

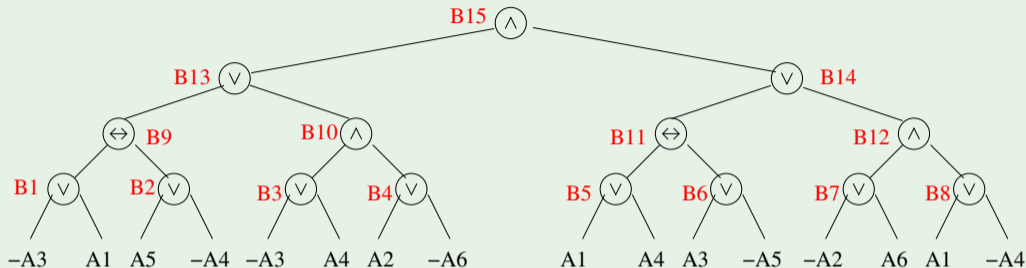
$l_i, l_j$  being literals and  $B$  being a “new” variable.

- Worst-case **linear**!
- $Atoms(CNF_{label}(\varphi)) \supseteq Atoms(\varphi)$
- $CNF_{label}(\varphi)$  is **equi-satisfiable** w.r.t.  $\varphi$ :  
 $M(CNF(\varphi)) \neq \emptyset$  iff  $M(\varphi) \neq \emptyset$
- Much more used than classic conversion in practice.

## Labeling CNF conversion $CNF_{label}(\varphi)$ (cont.)

$CNF(B \leftrightarrow (l_i \vee l_j))$	$\iff$	$(\neg B \vee l_i \vee l_j) \wedge$ $(B \vee \neg l_i) \wedge$ $(B \vee \neg l_j)$
$CNF(B \leftrightarrow (l_i \wedge l_j))$	$\iff$	$(\neg B \vee l_i) \wedge$ $(\neg B \vee l_j) \wedge$ $(B \vee \neg l_i \neg l_j)$
$CNF(B \leftrightarrow (l_i \leftrightarrow l_j))$	$\iff$	$(\neg B \vee \neg l_i \vee l_j) \wedge$ $(\neg B \vee l_i \vee \neg l_j) \wedge$ $(B \vee l_i \vee l_j) \wedge$ $(B \vee \neg l_i \vee \neg l_j)$

# Labeling CNF Conversion $CNF_{label}$ – Example



$$CNF(B_1 \leftrightarrow (\neg A_3 \vee A_1)) \wedge$$

... $\wedge$

$$CNF(B_8 \leftrightarrow (A_1 \vee \neg A_4)) \wedge$$

$$CNF(B_9 \leftrightarrow (B_1 \leftrightarrow B_2)) \wedge$$

... $\wedge$

$$CNF(B_{12} \leftrightarrow (B_7 \wedge B_8)) \wedge$$

$$CNF(B_{13} \leftrightarrow (B_9 \vee B_{10})) \wedge$$

$$CNF(B_{14} \leftrightarrow (B_{11} \vee B_{12})) \wedge$$

$$CNF(B_{15} \leftrightarrow (B_{13} \wedge B_{14})) \wedge$$

$B_{15}$

$$(\neg B_1 \vee \neg A_3 \vee A_1) \wedge (B_1 \vee A_3) \wedge (B_1 \vee \neg A_1) \wedge$$

... $\wedge$

$$(\neg B_8 \vee A_1 \vee \neg A_4) \wedge (B_8 \vee \neg A_1) \wedge (B_8 \vee A_4) \wedge$$

$$(\neg B_9 \vee \neg B_1 \vee B_2) \wedge (\neg B_9 \vee B_1 \vee \neg B_2) \wedge$$

$$(B_9 \vee B_1 \vee B_2) \wedge (B_9 \vee \neg B_1 \vee \neg B_2) \wedge$$

= ... $\wedge$

$$(B_{12} \vee \neg B_7 \vee \neg B_8) \wedge (\neg B_{12} \vee B_7) \wedge (\neg B_{12} \vee B_8) \wedge$$

$$(\neg B_{13} \vee B_9 \vee B_{10}) \wedge (B_{13} \vee \neg B_9) \wedge (B_{13} \vee \neg B_{10}) \wedge$$

$$(\neg B_{14} \vee B_{11} \vee B_{12}) \wedge (B_{14} \vee \neg B_{11}) \wedge (B_{14} \vee \neg B_{12}) \wedge$$

$$(B_{15} \vee \neg B_{13} \vee \neg B_{14}) \wedge (\neg B_{15} \vee B_{13}) \wedge (\neg B_{15} \vee B_{14}) \wedge$$

$B_{15}$

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# Propositional Reasoning: Generalities

- Automated Reasoning in Propositional Logic fundamental task
  - AI, formal verification, circuit synthesis, operational research,....
- Important in AI:  $KB \models \alpha$ : entail fact  $\alpha$  from some knowledge base  $KB$  (aka Model Checking:  $M(KB) \subseteq M(\alpha)$ )
  - typically  $\|KB\| \gg \|\alpha\|$
  - sometimes  $KB$  set of variable implications  $(A_1 \wedge \dots \wedge A_k) \rightarrow B$
- All propositional reasoning tasks reduced to satisfiability (SAT)
  - $KB \models \alpha \implies \text{SAT}(KB \wedge \neg\alpha) = \text{false}$
  - input formula CNF-ized and fed to a SAT solver
- Current SAT solvers dramatically efficient:
  - handle industrial problems with  $10^6 - 10^7$  variables & clauses!
  - used as backend engines in a variety of systems (not only AI)

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# The Resolution Rule

- **Resolution**: deduction of a new clause from a pair of clauses with exactly one incompatible variable (**resolvent**):

$$\frac{(\underbrace{l_1 \vee \dots \vee l_k}_{\text{common}} \vee \underbrace{l}_{\text{resolvent}} \vee \underbrace{l'_{k+1} \vee \dots \vee l'_m}_{C'}) \quad (\underbrace{l_1 \vee \dots \vee l_k}_{\text{common}} \vee \underbrace{\neg l}_{\text{resolvent}} \vee \underbrace{l''_{k+1} \vee \dots \vee l''_n}_{C'})}{(\underbrace{l_1 \vee \dots \vee l_k}_{\text{common}} \vee \underbrace{l'_{k+1} \vee \dots \vee l'_m}_{C'} \vee \underbrace{l''_{k+1} \vee \dots \vee l''_n}_{C'})}$$

- Ex: 
$$\frac{(A \vee B \vee C \vee D \vee E) \quad (A \vee B \vee \neg C \vee F)}{(A \vee B \vee D \vee E \vee F)}$$

- Note: many standard inference rules subcases of resolution:  
(recall that  $\alpha \rightarrow \beta \iff \neg\alpha \vee \beta$ )

$$\frac{A \rightarrow B \quad B \rightarrow C}{A \rightarrow C} \text{ (trans.)} \quad \frac{A \quad A \rightarrow B}{B} \text{ (m. ponens)} \quad \frac{\neg B \quad A \rightarrow B}{\neg A} \text{ (m. tollens)}$$

# Basic Propositional Inference: Resolution

- Assume input formula in CNF
  - if not, apply Tseitin CNF-ization first

⇒  $\varphi$  is represented as a set of clauses

- **Search** for a refutation of  $\varphi$  (is  $\varphi$  unsatisfiable?)
  - recall:  $\alpha \models \beta$  iff  $\alpha \wedge \neg\beta$  unsatisfiable
- Basic idea: **apply iteratively the resolution rule to pairs of clauses with a conflicting literal, producing novel clauses, until either**
  - a false clause is generated, or
  - the resolution rule is no more applicable
- **Correct**: if returns an empty clause, then  $\varphi$  unsat ( $\alpha \models \beta$ )
- **Complete**: if  $\varphi$  unsat ( $\alpha \models \beta$ ), then it returns an empty clause
- **Time-inefficient**
- **Very Memory-inefficient (exponential in memory)**
- Many different strategies

## Very-Basic PL-Resolution Procedure

**function** PL-RESOLUTION( $KB, \alpha$ ) **returns** *true* or *false*

**inputs:**  $KB$ , the knowledge base, a sentence in propositional logic  
 $\alpha$ , the query, a sentence in propositional logic

$clauses \leftarrow$  the set of clauses in the CNF representation of  $KB \wedge \neg\alpha$

$new \leftarrow \{ \}$

**loop do**

**for each** pair of clauses  $C_i, C_j$  **in**  $clauses$  **do**

$resolvents \leftarrow$  PL-RESOLVE( $C_i, C_j$ )

**if**  $resolvents$  contains the empty clause **then return** *true*

$new \leftarrow new \cup resolvents$

**if**  $new \subseteq clauses$  **then return** *false*

$clauses \leftarrow clauses \cup new$

# Improvements: Subsumption & Unit Propagation

General “set” notation ( $\Gamma$  clause set):

$$\frac{\Gamma, \phi_1, \dots, \phi_n}{\Gamma, \phi'_1, \dots, \phi'_n} \left( \text{e.g.,} \quad \frac{\Gamma, C_1 \vee p, C_2 \vee \neg p}{\Gamma, C_1 \vee p, C_2 \vee \neg p, C_1 \vee C_2,} \right)$$

- Removal of valid clauses:

$$\frac{\Gamma \wedge (p \vee \neg p \vee C)}{\Gamma}$$

- Clause Subsumption ( $C$  clause):

$$\frac{\Gamma \wedge C \wedge (C \vee \bigvee_i l_i)}{\Gamma \wedge (C)}$$

- Unit Resolution:

$$\frac{\Gamma \wedge (l) \wedge (\neg l \vee \bigvee_i l_i)}{\Gamma \wedge (l) \wedge (\bigvee_i l_i)}$$

- Unit Subsumption:

$$\frac{\Gamma \wedge (l) \wedge (l \vee \bigvee_i l_i)}{\Gamma \wedge (l)}$$

- Unit Propagation = Unit Resolution + Unit Subsumption

“Deterministic” rule: applied **before** other “non-deterministic” rules!

## Remark

### What happens with more than 1 resolvent?

- Common mistake: the following is not a correct application of the resolution rule:

$$\frac{\Gamma, (C_1 \vee l_1 \vee l_2), (C_2 \vee \neg l_1 \vee \neg l_2)}{\Gamma, (C_1 \vee l_1 \vee l_2), (C_2 \vee \neg l_1 \vee \neg l_2), (C_1 \vee C_2)}$$

- Rather, a correct application would be:

$$\frac{\Gamma, (C_1 \vee l_1 \vee l_2), (C_2 \vee \neg l_1 \vee \neg l_2)}{\Gamma, (C_1 \vee l_1 \vee l_2), (C_2 \vee \neg l_1 \vee \neg l_2), (C_1 \vee l_2 \vee C_2 \vee \neg l_2)}$$

... but  $(C_1 \vee C_2 \vee \neg l_1 \vee \neg l_2)$  is valid and should be removed

⇒ no clause is produced

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# The Davis-Putnam-Longemann-Loveland Procedure

- Tries to build an assignment  $\mu$  satisfying  $\varphi$
- At each step assigns a truth value to (all instances of) **one atom**
- Performs **deterministic choices** (mostly unit-propagation) first
- The grandfather of the most efficient SAT solvers
- Correct and complete
- Much more efficient than PL-Resolution
- Requires **polynomial space**

## The DPLL Procedure [cont.]

**function** DPLL-SATISFIABLE?(*s*) **returns** *true* or *false*

**inputs:** *s*, a sentence in propositional logic

*clauses*  $\leftarrow$  the set of clauses in the CNF representation of *s*

*symbols*  $\leftarrow$  a list of the proposition symbols in *s*

**return** DPLL(*clauses*, *symbols*, { })

---

**function** DPLL(*clauses*, *symbols*, *model*) **returns** *true* or *false*

**if** every clause in *clauses* is true in *model* **then return** *true*

**if** some clause in *clauses* is false in *model* **then return** *false*

*P*, *value*  $\leftarrow$  FIND-PURE-SYMBOL(*symbols*, *clauses*, *model*)

**if** *P* is non-null **then return** DPLL(*clauses*, *symbols* - *P*, *model*  $\cup$  {*P*=*value*})

*P*, *value*  $\leftarrow$  FIND-UNIT-CLAUSE(*clauses*, *model*)

**if** *P* is non-null **then return** DPLL(*clauses*, *symbols* - *P*, *model*  $\cup$  {*P*=*value*})

*P*  $\leftarrow$  FIRST(*symbols*); *rest*  $\leftarrow$  REST(*symbols*)

**return** DPLL(*clauses*, *rest*, *model*  $\cup$  {*P*=*true*}) **or**

DPLL(*clauses*, *rest*, *model*  $\cup$  {*P*=*false*})

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Pure-Symbol Rule out of date, no more used in modern solvers.



## The DPLL Procedure [cont.]

**function** DPLL-SATISFIABLE?(*s*) **returns** *true* or *false*

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---

**function** DPLL(*clauses*, *symbols*, *model*) **returns** *true* or *false*

**if** every clause in *clauses* is true in *model* **then return** *true*

**if** some clause in *clauses* is false in *model* **then return** *false*

~~*P*, *value*  $\leftarrow$  FIND-PURE-SYMBOL(*symbols*, *clauses*, *model*)~~

~~**if** *P* is non-null **then return** DPLL(*clauses*, *symbols* - *P*, *model*  $\cup$  {*P*=*value*})~~

*P*, *value*  $\leftarrow$  FIND-UNIT-CLAUSE(*clauses*, *model*)

**if** *P* is non-null **then return** DPLL(*clauses*, *symbols* - *P*, *model*  $\cup$  {*P*=*value*})

*P*  $\leftarrow$  FIRST(*symbols*); *rest*  $\leftarrow$  REST(*symbols*)

**return** DPLL(*clauses*, *rest*, *model*  $\cup$  {*P*=*true*}) **or**

DPLL(*clauses*, *rest*, *model*  $\cup$  {*P*=*false*})

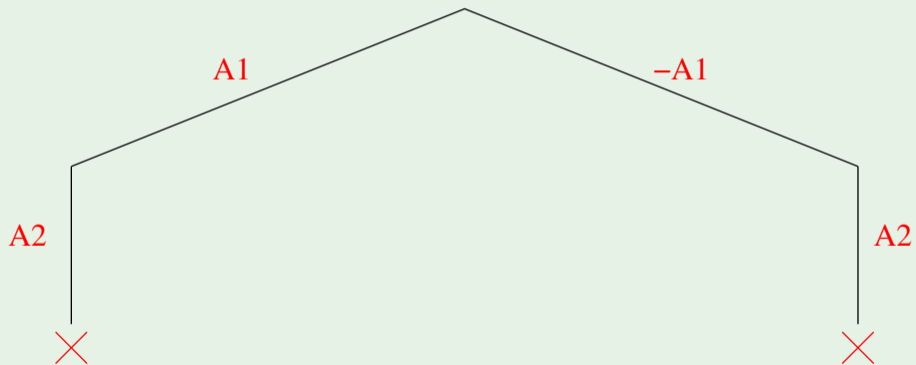
(© S. Russell & P. Norwig, AIMA)

Pure-Symbol Rule out of date, no more used in modern solvers.

# DPLL: Example

## DPLL search tree

$$\varphi = (A_1 \vee A_2) \wedge (A_1 \vee \neg A_2) \wedge (\neg A_1 \vee A_2) \wedge (\neg A_1 \vee \neg A_2)$$

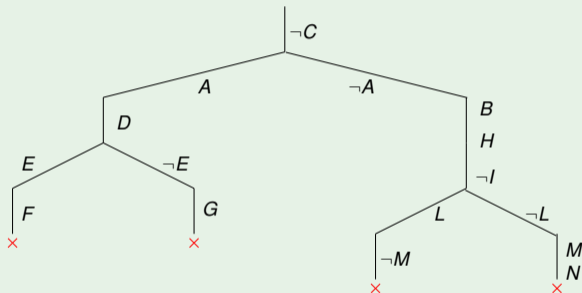


# DPLL – example

## DPLL (without pure-literal rule)

Here “choose-literal” selects variable in alphabetic order, selecting true first.

$(\neg C) \wedge$   
 $(B \vee A \vee C) \wedge$   
 $(\neg A \vee D) \wedge$   
 $(\neg E \vee \neg A \vee F) \wedge$   
 $(\neg E \vee \neg F \vee \neg A) \wedge$   
 $(G \vee \neg A \vee E) \wedge$   
 $(E \vee \neg G \vee \neg A) \wedge$   
 $(A \vee H \vee C) \wedge$   
 $(\neg H \vee \neg I \vee A) \wedge$   
 $(I \vee L \vee M) \wedge$   
 $(\neg L \vee C \vee \neg M) \wedge$   
 $(A \vee \neg L \vee M) \wedge$   
 $(L \vee N \vee \neg H) \wedge$   
 $(I \vee L \vee \neg N)$



$\Rightarrow$  UNSAT

# Modern CDCL SAT Solvers

- Non-recursive, stack-based implementations
- Based on **Conflict-Driven Clause-Learning (CDCL)** schema
  - inspired to conflict-driven backjumping and learning in CSPs
  - learns implied clauses as nogoods
- **Random restarts**
  - abandon the current search tree and restart on top level
  - previously-learned clauses maintained
- Smart **literal selection heuristics** (ex: **VSIDS**)
  - “static”: scores updated only at the end of a branch
  - “local”: privileges variable in recently learned clauses
- Smart **preprocessing/inprocessing** technique to simplify formulas
- **Smart indexing** techniques (e.g. **2-watched literals**)
  - efficiently do/undo assignments and reveal unit clauses
- Allow **Incremental Calls** (stack-based interface)
  - allow for reusing previous search on “similar” problems

Can handle industrial problems with  $10^6 - 10^7$  variables and clauses!

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# Horn Formulas

- A **Horn clause** is a clause containing at most one positive literal
  - a **definite clause** is a clause containing exactly one positive literal
  - a **goal clause** is a clause containing no positive literal
- A **Horn formula** is a conjunction/set of Horn clauses

- Ex:
  - $A_1 \vee \neg A_2$  // definite
  - $A_2 \vee \neg A_3 \vee \neg A_4$  // definite
  - $\neg A_5 \vee \neg A_3 \vee \neg A_4$  // goal
  - $A_3$  // definite

- Intuition: implications between positive Boolean variables:

$$\begin{array}{l} A_2 \rightarrow A_1 \\ (A_3 \wedge A_4) \rightarrow A_2 \\ (A_5 \wedge A_3 \wedge A_4) \rightarrow \perp \\ A_3 \end{array}$$

- Often allow to represent knowledge-base entailment  $KB \models \alpha$ :
  - **knowledge base KB** written as sets of definite clauses  
ex:  $In11$ ;  $(\neg In11 \vee \neg MoveFrom11To12 \vee In12)$ ;
  - goal  $\neg \alpha$  as a goal clause  
ex:  $\neg In12$

# Tractability of Horn Formulas

## Property

Checking the satisfiability of Horn formulas requires polynomial time:

- Hint:

- 1 Eliminate unit clauses by propagating their value;
- 2 If an empty clause is generated, return unsat
- 3 Otherwise, every clause contains at least one negative literal

⇒ Assign all variables to  $\perp$ ; return the assignment

- Alternatively: run DPLL/CDCL, selecting negative literals first

# A simple polynomial procedure for Horn-SAT

```
function Horn_SAT(formula  $\varphi$ , assignment &  $\mu$ ) {  
  Unit_Propagate( $\varphi$ ,  $\mu$ );  
  if ( $\varphi == \perp$ )  
    then return UNSAT;  
  else {  
     $\mu := \mu \cup \bigcup_{A_i \notin \mu} \{\neg A_i\}$ ;  
    return SAT;  
  } }  
}
```

```
function Unit_Propagate(formula &  $\varphi$ , assignment &  $\mu$ )  
  while ( $\varphi \neq \top$  and  $\varphi \neq \perp$  and {a unit clause ( $l$ ) occurs in  $\varphi$ ) do {  
     $\varphi = \text{assign}(\varphi, l)$ ;  
     $\mu := \mu \cup \{l\}$ ;  
  } }  
}
```



# Example

$$\begin{array}{l} \neg A_1 \vee A_2 \vee \neg A_3 \\ A_1 \vee \neg A_3 \vee \neg A_4 \\ \neg A_2 \vee \neg A_4 \\ A_3 \vee \neg A_4 \\ A_4 \end{array}$$

# Example

$$\begin{array}{l} \neg A_1 \vee A_2 \vee \neg A_3 \\ A_1 \vee \neg A_3 \vee \neg A_4 \\ \neg A_2 \vee \neg A_4 \\ A_3 \vee \neg A_4 \\ A_4 \end{array}$$

$$\mu := \{A_4 := \text{T}\}$$

# Example

$$\begin{array}{l} \neg A_1 \vee A_2 \vee \neg A_3 \\ A_1 \vee \neg A_3 \vee \neg A_4 \\ \neg A_2 \vee \neg A_4 \\ A_3 \vee \neg A_4 \\ A_4 \end{array}$$

$$\mu := \{A_4 := \text{T}, A_3 := \text{T}\}$$

# Example

$$\begin{array}{l} \neg A_1 \vee A_2 \vee \neg A_3 \\ A_1 \vee \neg A_3 \vee \neg A_4 \\ \neg A_2 \vee \neg A_4 \\ A_3 \vee \neg A_4 \\ A_4 \end{array}$$

$$\mu := \{A_4 := \top, A_3 := \top, A_2 := \perp\}$$

# Example

$$\begin{array}{l} \neg A_1 \vee A_2 \vee \neg A_3 \quad \times \\ A_1 \vee \neg A_3 \vee \neg A_4 \\ \neg A_2 \vee \neg A_4 \\ A_3 \vee \neg A_4 \\ A_4 \end{array}$$

$$\mu := \{A_4 := \top, A_3 := \top, A_2 := \perp, A_1 := \top\} \implies \text{UNSAT}$$

## Example 2

$$\begin{array}{l} A_1 \vee \neg A_2 \\ A_2 \vee \neg A_5 \vee \neg A_4 \\ A_4 \vee \neg A_3 \\ A_3 \end{array}$$

## Example 2

$$\begin{array}{l} A_1 \quad \vee \neg A_2 \\ A_2 \quad \vee \neg A_5 \quad \vee \neg A_4 \\ A_4 \quad \vee \neg A_3 \\ A_3 \end{array}$$
$$\mu := \{A_3 := \top\}$$

## Example 2

$$\begin{array}{l} A_1 \quad \vee \neg A_2 \\ A_2 \quad \vee \neg A_5 \quad \vee \neg A_4 \\ A_4 \quad \vee \neg A_3 \\ A_3 \end{array}$$
$$\mu := \{A_3 := \top, A_4 := \top\}$$



## Example 2

$$\begin{array}{l} A_1 \vee \neg A_2 \\ A_2 \vee \neg A_5 \vee \neg A_4 \\ A_4 \vee \neg A_3 \\ A_3 \end{array}$$
$$\mu := \{A_3 := \text{T}, A_4 := \text{T}\} \implies \text{SAT}$$

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# Local Search with SAT

- Similar to Local Search for CSPs
- Input: set of clauses
- Use total truth assignments
  - allow states with unsatisfied clauses
  - “neighbour states” differ for one variable truth value
  - steps: reassign variable truth values
- Cost: # of unsatisfied clauses
- Stochastic local search [see Ch. 4] applies to SAT as well
  - random walk, simulated annealing, GAs, taboo search, ...
- The WalkSAT stochastic local search
  - Clause selection: randomly select an unsatisfied clause  $C$
  - Variable selection:
    - prob.  $p$ : flip variable from  $C$  at random
    - prob.  $1-p$ : flip variable from  $C$  causing a minimum number of unsat clauses
- Note: can detect only satisfiability, not unsatisfiability
- Many variants

# The WalkSAT Procedure

**function** WALKSAT(*clauses*, *p*, *max\_flips*) **returns** a satisfying model or *failure*

**inputs:** *clauses*, a set of clauses in propositional logic

*p*, the probability of choosing to do a “random walk” move, typically around 0.5

*max\_flips*, number of flips allowed before giving up

*model*  $\leftarrow$  a random assignment of *true/false* to the symbols in *clauses*

**for** *i* = 1 **to** *max\_flips* **do**

**if** *model* satisfies *clauses* **then return** *model*

*clause*  $\leftarrow$  a randomly selected clause from *clauses* that is false in *model*

**with probability** *p* flip the value in *model* of a randomly selected symbol from *clause*

**else** flip whichever symbol in *clause* maximizes the number of satisfied clauses

**return** *failure*

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## A Quote

You can think about deep learning as equivalent to ... our visual cortex or auditory cortex. But, of course, true intelligence is a lot more than just that, you have to recombine it into higher-level thinking and symbolic reasoning, a lot of the things classical AI tried to deal with in the 80s.

...

We would like to build up to this symbolic level of reasoning - maths, language, and logic. So that's a big part of our work.

Demis Hassabis, CEO of Google Deepmind

# Knowledge Representation and Reasoning

- **Knowledge Representation & Reasoning (KR&R)**: the field of AI dedicated to representing knowledge of the world in a form a computer system can utilize to solve complex tasks
- The class of systems/agents that derive from this approach are called **knowledge based (KB) systems/agents**
- A KB agent maintains a **knowledge base (KB)** of facts
  - collection of **domain-specific facts** believed by the agent
  - expressed in a **formal language** (e.g. propositional logic)
  - represent the agent's **representation of the world**
  - initially contains the **background knowledge**
  - KB queries and updates via **logical entailment**, performed by an **inference engine**
- Inference engine **allows for inferring actions and new knowledge**
  - **domain-independent algorithms**, can answer any question





# Reasoning

- **Reasoning**: formal manipulation of the symbols representing a collection of beliefs to produce representations of new ones
- **Logical entailment** ( $KB \models \alpha$ ) is the fundamental operation -
- **Ex:**
  - (KB acquired fact): “Patient x is allergic to medication m”
  - (KB general rule): “Anybody allergic to m is also allergic to m’.”
  - (KB general rule): “If x is allergic to m’, do not prescribe m’ for x.”
  - (query): “Prescribe m’ for x?”
  - (answer) **No** (because patient x is allergic to medication m’)
- Other forms of reasoning (last part of this course)
  - **Probabilistic reasoning**
- Other forms of reasoning (not addressed in this course)
  - **Abductive reasoning** (aka **diagnosis**): given  $KB$  and  $\beta$ , conjecture hypotheses  $\alpha$  s.t.  $(KB \wedge \alpha) \models \beta$
  - **Abductive reasoning**: from a set of observation find a general rule

# Knowledge-Based Agents (aka Logic Agents)

- **Logic agents:** combine domain knowledge with current percepts to infer hidden aspects of current state prior to selecting actions
  - Crucial in partially observable environments
- KB Agent must be able to:
  - represent states and actions
  - incorporate new percepts
  - update internal representation of the world
  - deduce hidden properties of the world
  - deduce appropriate actions
- Agents can be described at different levels
  - **knowledge level (declarative approach):** behaviour completely described by the sentences stored in the KB
  - **implementation level (procedural approach):** behaviour described as program code
- **Declarative approach** to building an agent (or other system):
  - **Tell** the KB what it needs to know (update KB)
  - **Ask** what to do (answers should follow logically from KB & query)

# Knowledge-Based Agent: General Schema

- Given a percept, the agent
  - Tells the KB of the percept at time step  $t$
  - ASKs the KB for the best action to do at time step  $t$
  - Tells the KB that it has in fact taken that action
- Details hidden in three functions:  
MAKE-PERCEPT-SENTENCE, MAKE-ACTION-QUERY, MAKE-ACTION-SENTENCE
  - construct logic sentences
  - implement the interface between sensors/actuators and KRR core
- Tell and Ask may require complex logical inference

```
function KB-AGENT(percept) returns an action  
  persistent: KB, a knowledge base  
              t, a counter, initially 0, indicating time  
  
  TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))  
  action ← ASK(KB, MAKE-ACTION-QUERY(t))  
  TELL(KB, MAKE-ACTION-SENTENCE(action, t))  
  t ← t + 1  
  return action
```

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# Example: The Wumpus World

## Task Environment: PEAS Description

### Performance measure:

- gold: +1000, death: -1000
- step: -1, using the arrow: -10

### Environment:

- squares adjacent to Wumpus are stenchy
- squares adjacent to pit are breezy
- glitter iff gold is in the same square
- shooting kills Wumpus if you are facing it
- shooting uses up the only arrow
- grabbing picks up gold if in same square
- releasing drops the gold in same square

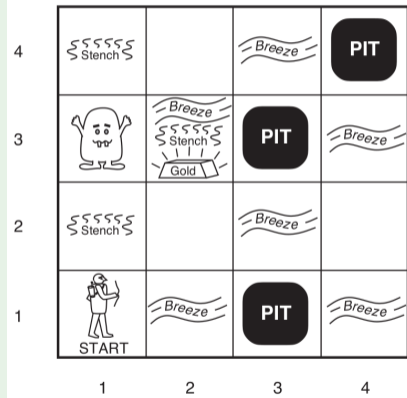
### Actuators:

- Left turn, Right turn, Forward, Grab, Release, Shoot

### Sensors:

- Stench, Breeze, Glitter, Bump, Scream

### One possible configuration:



(© S. Russell & P. Norwig, AIMA)

# Wumpus World: Characterization

- **Fully Observable?** No: only local perception
- **Deterministic?** Yes: outcomes exactly specified
- **Episodic?** No: actions can have long-term consequences
- **Static?** Yes: Wumpus and Pits do not move
- **Discrete?** Yes
- **Single-agent?** Yes (Wumpus is essentially a natural feature)

## Example: Exploring the Wumpus World

- The KB initially contains the rules of the environment.
- Agent is initially in 1,1
- Percepts:  
no stench, no breeze

⇒ [1,2] and [2,1] OK

OK			
OK A	OK		

A: Agent; B: Breeze; G: Glitter; S: Stench  
OK: safe square; W: Wumpus; P: Pit; BGS: bag of gold

## Example: Exploring the Wumpus World

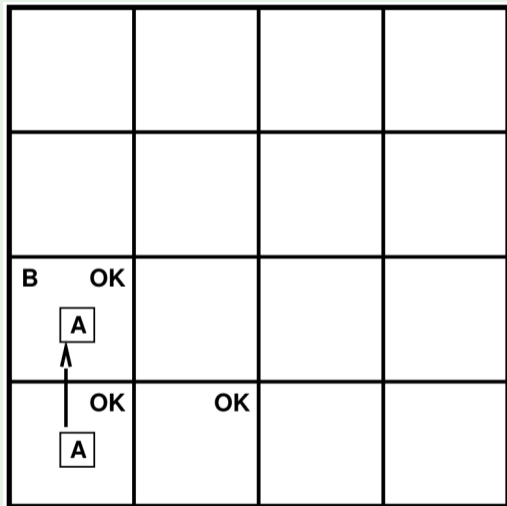
- Agent moves to [2,1]

- perceives a breeze

⇒ Pit in [3,1] or [2,2]

- perceives no stench

⇒ no Wumpus in [3,1], [2,2]



A: Agent; B: Breeze; G: Glitter; S: Stench

OK: safe square; W: Wumpus; P: Pit; BGS: bag of gold



## Example: Exploring the Wumpus World

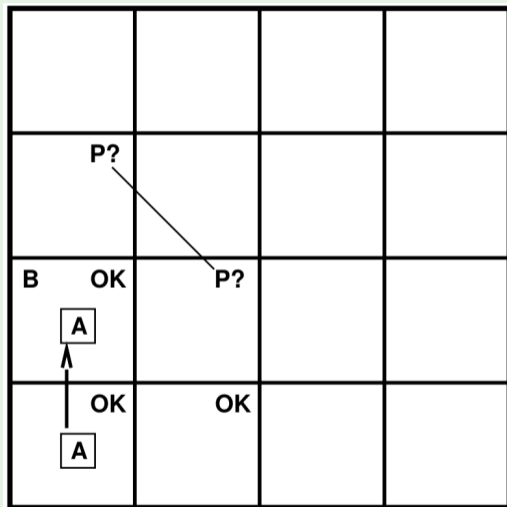
- Agent moves to [2,1]

- perceives a breeze

⇒ Pit in [3,1] or [2,2]

- perceives no stench

⇒ no Wumpus in [3,1], [2,2]

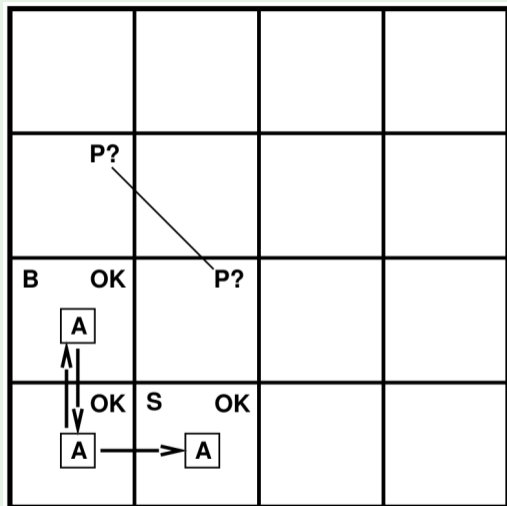


A: Agent; B: Breeze; G: Glitter; S: Stench

OK: safe square; W: Wumpus; P: Pit; BGS: bag of gold

## Example: Exploring the Wumpus World

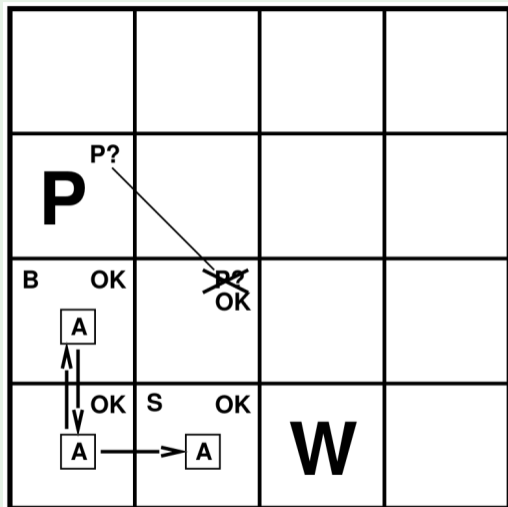
- Agent moves to [1,1]-[1,2]
- perceives no breeze
- ⇒ no Pit in [1,3], [2,2]
- ⇒ [2,2] OK
- ⇒ pit in [3,1]
- perceives a stench
- ⇒ Wumpus in ~~[2,2]~~ or [1,3]!



A: Agent; B: Breeze; G: Glitter; S: Stench  
OK: safe square; W: Wumpus; P: Pit; BGS: bag of gold

## Example: Exploring the Wumpus World

- Agent moves to [1,1]-[1,2]
- perceives no breeze
- ⇒ no Pit in [1,3], [2,2]
- ⇒ [2,2] OK
- ⇒ pit in [3,1]
- perceives a stench
- ⇒ Wumpus in ~~[2,2]~~ or [1,3]!



A: Agent; B: Breeze; G: Glitter; S: Stench  
OK: safe square; W: Wumpus; P: Pit; BGS: bag of gold

## Example: Exploring the Wumpus World

- Agent moves to [2,2]

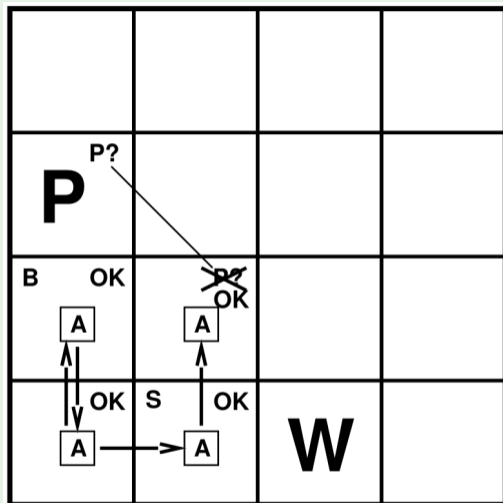
- perceives no breeze

⇒ no pit in [3,2], [2,3]

- perceives no stench

⇒ no Wumpus in [3,2], [2,3]

⇒ [3,2] and [2,3] OK



A: Agent; B: Breeze; G: Glitter; S: Stench

OK: safe square; W: Wumpus; P: Pit; BGS: bag of gold

## Example: Exploring the Wumpus World

- Agent moves to [2,2]

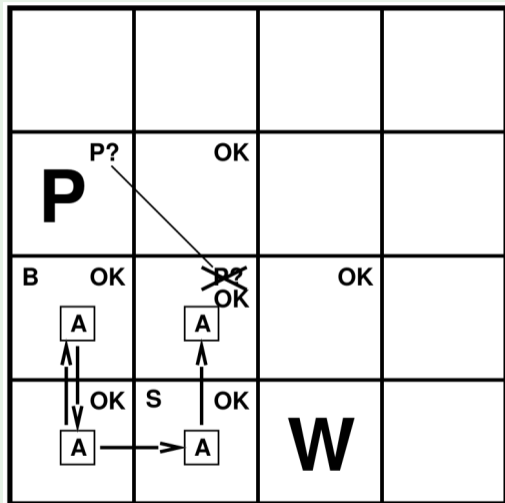
- perceives no breeze

⇒ no pit in [3,2], [2,3]

- perceives no stench

⇒ no Wumpus in [3,2], [2,3]

⇒ [3,2] and [2,3] OK



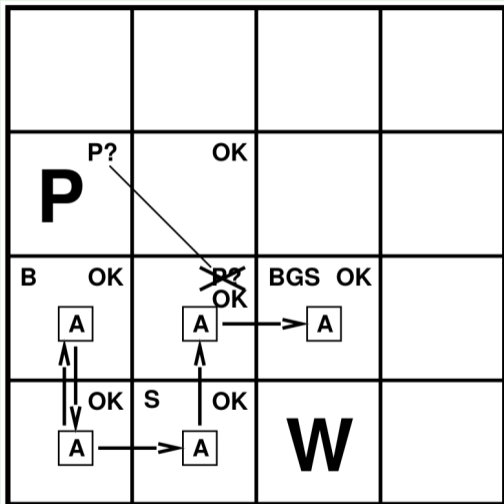
**A**: Agent; **B**: Breeze; **G**: Glitter; **S**: Stench

**OK**: safe square; **W**: Wumpus; **P**: Pit; **BGS**: bag of gold

## Example: Exploring the Wumpus World

- Agent moves to [2,3]
- perceives a glitter

⇒ bag of gold!

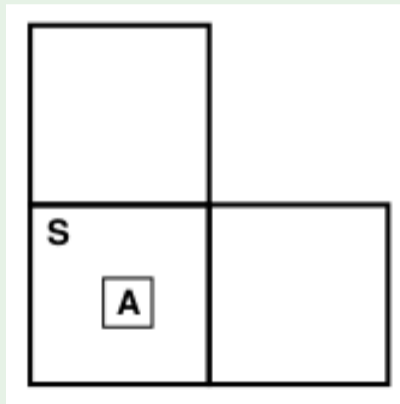


A: Agent; B: Breeze; G: Glitter; S: Stench  
OK: safe square; W: Wumpus; P: Pit; BGS: bag of gold

## Example 2: Exploring the Wumpus World [see Ch 13]

### Alternative scenario: apply coercion

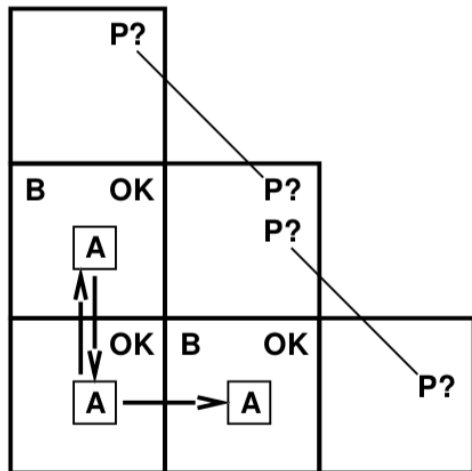
- Feel stench in [1,1]
- ⇒ Wumpus [1,2] or [2,1]
- ⇒ **Cannot move**
- **Apply coercion**: shoot ahead
  - Wumpus was there
    - ⇒ Wumpus dead
    - ⇒ **Safe**
  - Wumpus wasn't there
    - ⇒ **Safe**



## Example 3: Exploring the Wumpus World [see Ch. 13]

### Alternative scenario: probabilistic solution (hints)

- Feel breeze in [1,2] and [2,1]
- ⇒ pit in [1,3] or [2,2] or [3,1]
- ⇒ **no 100% safe action**
- Probability analysis [see Ch 13] (assuming pits uniformly distributed):
  - $P(\text{pit} \in [2, 2]) = 0.86$
  - $P(\text{pit} \in [1, 3]) = 0.31$
  - $P(\text{pit} \in [3, 1]) = 0.31$
- ⇒ **better choose [1,3] or [3,1]**





# Outline

- 1 Propositional Logic
- 2 Propositional Reasoning
  - Resolution
  - DPLL
  - Reasoning with Horn Formulas
  - Local Search
- 3 Agents Based on Knowledge Representation & Reasoning
  - Knowledge-Based Agents
  - Example: the Wumpus World
- 4 Agents Based on Propositional Reasoning**
  - Propositional Logic Agents
  - Example: the Wumpus World

# Outline

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# Propositional Logic Agents

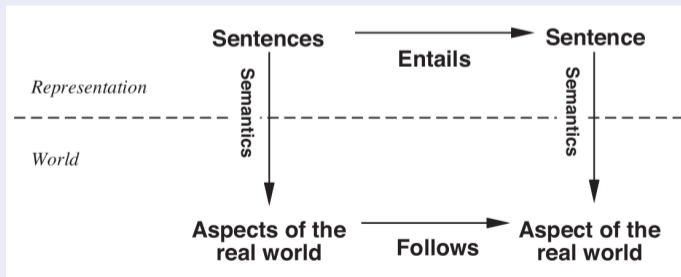
- Kind of Logic agents
- Language: **propositional logic, first-order logic, ...**
  - represent KB as set of propositional formulas
  - percepts and actions are (collections of ) propositional atoms
  - in practice: **sets of clauses**
- Perform propositional logic inference
  - model checking, entailment
  - in practice: **incremental calls to a SAT solver**

# Representation vs. World

Reasoning process (propositional entailment) sound

⇒ if KB is true in the real world, then any sentence  $\alpha$  derived from KB by a sound inference procedure is also true in the real world

- sentences are configurations of the agent
  - reasoning constructs new configurations from old ones
- ⇒ the new configurations represent aspects of the world that actually follow from the aspects that the old configurations represent



# Reasoning as Entailment

## Scenario in Wumpus World

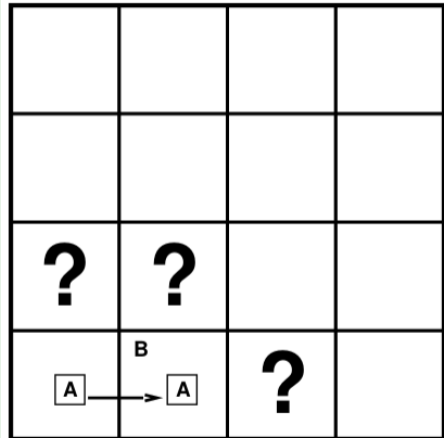
Consider pits (and breezes) only:

- initial:  $\neg P_{[1,1]}$
- after detecting nothing in  $[1,1]$ :  $\neg B_{[1,1]}$
- move to  $[2,1]$ , detect breeze:  $B_{[2,1]}$

Q: are there pits in  $[1,2]$ ,  $[2,1]$ ,  $[3,1]$ ?

- 3 variables:  $P_{[1,2]}, P_{[2,1]}, P_{[3,1]}$ ,  
⇒ 8 possible models

- Query  $\alpha_1$ :  $KB \models \neg P_{[1,2]}$ ?
- Query  $\alpha_2$ :  $KB \models \neg P_{[2,1]}$ ?
- Query  $\alpha_3$ :  $KB \models \neg P_{[3,1]}$ ?

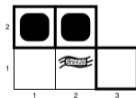
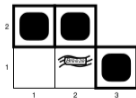
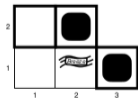
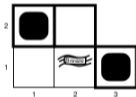
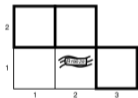
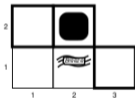
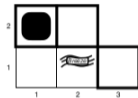
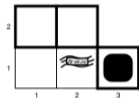


A: Agent; B: Breeze; G: Glitter; S: Stench

OK: safe square; W: Wumpus; P: pit; BGS: bag of gold

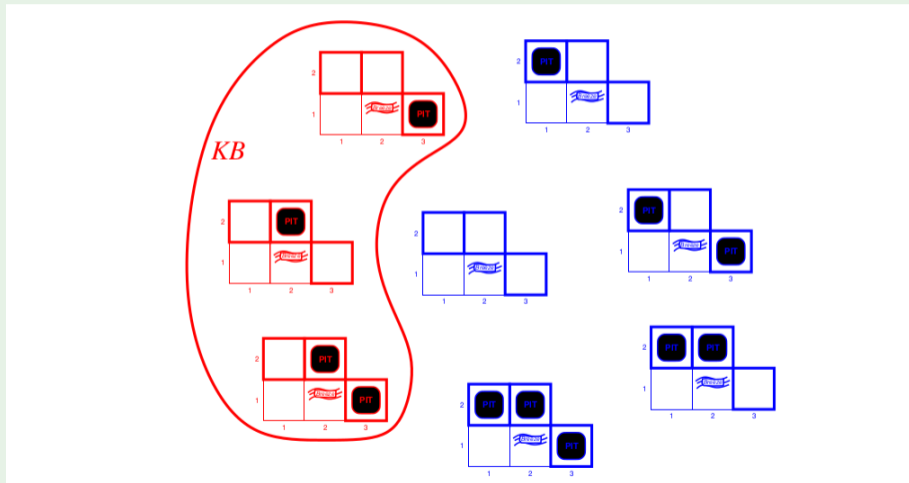
# Reasoning as Entailment [cont.]

8 possible models



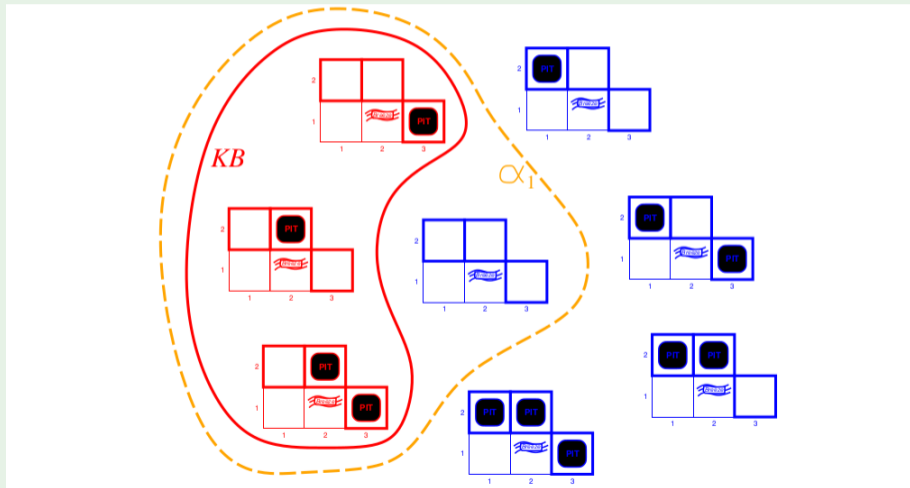
# Reasoning as Entailment [cont.]

KB: Wumpus World rules + observations  $\implies$  3 models



# Reasoning as Entailment [cont.]

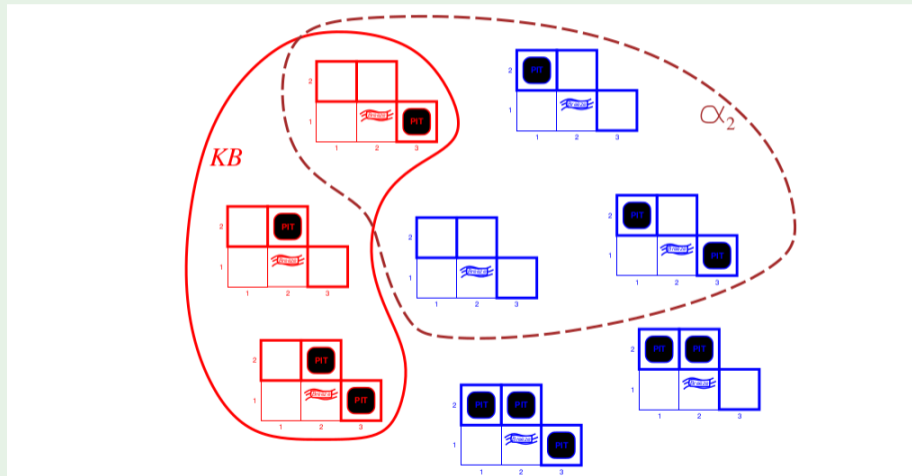
Query  $\alpha_1 : \neg P_{[1,2]} \implies KB \models \alpha_1$  (i.e.  $M(KB) \subseteq M(\alpha_1)$ )





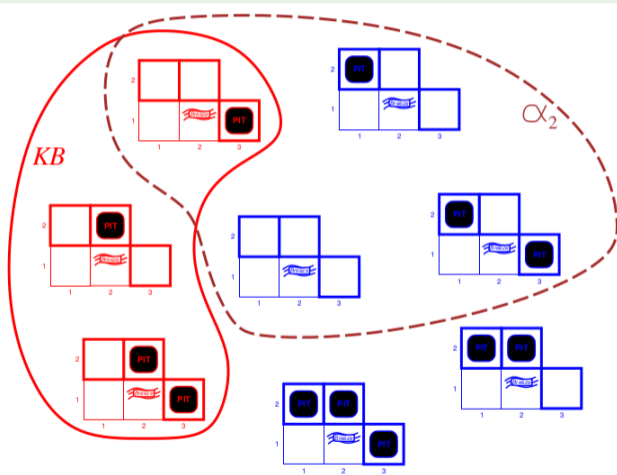
# Reasoning as Entailment [cont.]

Query  $\alpha_2 : \neg P_{[2,2]} \implies KB \not\models \alpha_2$  (i.e.  $M(KB) \not\subseteq M(\alpha_2)$ )



# Reasoning as Entailment [cont.]

In practice:  $DPLL(CNF(KB \wedge \neg\alpha_2)) = \text{sat}$



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## Example: Exploring the Wumpus World

KB initially contains (the CNFized versions of) the following formulas,  $\forall i, j \in [1..4]$ :

- breeze iff pit in neighbours

$$B_{[i,j]} \leftrightarrow (P_{[i,j-1]} \vee P_{[i+1,j]} \vee P_{[i,j+1]} \vee P_{[i-1,j]})$$

- stench iff Wumpus in neighbours

$$S_{[i,j]} \leftrightarrow (W_{[i,j-1]} \vee W_{[i+1,j]} \vee W_{[i,j+1]} \vee W_{[i-1,j]})$$

- safe iff no Wumpus and no pit there  $OK_{[i,j]} \leftrightarrow (\neg W_{[i,j]} \wedge \neg P_{[i,j]})$

- glitter iff pile of gold there

$$G_{[i,j]} \leftrightarrow BGS_{[i,j]}$$




- in  $[1, 1]$  no Wumpus and no pit  $\implies$  safe

$$\neg P_{[1,1]}, \neg W_{[1,1]}, OK_{[1,1]}$$

(implicit:  $P_{[i,j]}, W_{[i,j]}, P_{[i,j]}$  false if  $i, j \notin [1..4]$ )

**A**: Agent; **B**: Breeze; **G**: Glitter; **S**: Stench

**OK**: safe square; **W**: Wumpus; **P**: pit; **BGS**: bag of gold

			
OK 			

## Example: Exploring the Wumpus World

- KB initially contains:

$\neg P_{[1,1]}, \neg W_{[1,1]}, OK_{[1,1]}$

$B_{[1,1]} \leftrightarrow (P_{[1,2]} \vee P_{[2,1]})$

$S_{[1,1]} \leftrightarrow (W_{[1,2]} \vee W_{[2,1]})$

$OK_{[1,2]} \leftrightarrow (\neg W_{[1,2]} \wedge \neg P_{[1,2]})$

$OK_{[2,1]} \leftrightarrow (\neg W_{[2,1]} \wedge \neg P_{[2,1]})$

...

- Agent is initially in 1,1
- Percepts (no stench, no breeze):  $\neg S_{[1,1]}, \neg B_{[1,1]}$

$\Rightarrow \neg W_{[1,2]}, \neg W_{[2,1]}, \neg P_{[1,2]}, \neg P_{[2,1]}$

$\Rightarrow OK_{[1,2]}, OK_{[2,1]}$  ([1,2]&[2,1] OK)

- Add all them to KB

OK			
OK A	OK		

A: Agent; B: Breeze; G: Glitter; S: Stench

OK: safe square; W: Wumpus; P: pit; BGS: glitter, bag of gold

## Example: Exploring the Wumpus World

- KB initially contains:

$\neg P_{[1,1]}, \neg W_{[1,1]}, OK_{[1,1]}$

$B_{[2,1]} \leftrightarrow (P_{[1,1]} \vee P_{[2,2]} \vee P_{[3,1]})$

$S_{[2,1]} \leftrightarrow (W_{[1,1]} \vee W_{[2,2]} \vee W_{[3,1]})$

...

- Agent moves to [2,1]

- perceives a breeze:  $B_{[2,1]}$

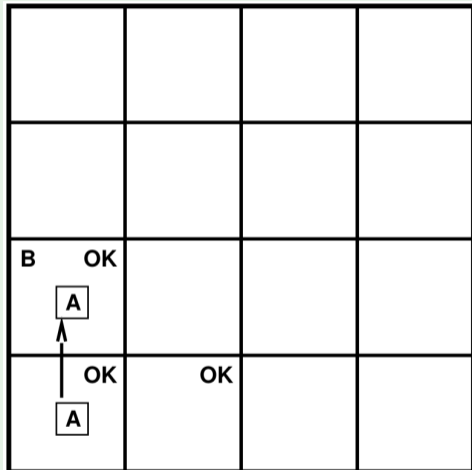
⇒  $(P_{[3,1]} \vee P_{[2,2]})$  (pit in [3,1] or [2,2])

- perceives no stench  $\neg S_{[2,1]}$

⇒  $\neg W_{[3,1]}, \neg W_{[2,2]}$

(no Wumpus in [3,1], [2,2])

- Add all them to KB



A: Agent; B: Breeze; G: Glitter; S: Stench

OK: safe square; W: Wumpus; P: pit; BGS: glitter, bag of gold

## Example: Exploring the Wumpus World

- KB initially contains:

$\neg P_{[1,1]}, \neg W_{[1,1]}, OK_{[1,1]}$

$B_{[2,1]} \leftrightarrow (P_{[1,1]} \vee P_{[2,2]} \vee P_{[3,1]})$

$S_{[2,1]} \leftrightarrow (W_{[1,1]} \vee W_{[2,2]} \vee W_{[3,1]})$

...

- Agent moves to [2,1]

- perceives a breeze:  $B_{[2,1]}$

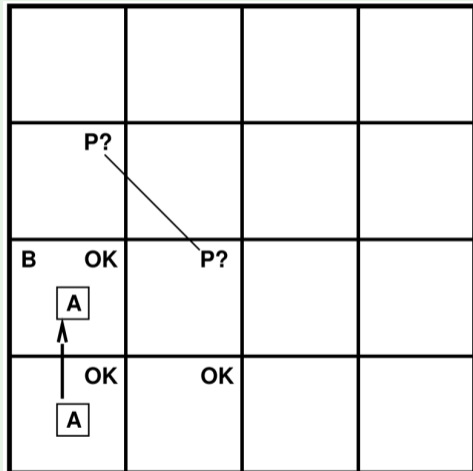
⇒  $(P_{[3,1]} \vee P_{[2,2]})$  (pit in [3,1] or [2,2])

- perceives no stench  $\neg S_{[2,1]}$

⇒  $\neg W_{[3,1]}, \neg W_{[2,2]}$

(no Wumpus in [3,1], [2,2])

- Add all them to KB



A: Agent; B: Breeze; G: Glitter; S: Stench

OK: safe square; W: Wumpus; P: pit; BGS: glitter, bag of gold

# Example: Exploring the Wumpus World

- KB initially contains:

$\neg P_{[1,1]}, \neg W_{[1,1]}, OK_{[1,1]}$

$(P_{[3,1]} \vee P_{[2,2]}), \neg W_{[3,1]}, \neg W_{[2,2]}$

$B_{[1,2]} \leftrightarrow (P_{[1,1]} \vee P_{[2,2]} \vee P_{[1,3]})$

$S_{[1,2]} \leftrightarrow (W_{[1,1]} \vee W_{[2,2]} \vee W_{[1,3]})$

$OK_{[2,2]} \leftrightarrow (\neg W_{[2,2]} \wedge \neg P_{[2,2]})$

- Agent moves to [1,1]-[1,2]

- perceives no breeze:  $\neg B_{[1,2]}$

$\Rightarrow \neg P_{[2,2]}, \neg P_{[1,3]}$  (no pit in [2,2], [1,3])

$\Rightarrow P_{[3,1]}$  (pit in [3,1])

- perceives a stench:  $S_{[1,2]}$

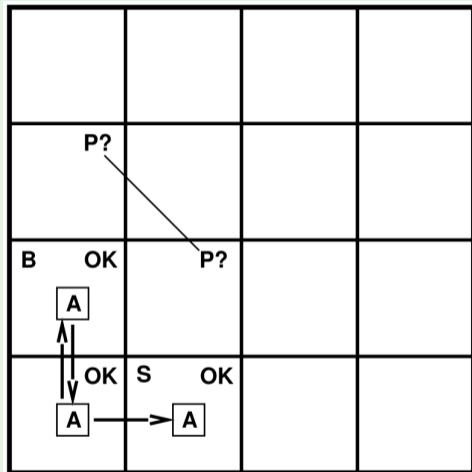
$\Rightarrow W_{[1,3]}$  (Wumpus in [1,3]!)

$\Rightarrow OK_{[2,2]}$  ([2,2] OK)

- Add all them to KB

A: Agent; B: Breeze; G: Glitter; S: Stench

OK: safe square; W: Wumpus; P: pit; BGS: glitter, bag of gold





## Example: Exploring the Wumpus World

- KB initially contains:

$\neg P_{[1,1]}, \neg W_{[1,1]}, OK_{[1,1]}$

$(P_{[3,1]} \vee P_{[2,2]}), \neg W_{[3,1]}, \neg W_{[2,2]}$

$B_{[1,2]} \leftrightarrow (P_{[1,1]} \vee P_{[2,2]} \vee P_{[1,3]})$

$S_{[1,2]} \leftrightarrow (W_{[1,1]} \vee W_{[2,2]} \vee W_{[1,3]})$

$OK_{[2,2]} \leftrightarrow (\neg W_{[2,2]} \wedge \neg P_{[2,2]})$

- Agent moves to [1,1]-[1,2]

- perceives no breeze:  $\neg B_{[1,2]}$

$\Rightarrow \neg P_{[2,2]}, \neg P_{[1,3]}$  (no pit in [2,2], [1,3])

$\Rightarrow P_{[3,1]}$  (pit in [3,1])

- perceives a stench:  $S_{[1,2]}$

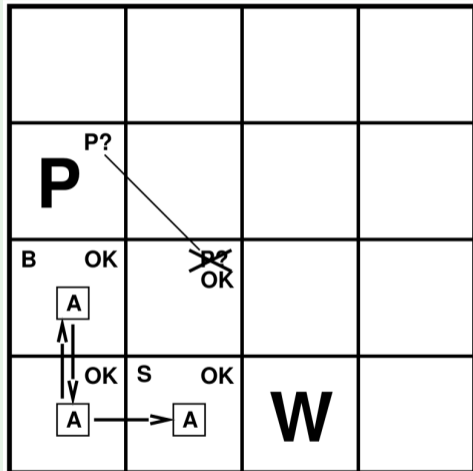
$\Rightarrow W_{[1,3]}$  (Wumpus in [1,3]!)

$\Rightarrow OK_{[2,2]}$  ([2,2] OK)

- Add all them to KB

A: Agent; B: Breeze; G: Glitter; S: Stench

OK: safe square; W: Wumpus; P: pit; BGS: glitter, bag of gold



## Example: Exploring the Wumpus World

- KB initially contains:

$$B_{[2,2]} \leftrightarrow (P_{[2,1]} \vee P_{[3,2]} \vee P_{[2,3]} \vee P_{[1,2]})$$

$$S_{[2,2]} \leftrightarrow (W_{[2,1]} \vee W_{[3,2]} \vee W_{[2,3]} \vee W_{[1,2]})$$

$$OK_{[3,2]} \leftrightarrow (\neg W_{[3,2]} \wedge \neg P_{[3,2]})$$

$$OK_{[2,3]} \leftrightarrow (\neg W_{[2,3]} \wedge \neg P_{[2,3]})$$

- Agent moves to [2,2]

- perceives no breeze:  $\neg B_{[2,2]}$

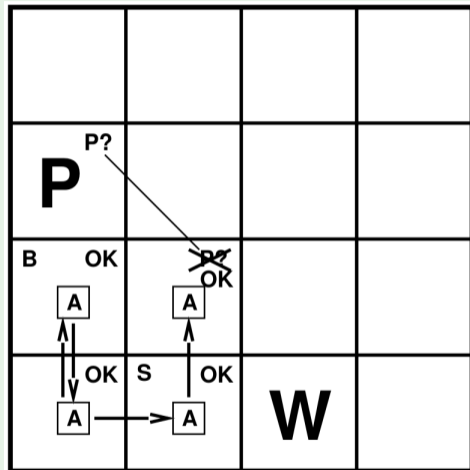
⇒  $\neg P_{[3,2]}, \neg P_{[2,3]}$  (no pit in [3,2], [2,3])

- perceives no stench:  $\neg S_{[2,2]}$

⇒  $\neg W_{[3,2]}, \neg W_{[2,3]}$  (no Wumpus in [3,2], [2,3])

⇒  $OK_{[3,2]}, OK_{[2,3]}$ , ([3,2] and [2,3] OK)

- Add all them to KB



A: Agent; B: Breeze; G: Glitter; S: Stench

OK: safe square; W: Wumpus; P: pit; BGS: glitter, bag of gold

## Example: Exploring the Wumpus World

- KB initially contains:

$$B_{[2,2]} \leftrightarrow (P_{[2,1]} \vee P_{[3,2]} \vee P_{[2,3]} \vee P_{[1,2]})$$

$$S_{[2,2]} \leftrightarrow (W_{[2,1]} \vee W_{[3,2]} \vee W_{[2,3]} \vee W_{[1,2]})$$

$$OK_{[3,2]} \leftrightarrow (\neg W_{[3,2]} \wedge \neg P_{[3,2]})$$

$$OK_{[2,3]} \leftrightarrow (\neg W_{[2,3]} \wedge \neg P_{[2,3]})$$

- Agent moves to [2,2]

- perceives no breeze:  $\neg B_{[2,2]}$

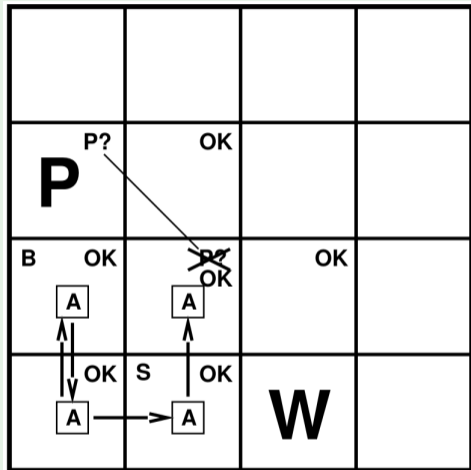
⇒  $\neg P_{[3,2]}, \neg P_{[2,3]}$  (no pit in [3,2], [2,3])

- perceives no stench:  $\neg S_{[2,2]}$

⇒  $\neg W_{[3,2]}, \neg W_{[2,3]}$  (no Wumpus in [3,2], [2,3])

⇒  $OK_{[3,2]}, OK_{[2,3]}$ , ([3,2] and [2,3] OK)

- Add all them to KB



A: Agent; B: Breeze; G: Glitter; S: Stench

OK: safe square; W: Wumpus; P: pit; BGS: glitter, bag of gold

# Example: Exploring the Wumpus World

- KB initially contains:

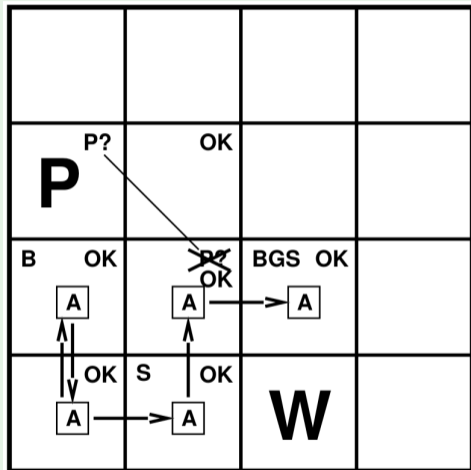
$G_{[2,3]} \leftrightarrow BGS_{[2,3]}$

- Agent moves to [2,3]

- perceives a glitter:  $G_{[2,3]}$

⇒  $BGS_{[2,3]}$  (bag of gold!)

- Add it them to KB



**A**: Agent; **B**: Breeze; **G**: Glitter; **S**: Stench

**OK**: safe square; **W**: Wumpus; **P**: pit; **BGS**: glitter, bag of gold

## Exercise

Consider the previous example.

- 1 Convert all formulas from KB into CNF
- 2 Execute all steps in the example as resolution calls
- 3 Execute all steps in the example as DPLL calls