Fundamentals of Artificial Intelligence

Laboratory

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Exercise 13.1

- Given the full joint distribution shown in the figure below, calculate the following:

  a) **P**(toothache)
  b) **P**(Cavity)
  c) **P**(Toothache | cavity)
  d) **P**(Cavity | toothache ∨ catch)

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

Figure 13.3 A full joint distribution for the $Toothache$, $Cavity$, $Catch$ world.

Exercise 13.1

The main point of this exercise is to understand the various notations of bold versus non-bold P, and uppercase versus lowercase variable names. The rest is easy, involving a small matter of addition.

a. This asks for the probability that Toothache is true.

   $P(toothache) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2$

b. This asks for the vector of probability values for the random variable Cavity.

   It has two values, which we list in the order ⟨true, false⟩.

   First add up $0.108 + 0.012 + 0.072 + 0.008 = 0.2$. Then we have $\mathbf{P}(Cavity) = \langle 0.2, 0.8 \rangle$.

c. This asks for the vector of probability values for Toothache, given that Cavity is true.

   $\mathbf{P}(Toothache \mid cavity) = \langle (0.108 + 0.012) / 0.2, (0.072 + 0.008) / 0.2 \rangle = \langle 0.6, 0.4 \rangle$

d. This asks for the vector of probability values for Cavity, given that either Toothache or Catch is true.

   First compute $P(toothache \vee catch) = 0.108 + 0.012 + 0.016 + 0.064 + 0.072 + 0.144 = 0.416$.

   Then $\mathbf{P}(Cavity \mid toothache \vee catch) = \langle (0.108 + 0.012 + 0.072) / 0.416, (0.016 + 0.064 + 0.144) / 0.416 \rangle$

   $= \langle 0.4615, 0.5384 \rangle$

Exercise 13.2

Assume the following facts are known from medicine literature:

- 3 persons over 1000 suffer of dengue
- 5% of persons have vomit
- One person with dengue has vomit with probability 0.8

Given that a person has vomit, compute the probability of having dengue.

Exercise 13.2

Assume the following facts are known from medicine literature:

- 3 persons over 1000 suffer of dengue
- 5% of persons have vomit
- One person with dengue has vomit with probability 0.8

Given that a person has vomit, compute the probability of having dengue.

P(dengue) = 0.003

P(vomit) = 0.05

P(vomit|dengue) = 0.8

P(dengue|vomit) = (P(vomit|dengue) * P(dengue)) / P(vomit) = (0.8 * 0.003) / 0.05 = 0.048

Exercise 13.3

- Consider two medical tests, A and B, for a virus.
- Test A is 95% effective at recognizing the virus when it is present, but has a 10% false positive rate (indicating that the virus is present, when it is not).
- Test B is 90% effective at recognizing the virus, but has a 5% false positive rate.
- The two tests use independent methods of identifying the virus.
- The virus is carried by 1% of all people.
- Say that a person is tested for the virus using only one of the tests, and that test comes back positive for carrying the virus.
- Which test returning positive is more indicative of someone really carrying the virus? Justify your answer mathematically.

Exercise 13.3

Let V be the statement that the patient has the virus, and A and B the statements that the medical tests A and B returned positive, respectively.

The problem statement gives:

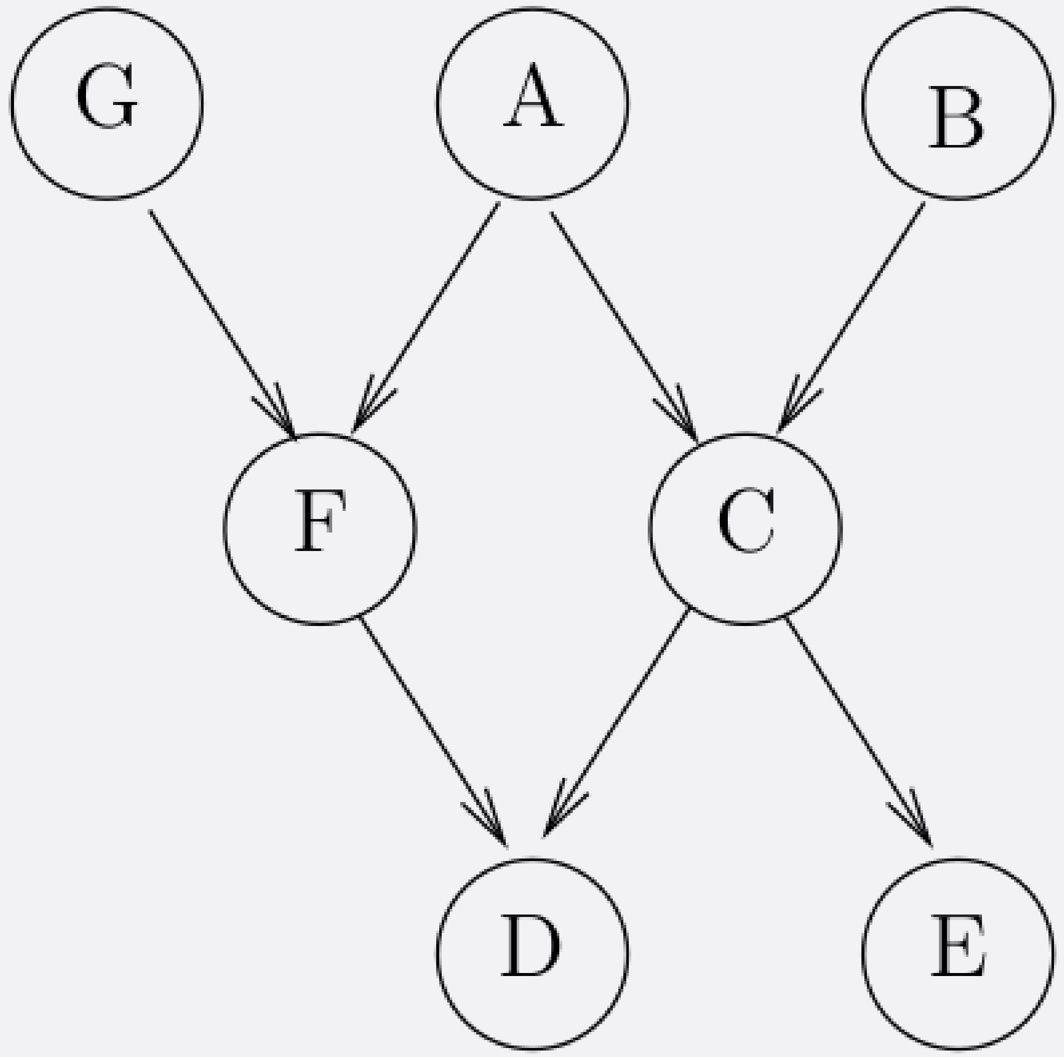
$P(V) = 0.01$
$P(A|V) = 0.95$
$P(A|\neg V) = 0.10$
$P(B|V) = 0.90$
$P(B|\neg V) = 0.05$

The test whose positive result is more indicative of the virus being present is the one whose posterior probability, $P(V|A)$ or $P(V|B)$ is largest.

One can compute these probabilities directly from the information given, finding that
$P(V|A) = (P(A|V)*P(V))/P(A) = (0.95 * 0.01) / (((0.95 * 0.01) + (0.10 * 0.99)) = 0.0876$
$P(V|B) = (P(B|V)*P(V))/P(B) = (0.90 * 0.01) / (((0.90 * 0.01) + (0.05 * 0.99)) = 0.1538$
so B is more indicative.

Exercise 13.4

Consider the following DAG of a Bayesian network.



For each of the following facts, say if it is true or false.

(1) P(C|ABF) = P(C|AB)
True, due to local semantics.

(2) P(C|ABD) = P(C|AB)
False

(3) P(C|DEF) = P(C|DE)
False

(4) P(C|ABDEFG) = P(C|ABDEF)
True, due to Markov blanket rule.

Exercise 13.5

Given the random propositional variables A, B, C and their joint probability distribution $\mathbf{P}(\mathbf{A}, \mathbf{B}, \mathbf{C})$ described as follows:

A	B	C	$\mathbf{P}(\mathbf{A}, \mathbf{B}, \mathbf{C})$
T	T	T	0.072
T	T	F	0.036
T	F	T	0.256
T	F	F	0.192
F	T	T	0.008
F	T	F	0.084
F	F	T	0.064
F	F	F	0.288

(a) Using marginalization, compute the probability $P(B)$

(b) Using normalization, compute the probability $P(A|B, C)$

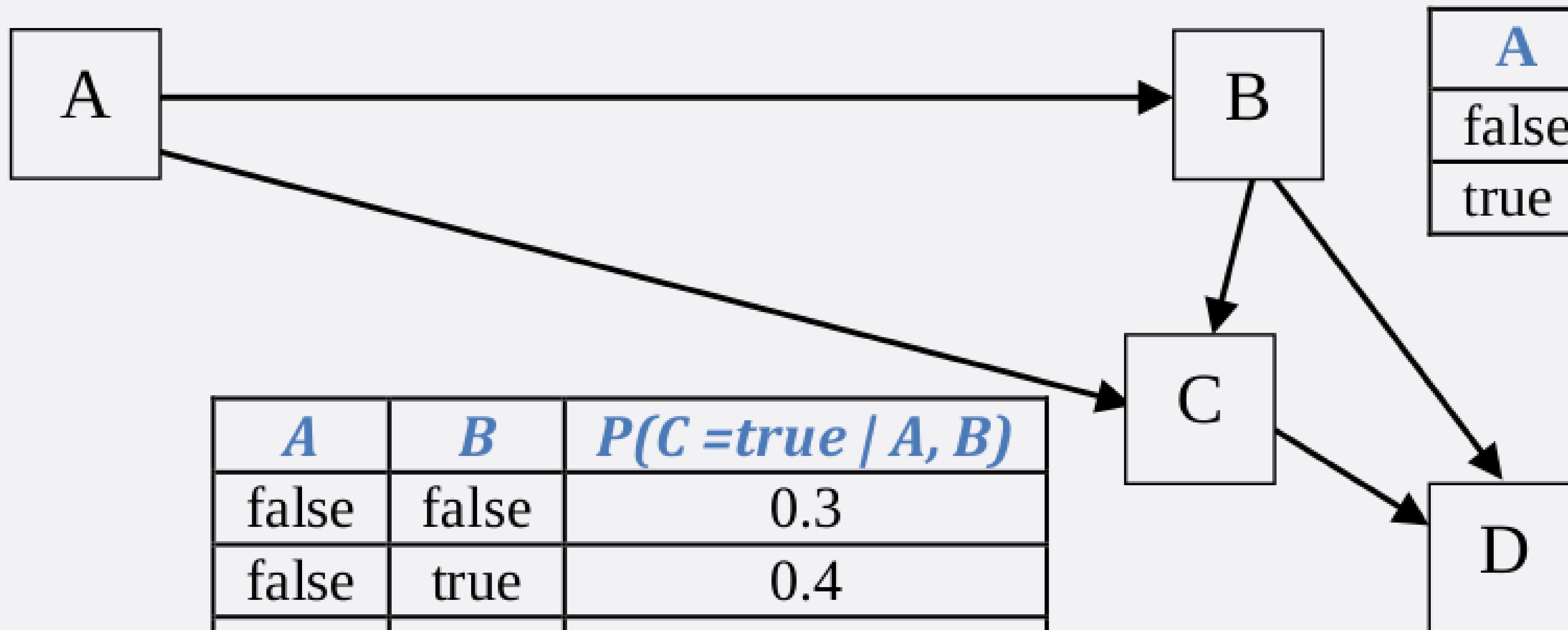
P(B) = P(A,B,C) + P(A,B,¬C) + P(¬A,B,C) + P(¬A,B,¬C) = 0.072 + 0.036 + 0.008 + 0.084 = 0.2

P(A|B,C) = α * P(A,B,C) = (1 / (P(A,B,C) + P(¬A,B,C))) * P(A,B,C) = (1 / (0.072 + 0.008)) * 0.072 = 0.9

Exercise 13.6

- Consider the following Bayesian Network, where variables **A-D** are all Boolean-valued:

P(A=true) = 0.7



A	P(B =true | A)
false	0.1
true	0.8

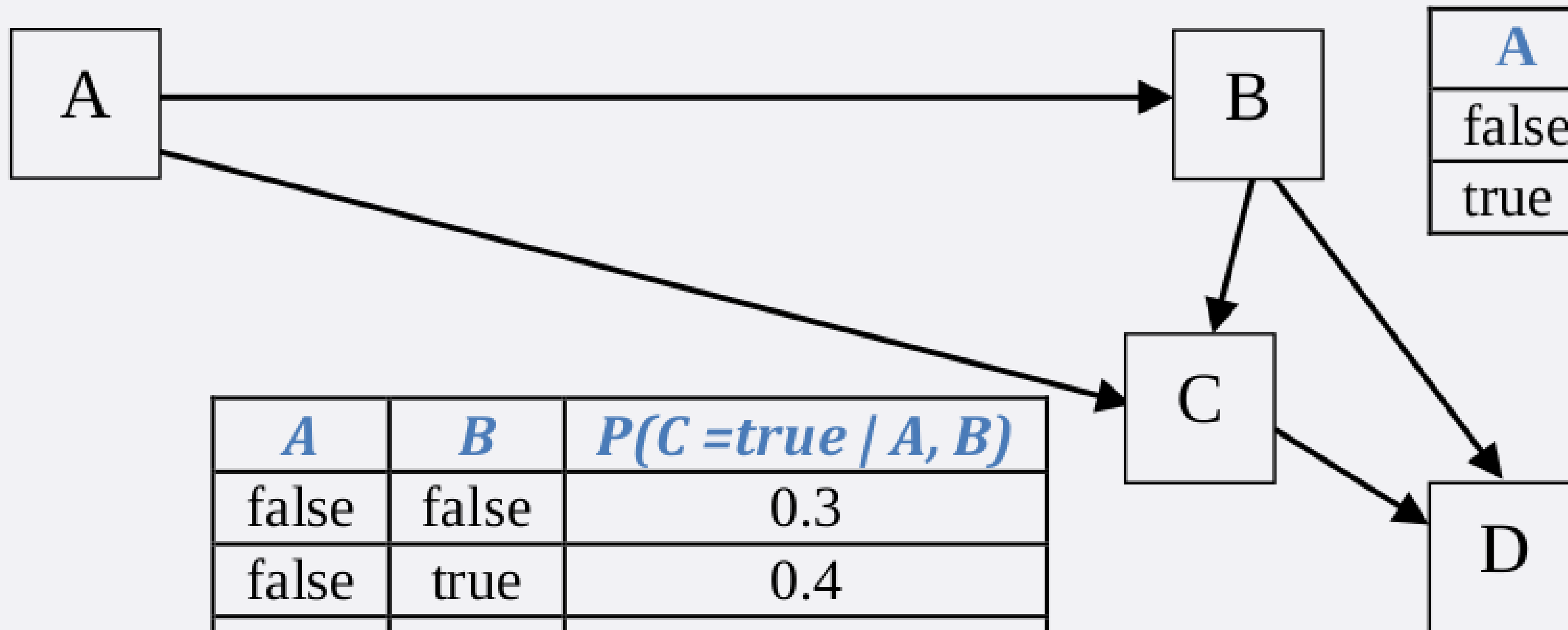
A	*B*	*P(C =true | A, B)*
false	false	0.3
false	true	0.4
true	false	0.5
true	true	0.6

B	*C*	*P(D =true | B,C)*
false	false	0.9
false	true	0.3
true	false	0.4
true	true	0.2

- Show your work for the following calculations.
  1. Compute *P*(A=true and B=true and C=true and D=true).
  2. Compute *P*(B=false and C=false and D=false).
  3. Compute *P*(A=true | B=true and C=true and D=false).
  4. Compute *P*(D=false | A=true and B=true and C=true).
     (Do this one without employing the Markov Blanket property, i.e., algebraically or numerically confirm that knowing A=true does not change the results from computing *P*(D=false | B=true and C=true).
  5. Compute *P*((A=true and D=true) or (B=true and C=true)).

Exercise 13.6

P(A=true) = 0.7



A	P(B =true | A)
false	0.1
true	0.8

A	*B*	*P(C =true | A, B)*
false	false	0.3
false	true	0.4
true	false	0.5
true	true	0.6

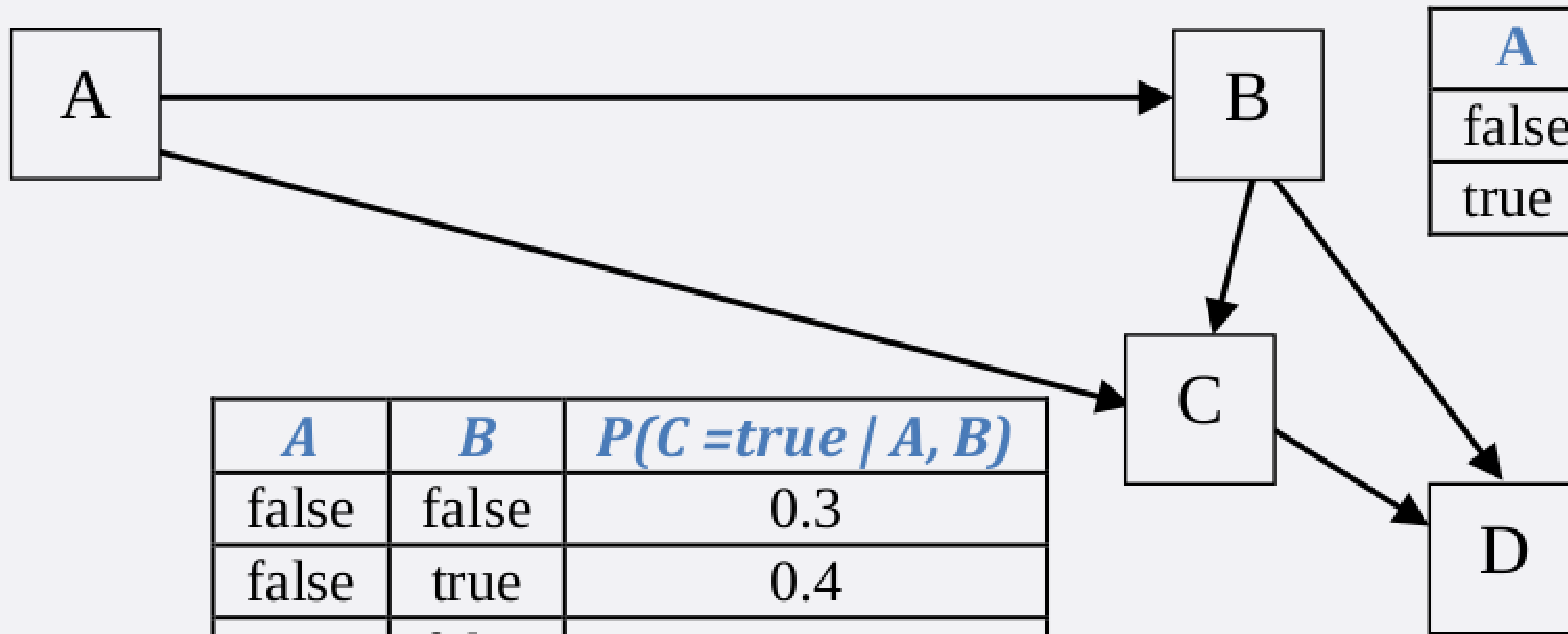
B	*C*	*P(D =true | B,C)*
false	false	0.9
false	true	0.3
true	false	0.4
true	true	0.2

1. Compute *P*(A=true and B=true and C=true and D=true).

P(A,B,C,D) = P(A) P(B|A) P(C|A,B) P(D|B,C) = 0.7 * 0.8 * 0.6 * 0.2 = 0.0672

Exercise 13.6

P(A=true) = 0.7



A	P(B =true | A)
false	0.1
true	0.8

A	*B*	*P(C =true | A, B)*
false	false	0.3
false	true	0.4
true	false	0.5
true	true	0.6

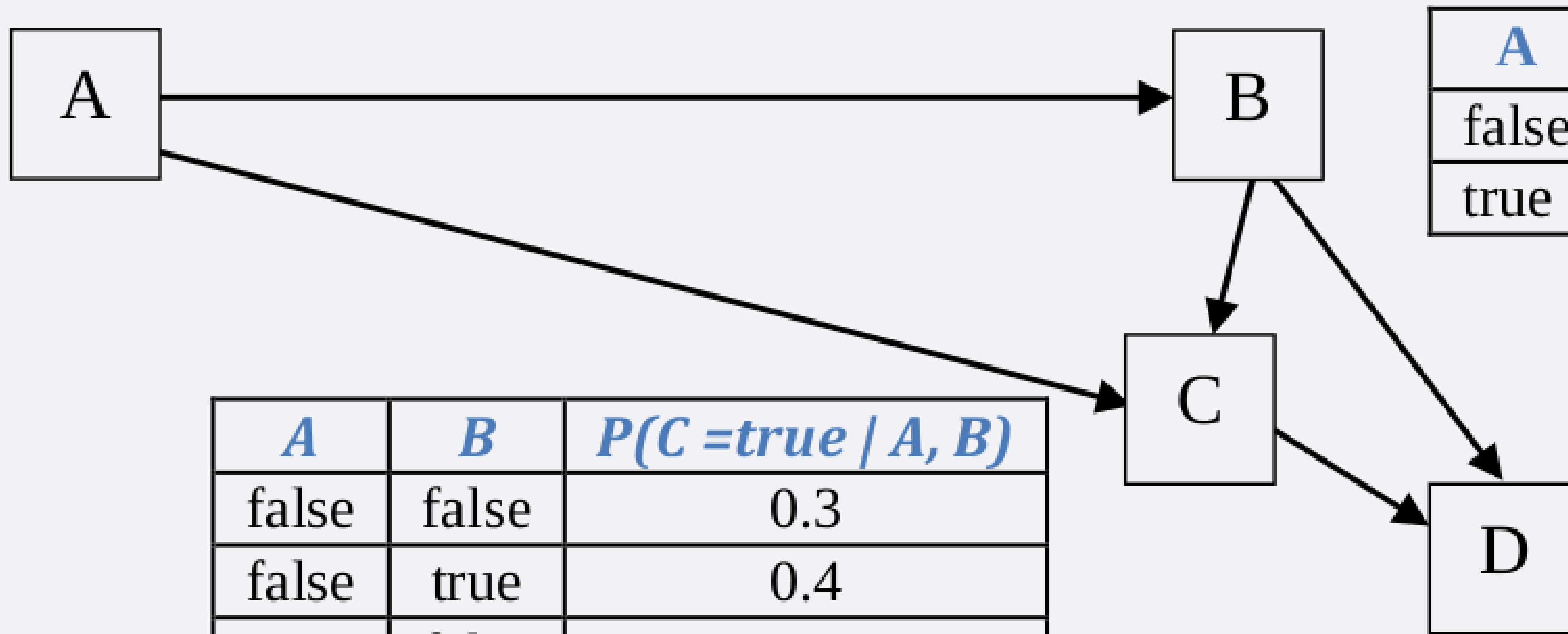
B	*C*	*P(D =true | B,C)*
false	false	0.9
false	true	0.3
true	false	0.4
true	true	0.2

2. Compute *P*(B=false and C=false and D=false).

$P(\neg B,\neg C,\neg D) = P(A,\neg B,\neg C,\neg D) + P(\neg A,\neg B,\neg C,\neg D)$
$P(A,\neg B,\neg C,\neg D) = P(A)\ P(\neg B|A)\ P(\neg C|A,\neg B)\ P(\neg D|\neg B,\neg C) = 0.7 * 0.2 * 0.5 * 0.1 = 0.007$
$P(\neg A,\neg B,\neg C,\neg D) = P(\neg A)\ P(\neg B|\neg A)\ P(\neg C|\neg A,\neg B)\ P(\neg D|\neg B,\neg C) = 0.3 * 0.9 * 0.7 * 0.1 = 0.0189$
$P(\neg B,\neg C,\neg D) = 0.007 + 0.0189 = 0.0259$

Exercise 13.6

P(A=true) = 0.7



A	P(B =true | A)
false	0.1
true	0.8

A	*B*	*P(C =true | A, B)*
false	false	0.3
false	true	0.4
true	false	0.5
true	true	0.6

B	*C*	*P(D =true | B,C)*
false	false	0.9
false	true	0.3
true	false	0.4
true	true	0.2

3. Compute *P*(A=true | B=true and C=true and D=false).

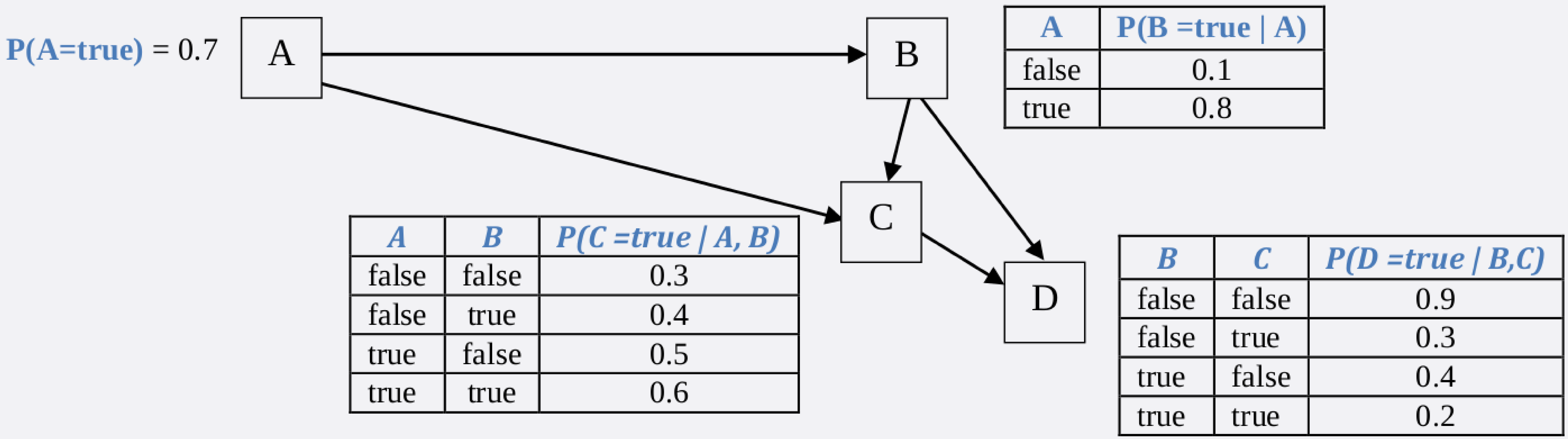
P(A|B,C,¬D) = P(A,B,C,¬D) / P(B,C,¬D)

P(A,B,C,¬D) = P(A) P(B|A) P(C|A,B) P(¬D|B,C) = 0.7 * 0.8 * 0.6 * 0.8 = 0.2688

P(B,C,¬D) = P(A,B,C,¬D) + P(¬A,B,C,¬D)
P(¬A,B,C,¬D) = P(¬A) P(B|¬A) P(C|¬A,B) P(D|B,C) = 0.3 * 0.1 * 0.4 * 0.8 = 0.0096
P(B,C,¬D) = 0.2688 + 0.0096 = 0.2784

P(A|B,C,¬D) = 0.2688 / 0.2784 = 0.9655

Exercise 13.6



A	P(B =true | A)
false	0.1
true	0.8

A	*B*	*P(C =true | A, B)*
false	false	0.3
false	true	0.4
true	false	0.5
true	true	0.6

B	*C*	*P(D =true | B,C)*
false	false	0.9
false	true	0.3
true	false	0.4
true	true	0.2

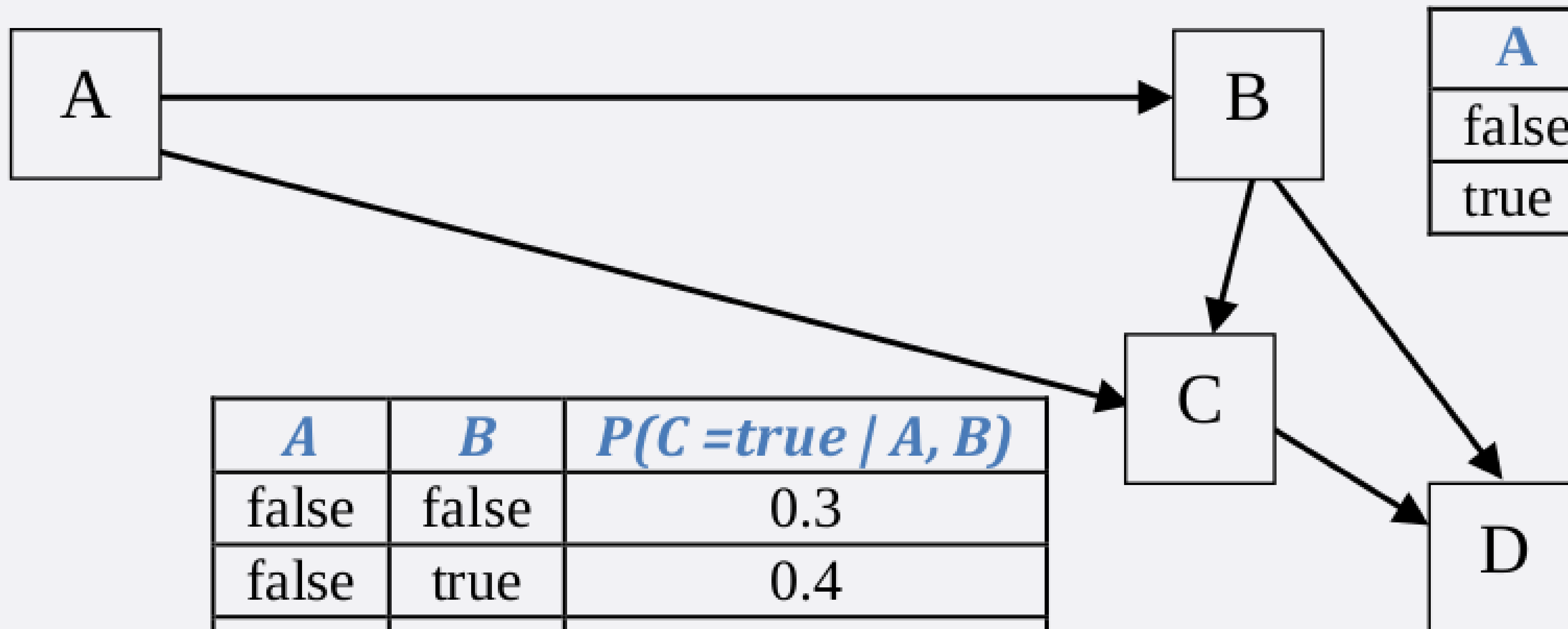
3. Compute *P*(A=true | B=true and C=true and D=false).

You could have also observed that due to the Markov Blanket Property, P(A|B,C,¬D) = P(A|B,C) which would have expanded into

P(A|B,C) = P(A,B,C) / P(B,C)
P(A,B,C) = P(A) P(B|A) P(C|A,B) = 0.7 * 0.8 * 0.6 = 0.336
P(B,C) = P(A,B,C) + P(¬A,B,C)
P(¬A,B,C) = 0.3 * 0.1 * 0.4 = 0.012
P(B,C) = 0.336 + 0.012 = 0.348
P(A|B,C) = 0.336 / 0.348 = 0.9655

Exercise 13.6

P(A=true) = 0.7



A	P(B =true | A)
false	0.1
true	0.8

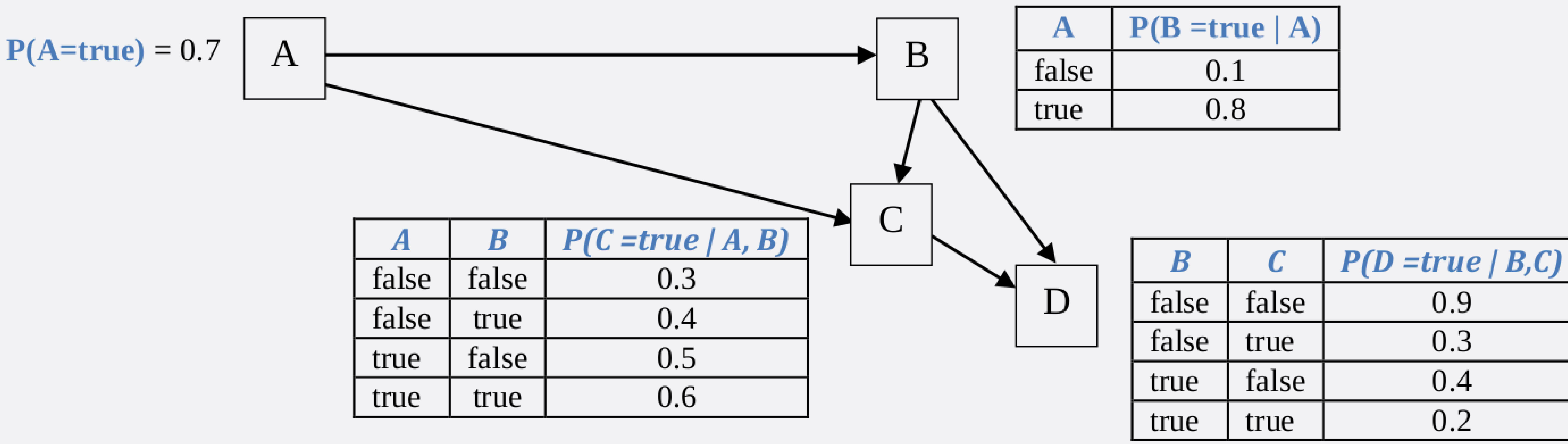
A	*B*	*P(C =true | A, B)*
false	false	0.3
false	true	0.4
true	false	0.5
true	true	0.6

B	*C*	*P(D =true | B,C)*
false	false	0.9
false	true	0.3
true	false	0.4
true	true	0.2

4. Compute *P*(D=false | A=true and B=true and C=true).
(Do this one without employing the Markov Blanket property, i.e., algebraically or numerically confirm that knowing A=true does not change the results from computing *P*(D=false | B=true and C=true).

P(¬D|A,B,C) = P(A,B,C,¬D) / P(A,B,C)
P(A,B,C,¬D) = 0.2688
P(A,B,C) = 0.336
P(¬D|A,B,C) = 0.2688 / 0.336 = 0.8

Exercise 13.6



A	P(B =true | A)
false	0.1
true	0.8

A	*B*	*P(C =true | A, B)*
false	false	0.3
false	true	0.4
true	false	0.5
true	true	0.6

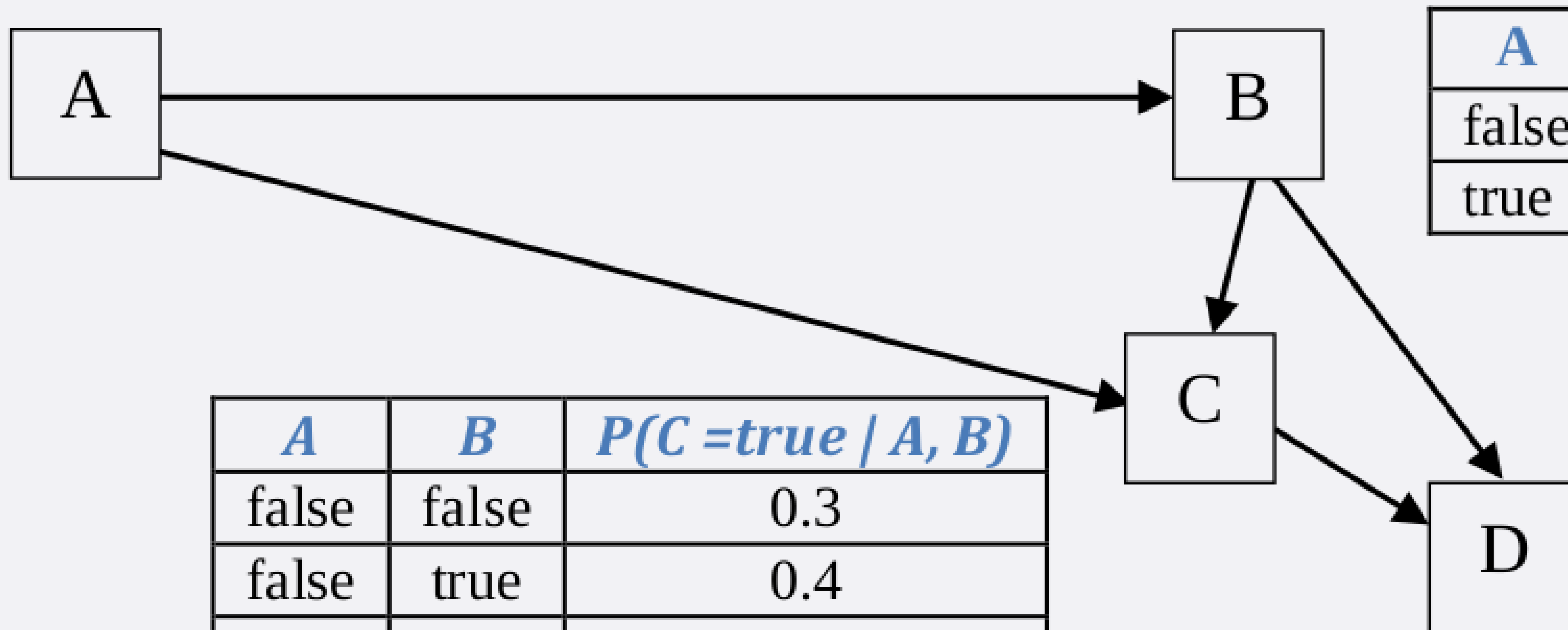
B	*C*	*P(D =true | B,C)*
false	false	0.9
false	true	0.3
true	false	0.4
true	true	0.2

4. Compute *P*(D=false | A=true and B=true and C=true).
   (Do this one without employing the Markov Blanket property, i.e., algebraically or numerically confirm that knowing A=true does not change the results from computing *P*(D=false | B=true and C=true).

$P(\neg D|B,C) = P(B,C,\neg D) / P(B,C)$
$P(B,C,\neg D) = 0.2784$
$P(B,C) = 0.348$
$P(\neg D|B,C) = 0.2784 / 0.348 = 0.8 = P(\neg D|A,B,C)$

Exercise 13.6

P(A=true) = 0.7



A	P(B =true | A)
false	0.1
true	0.8

A	*B*	*P(C =true | A, B)*
false	false	0.3
false	true	0.4
true	false	0.5
true	true	0.6

B	*C*	*P(D =true | B,C)*
false	false	0.9
false	true	0.3
true	false	0.4
true	true	0.2

5. Compute *P*((A=true and D=true) or (B=true and C=true)).

P((A,D)v(B,C)) = P(A,D) + P(B,C) - P(A,B,C,D)
P(A,D) = P(A,B,C,D) + P(A,¬B,C,D) + P(A,B,¬C,D) + P(A,¬B,¬C,D)
P(A,B,C,D) = 0.0672
P(A,¬B,C,D) = P(A) P(¬B|A) P(C|A,¬B) P(D|¬B,C) = 0.7 * 0.2 * 0.5 * 0.3 = 0.021
P(A,B,¬C,D) = 0.7 * 0.8 * 0.4 * 0.4 = 0.0896
P(A,¬B,¬C,D) = 0.7 * 0.2 * 0.5 * 0.9 = 0.063
P(A,D) = 0.0672 + 0.021 + 0.0896 + 0.063 = 0.2408
P(B,C) = 0.348
P((A,D)v(B,C)) = 0.2408 + 0.348 - 0.0672 = 0.5216

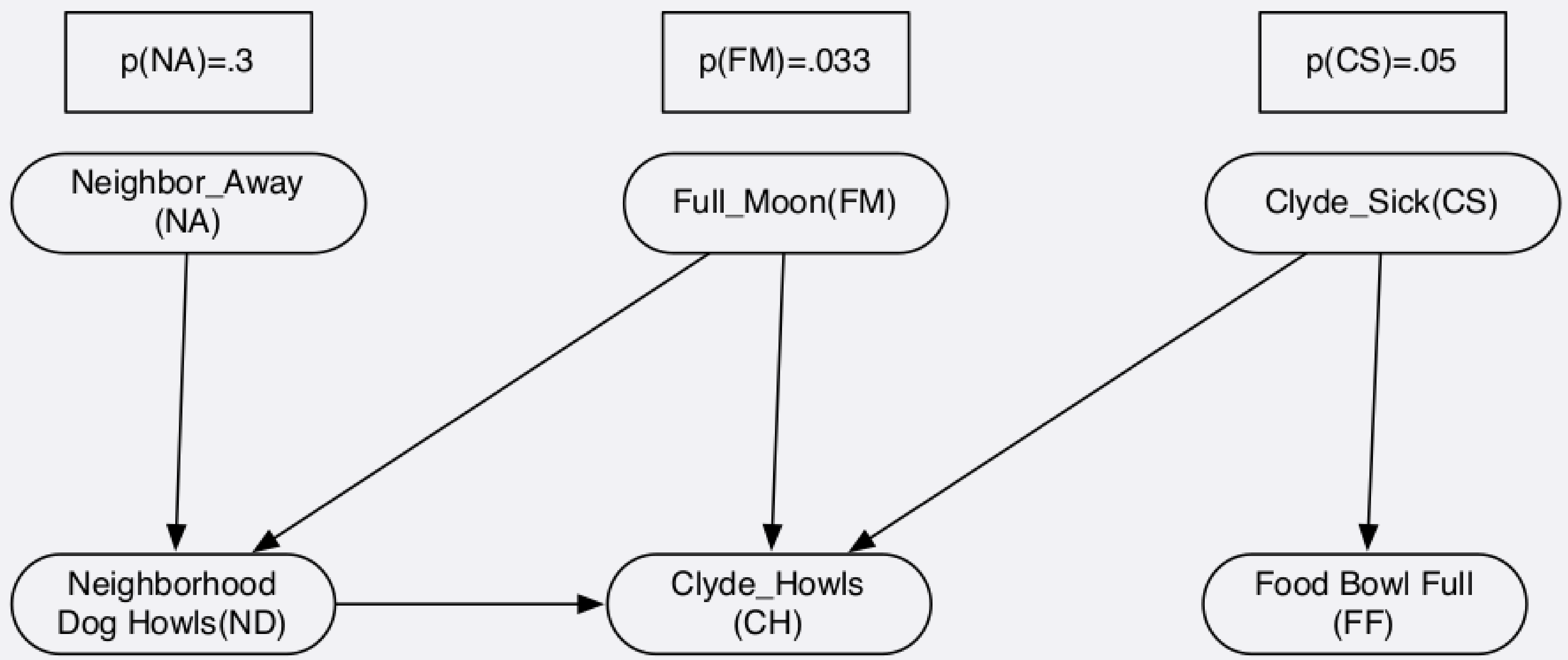
Exercise 13.7

- Your loyal dog Clyde has been howling for the last three hours and you want to decide whether or not to take him to the vet or just to put in ear plugs and go back to sleep. You know that Clyde often howls when there's a full moon, when he's genuinely sick, or occasionally when a particular neighborhood dog starts howling. That neighborhood dog sometimes howls at the full moon and sometimes howls when her owner isn't home, but is not affected by Clyde's howls. If Clyde's really sick he probably won't have eaten very much and should have a bunch of food left in his bowl - but he sometimes just isn't very hungry despite not being sick.

- Question - Create a Bayesian network for the scenario described above - use single letter names for each boolean variable (explaining what they mean of course). You can make up the exact numbers in the conditional probability tables (CPTs), but both the CPT values and the causal topology of the network should be reasonable. Briefly explain why you are setting up the topology as you are.

Exercise 13.7

In terms of the CPTs, you just needed to make sure that adding multiple causes increased the probability of a thing - for instance, if the neighbor's dog is howling AND Clyde is sick it should be more probable that Clyde is howling than if only a single one of those causes was true. Additionally, you should have included something around 1/28 for the prior probability of Full Moon (assuming you're on Earth, which is a reasonable assumption).



FM	NA	p(ND)
T	T	.8
T	F	.4
F	T	.5
F	F	0

FM	CS	ND	p(CH)
T	T	T	.99
T	T	F	.90
T	F	T	.65
T	F	F	.4
F	T	T	.75
F	T	F	.5
F	F	T	.2
F	F	F	0

CS	p(FF)
T	.6
F	.1