

Fundamentals of Artificial Intelligence

Laboratory

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Exercise 9.1

- Consider the following axioms:
 1. If something is intelligent, it has common sense
 2. Deep Blue does not have common sense
 3. (Conclusion) Deep Blue is not intelligent

Exercise 9.1

- Resolution

1. If something is intelligent, it has common sense
 $\forall x I(x) \Rightarrow H(x)$

2. Deep Blue does not have common sense
 $\neg H(D)$

3. (Conclusion) Deep Blue is not intelligent
 $\neg I(D)$

Exercise 9.1

- Resolution

1. If something is intelligent, it has common sense

$$\forall x I(x) \Rightarrow H(x)$$

$$\neg I(x) \vee H(x)$$

2. Deep Blue does not have common sense

$$\neg H(D)$$

3. (Conclusion) Deep Blue is not intelligent

$$\neg I(D)$$

$I(D)$ (negated conclusion)

Exercise 9.1

■ Resolution

1. If something is intelligent, it has common sense

$$\forall x I(x) \Rightarrow H(x)$$

$$\neg I(x) \vee H(x)$$

2. Deep Blue does not have common sense

$$\neg H(D)$$

3. (Conclusion) Deep Blue is not intelligent

$$\neg I(D)$$

$I(D)$ (negated conclusion)

4. [1, 2] $\neg I(D)$ {x/D}

Exercise 9.1

■ Resolution

1. If something is intelligent, it has common sense

$$\forall x I(x) \Rightarrow H(x)$$

$$\neg I(x) \vee H(x)$$

2. Deep Blue does not have common sense

$$\neg H(D)$$

3. (Conclusion) Deep Blue is not intelligent

$$\neg I(D)$$

$I(D)$ (negated conclusion)

4. [1, 2] $\neg I(D)$ {x/D}

5. [3, 4] \square

Exercise 9.2

- Consider the following axioms:
 1. $\text{Mother}(\text{Lulu}, \text{Fifi})$
 2. $\text{Alive}(\text{Lulu})$
 3. $\forall x \forall y \text{ Mother}(x,y) \Rightarrow \text{Parent}(x,y)$
 4. $\forall x \forall y (\text{Parent}(x,y) \wedge \text{Alive}(x)) \Rightarrow \text{Older}(x,y)$
 5. (Conclusion) $\text{Older}(\text{Lulu}, \text{Fifi})$

Exercise 9.2

- Consider the following axioms:

1. **Mother(Lulu, Fifi)**

2. **Alive(Lulu)**

3. $\forall x \forall y \text{Mother}(x,y) \Rightarrow \text{Parent}(x,y)$
 $\neg\text{Mother}(x,y) \vee \text{Parent}(x,y)$

4. $\forall x \forall y (\text{Parent}(x,y) \wedge \text{Alive}(x)) \Rightarrow \text{Older}(x,y)$
 $\neg\text{Parent}(x,y) \vee \neg\text{Alive}(x) \vee \text{Older}(x,y)$

5. (Conclusion) $\text{Older}(\text{Lulu}, \text{Fifi})$
 $\neg\text{Older}(\text{Lulu}, \text{Fifi})$

Exercise 9.2

- Consider the following axioms:

1. **Mother(Lulu, Fifi)**

2. **Alive(Lulu)**

3. $\forall x \forall y \text{ Mother}(x,y) \Rightarrow \text{Parent}(x,y)$
 $\neg\text{Mother}(x,y) \vee \text{Parent}(x,y)$

4. $\forall x \forall y (\text{Parent}(x,y) \wedge \text{Alive}(x)) \Rightarrow \text{Older}(x,y)$
 $\neg\text{Parent}(x,y) \vee \neg\text{Alive}(x) \vee \text{Older}(x,y)$

5. (Conclusion) $\text{Older}(\text{Lulu}, \text{Fifi})$
 $\neg\text{Older}(\text{Lulu}, \text{Fifi})$

6. [1, 3] $\text{Parent}(\text{Lulu}, \text{Fifi}) \quad \{\text{x/Lulu, y/Fifi}\}$

Exercise 9.2

- Consider the following axioms:

1. **Mother(Lulu, Fifi)**

2. **Alive(Lulu)**

3. $\forall x \forall y \text{Mother}(x,y) \Rightarrow \text{Parent}(x,y)$
 $\neg\text{Mother}(x,y) \vee \text{Parent}(x,y)$

4. $\forall x \forall y (\text{Parent}(x,y) \wedge \text{Alive}(x)) \Rightarrow \text{Older}(x,y)$
 $\neg\text{Parent}(x,y) \vee \neg\text{Alive}(x) \vee \text{Older}(x,y)$

5. (Conclusion) **Older(Lulu, Fifi)**
 $\neg\text{Older}(Lulu, Fifi)$

6. [1, 3] **Parent(Lulu,Fifi)** {x/Lulu, y/Fifi}

7. [4, 6] **$\neg\text{Alive}(Lulu) \vee \text{Older}(Lulu, Fifi)$** {x/Lulu, y/Fifi}

Exercise 9.2

- Consider the following axioms:

1. **Mother(Lulu, Fifi)**

2. **Alive(Lulu)**

3. $\forall x \forall y \text{Mother}(x,y) \Rightarrow \text{Parent}(x,y)$
 $\neg\text{Mother}(x,y) \vee \text{Parent}(x,y)$

4. $\forall x \forall y (\text{Parent}(x,y) \wedge \text{Alive}(x)) \Rightarrow \text{Older}(x,y)$
 $\neg\text{Parent}(x,y) \vee \neg\text{Alive}(x) \vee \text{Older}(x,y)$

5. (Conclusion) **Older(Lulu, Fifi)**
 $\neg\text{Older}(Lulu, Fifi)$

6. [1, 3] **Parent(Lulu,Fifi)** {x/Lulu, y/Fifi}

7. [4, 6] **$\neg\text{Alive}(Lulu) \vee \text{Older}(Lulu, Fifi)$** {x/Lulu, y/Fifi}

8. [2, 7] **Older(Lulu, Fifi)**

Exercise 9.2

- Consider the following axioms:

1. **Mother(Lulu, Fifi)**

2. **Alive(Lulu)**

3. $\forall x \forall y \text{Mother}(x,y) \Rightarrow \text{Parent}(x,y)$
 $\neg\text{Mother}(x,y) \vee \text{Parent}(x,y)$

4. $\forall x \forall y (\text{Parent}(x,y) \wedge \text{Alive}(x)) \Rightarrow \text{Older}(x,y)$
 $\neg\text{Parent}(x,y) \vee \neg\text{Alive}(x) \vee \text{Older}(x,y)$

5. (Conclusion) **Older(Lulu, Fifi)**
 $\neg\text{Older}(Lulu, Fifi)$

6. [1, 3] **Parent(Lulu,Fifi)** {x/Lulu, y/Fifi}

7. [4, 6] **$\neg\text{Alive}(Lulu) \vee \text{Older}(Lulu, Fifi)$** {x/Lulu, y/Fifi}

8. [2, 7] **Older(Lulu, Fifi)**

9. [5, 8] \square

Exercise 9.3

- Axioms:

1. Allergies(x) \Rightarrow Sneeze(x)
2. Cat(y) \wedge AllergicToCats(x) \Rightarrow Allergies(x)
3. Cat(Felix)
4. AllergicToCats(Mary)
5. (Conclusion) Sneeze(Mary)

Exercise 9.3

- Axioms:

1. Allergies(x) \Rightarrow Sneeze(x)

\neg Allergies(w) \vee Sneeze(w)

2. Cat(y) \wedge AllergicToCats(x) \Rightarrow Allergies(x)

\neg Cat(y) \vee \neg AllergicToCats(z) \vee Allergies(z)

3. **Cat(Felix)**

4. **AllergicToCats(Mary)**

5. (Conclusion) Sneeze(Mary)

\neg Sneeze(Mary)

Exercise 9.3

- Axioms:

1. $\neg \text{Allergies}(w) \vee \text{Sneeze}(w)$
2. $\neg \text{Cat}(y) \vee \neg \text{AllergicToCats}(z) \vee \text{Allergies}(z)$
3. $\text{Cat}(\text{Felix})$
4. $\text{AllergicToCats}(\text{Mary})$
5. (Conclusion) $\neg \text{Sneeze}(\text{Mary})$
6. [1, 2] $\neg \text{Cat}(y) \vee \text{Sneeze}(z) \vee \neg \text{AllergicToCats}(z)$ {w/z}

Exercise 9.3

- Axioms:

1. $\neg \text{Allergies}(w) \vee \text{Sneeze}(w)$
2. $\neg \text{Cat}(y) \vee \neg \text{AllergicToCats}(z) \vee \text{Allergies}(z)$
3. $\text{Cat}(\text{Felix})$
4. $\text{AllergicToCats}(\text{Mary})$
5. (Conclusion) $\neg \text{Sneeze}(\text{Mary})$
6. [1, 2] $\neg \text{Cat}(y) \vee \text{Sneeze}(z) \vee \neg \text{AllergicToCats}(z)$ {w/z}
7. [3, 6] $\text{Sneeze}(z) \vee \neg \text{AllergicToCats}(z)$ {y/Felix}

Exercise 9.3

- Axioms:

1. $\neg \text{Allergies}(w) \vee \text{Sneeze}(w)$
2. $\neg \text{Cat}(y) \vee \neg \text{AllergicToCats}(z) \vee \text{Allergies}(z)$
3. $\text{Cat}(\text{Felix})$
4. $\text{AllergicToCats}(\text{Mary})$
5. (Conclusion) $\neg \text{Sneeze}(\text{Mary})$
6. [1, 2] $\neg \text{Cat}(y) \vee \text{Sneeze}(z) \vee \neg \text{AllergicToCats}(z)$ {w/z}
7. [3, 6] $\text{Sneeze}(z) \vee \neg \text{AllergicToCats}(z)$ {y/Felix}
8. [4, 7] $\text{Sneeze}(\text{Mary})$ {z/Mary}

Exercise 9.3

- Axioms:

1. $\neg \text{Allergies}(w) \vee \text{Sneeze}(w)$
2. $\neg \text{Cat}(y) \vee \neg \text{AllergicToCats}(z) \vee \text{Allergies}(z)$
3. $\text{Cat}(\text{Felix})$
4. $\text{AllergicToCats}(\text{Mary})$
5. (Conclusion) $\neg \text{Sneeze}(\text{Mary})$
6. [1, 2] $\neg \text{Cat}(y) \vee \text{Sneeze}(z) \vee \neg \text{AllergicToCats}(z)$ {w/z}
7. [3, 6] $\text{Sneeze}(z) \vee \neg \text{AllergicToCats}(z)$ {y/Felix}
8. [4, 7] $\text{Sneeze}(\text{Mary})$ {z/Mary}
9. [5, 8] \square

Exercise 9.4

- Consider the following axioms:
 1. All hounds howl at night.
 2. Anyone who has any cats will not have any mice.
 3. Light sleepers do not have anything which howls at night.
 4. John has either a cat or a hound.
 5. (Conclusion) If John is a light sleeper, then John does not have any mice.

Exercise 9.4

- The conclusion can be proved using Resolution as shown in the next slides.
- The first step is to write each axiom as a well-formed formula in first-order predicate calculus. The clauses written for the above axioms are shown below, using LS(x) for “light sleeper”.

1. All hounds howl at night.

$$\forall x (\text{HOUND}(x) \Rightarrow \text{HOWL}(x))$$

2. Anyone who has any cats will not have any mice.

$$\forall x \forall y (\text{HAVE}(x,y) \wedge \text{CAT}(y) \Rightarrow \neg \exists z (\text{HAVE}(x,z) \wedge \text{MOUSE}(z)))$$

3. Light sleepers do not have anything which howls at night.

$$\forall x (\text{LS}(x) \Rightarrow \neg \exists y (\text{HAVE}(x,y) \wedge \text{HOWL}(y)))$$

4. John has either a cat or a hound.

$$\exists x (\text{HAVE}(\text{John},x) \wedge (\text{CAT}(x) \vee \text{HOUND}(x)))$$

5. (Conclusion) If John is a light sleeper, then John does not have any mice.

$$\text{LS}(\text{John}) \Rightarrow \neg \exists z (\text{HAVE}(\text{John},z) \wedge \text{MOUSE}(z))$$

Exercise 9.4

- The next step is to transform each well-formed formula into Prenex Normal Form, skolemize, and rewrite as clauses in conjunctive normal form.

- All hounds howl at night.

$$\forall x (\text{HOUND}(x) \Rightarrow \text{HOWL}(x))$$

$$\neg \text{HOUND}(x) \vee \text{HOWL}(x)$$

Exercise 9.4

- The next step is to transform each wff into Prenex Normal Form, skolemize, and rewrite as clauses in conjunctive normal form; these transformations are shown below.

2. Anyone who has any cats will not have any mice.

$$\forall x \forall y (\text{HAVE}(x,y) \wedge \text{CAT}(y) \Rightarrow \neg \exists z (\text{HAVE}(x,z) \wedge \text{MOUSE}(z)))$$

$$\forall x \forall y (\text{HAVE}(x,y) \wedge \text{CAT}(y) \Rightarrow \forall z \neg (\text{HAVE}(x,z) \wedge \text{MOUSE}(z)))$$

$$\forall x \forall y \forall z (\neg (\text{HAVE}(x,y) \wedge \text{CAT}(y)) \vee \neg (\text{HAVE}(x,z) \wedge \text{MOUSE}(z)))$$

$$\neg \text{HAVE}(x,y) \vee \neg \text{CAT}(y) \vee \neg \text{HAVE}(x,z) \vee \neg \text{MOUSE}(z)$$

Exercise 9.4

- The next step is to transform each wff into Prenex Normal Form, skolemize, and rewrite as clauses in conjunctive normal form; these transformations are shown below.

3. Light sleepers do not have anything which howls at night.

$$\forall x (LS(x) \Rightarrow \neg \exists y (\text{HAVE}(x,y) \wedge \text{HOWL}(y)))$$

$$\forall x (LS(x) \Rightarrow \forall y \neg(\text{HAVE}(x,y) \wedge \text{HOWL}(y)))$$

$$\forall x \forall y (LS(x) \Rightarrow \neg \text{HAVE}(x,y) \vee \neg \text{HOWL}(y))$$

$$\forall x \forall y (\neg LS(x) \vee \neg \text{HAVE}(x,y) \vee \neg \text{HOWL}(y))$$

$$\neg LS(x) \vee \neg \text{HAVE}(x,y) \vee \neg \text{HOWL}(y)$$

Exercise 9.4

- The next step is to transform each wff into Prenex Normal Form, skolemize, and rewrite as clauses in conjunctive normal form; these transformations are shown below.

4. John has either a cat or a hound.

$$\exists x (\text{HAVE}(\text{John},x) \wedge (\text{CAT}(x) \vee \text{HOUND}(x)))$$

$$\text{HAVE}(\text{John},a) \wedge (\text{CAT}(a) \vee \text{HOUND}(a))$$

Exercise 9.4

- The next step is to transform each wff into Prenex Normal Form, skolemize, and rewrite as clauses in conjunctive normal form; these transformations are shown below.

5. (Conclusion) If John is a light sleeper, then John does not have any mice.

$\text{LS}(\text{John}) \Rightarrow \neg \exists z (\text{HAVE}(\text{John}, z) \wedge \text{MOUSE}(z))$

$\neg[\text{LS}(\text{John}) \Rightarrow \neg \exists z (\text{HAVE}(\text{John}, z) \wedge \text{MOUSE}(z))]$ (negated conclusion)

$\neg[\neg \text{LS}(\text{John}) \vee \neg \exists z (\text{HAVE}(\text{John}, z) \wedge \text{MOUSE}(z))]$

$\text{LS}(\text{John}) \wedge \exists z (\text{HAVE}(\text{John}, z) \wedge \text{MOUSE}(z))$

$\text{LS}(\text{John}) \wedge \text{HAVE}(\text{John}, b) \wedge \text{MOUSE}(b)$

Exercise 9.4

- The set of CNF clauses for this problem is thus as follows:

1. $\neg\text{HOUND}(x) \vee \text{HOWL}(x)$

2. $\neg\text{HAVE}(x,y) \vee \neg\text{CAT}(y) \vee \neg\text{HAVE}(x,z) \vee \neg\text{MOUSE}(z)$

3. $\neg\text{LS}(x) \vee \neg\text{HAVE}(x,y) \vee \neg\text{HOWL}(y)$

4.

a. $\text{HAVE}(\text{John},a)$

b. $\text{CAT}(a) \vee \text{HOUND}(a)$

5.

a. $\text{LS}(\text{John})$

b. $\text{HAVE}(\text{John},b)$

c. $\text{MOUSE}(b)$

Exercise 9.4

- Now we proceed to prove the conclusion by resolution using the above clauses. Each result clause is numbered; the numbers of its parent clauses are shown to its left.
1. $\neg\text{HOUND}(x) \vee \text{HOWL}(x)$
 2. $\neg\text{HAVE}(x,y) \vee \neg\text{CAT}(y) \vee \neg\text{HAVE}(x,z) \vee \neg\text{MOUSE}(z)$
 3. $\neg\text{LS}(x) \vee \neg\text{HAVE}(x,y) \vee \neg\text{HOWL}(y)$
 4.
 - a. $\text{HAVE}(\text{John},a)$
 - b. $\text{CAT}(a) \vee \text{HOUND}(a)$
 5.
 - a. $\text{LS}(\text{John})$
 - b. $\text{HAVE}(\text{John},b)$
 - c. $\text{MOUSE}(b)$
 6. [1, 4(b)] $\text{CAT}(a) \vee \text{HOWL}(a)$

Exercise 9.4

- Now we proceed to prove the conclusion by resolution using the above clauses. Each result clause is numbered; the numbers of its parent clauses are shown to its left.

1. $\neg\text{HOUND}(x) \vee \text{HOWL}(x)$
2. $\neg\text{HAVE}(x,y) \vee \neg\text{CAT}(y) \vee \neg\text{HAVE}(x,z) \vee \neg\text{MOUSE}(z)$
3. $\neg\text{LS}(x) \vee \neg\text{HAVE}(x,y) \vee \neg\text{HOWL}(y)$
4.
 - a. $\text{HAVE}(\text{John},a)$
 - b. $\text{CAT}(a) \vee \text{HOUND}(a)$
5.
 - a. $\text{LS}(\text{John})$
 - b. $\text{HAVE}(\text{John},b)$
 - c. $\text{MOUSE}(b)$
6. [1, 4(b)] $\text{CAT}(a) \vee \text{HOWL}(a)$
7. [2, 5(c)] $\neg\text{HAVE}(x,y) \vee \neg\text{CAT}(y) \vee \neg\text{HAVE}(x,b)$

Exercise 9.4

- Now we proceed to prove the conclusion by resolution using the above clauses. Each result clause is numbered; the numbers of its parent clauses are shown to its left.

1. $\neg\text{HOUND}(x) \vee \text{HOWL}(x)$
2. $\neg\text{HAVE}(x,y) \vee \neg\text{CAT}(y) \vee \neg\text{HAVE}(x,z) \vee \neg\text{MOUSE}(z)$
3. $\neg\text{LS}(x) \vee \neg\text{HAVE}(x,y) \vee \neg\text{HOWL}(y)$
4.
 - a. $\text{HAVE}(\text{John},a)$
 - b. $\text{CAT}(a) \vee \text{HOUND}(a)$
5.
 - a. $\text{LS}(\text{John})$
 - b. $\text{HAVE}(\text{John},b)$
 - c. $\text{MOUSE}(b)$
6. [1, 4(b)] $\text{CAT}(a) \vee \text{HOWL}(a)$
7. [2, 5(c)] $\neg\text{HAVE}(x,y) \vee \neg\text{CAT}(y) \vee \neg\text{HAVE}(x,b)$
8. [7, 5(b)] $\neg\text{HAVE}(\text{John},y) \vee \neg\text{CAT}(y)$

Exercise 9.4

- Now we proceed to prove the conclusion by resolution using the above clauses. Each result clause is numbered; the numbers of its parent clauses are shown to its left.

1. $\neg\text{HOUND}(x) \vee \text{HOWL}(x)$
2. $\neg\text{HAVE}(x,y) \vee \neg\text{CAT}(y) \vee \neg\text{HAVE}(x,z) \vee \neg\text{MOUSE}(z)$
3. $\neg\text{LS}(x) \vee \neg\text{HAVE}(x,y) \vee \neg\text{HOWL}(y)$
4.
 - a. $\text{HAVE}(\text{John},a)$
 - b. $\text{CAT}(a) \vee \text{HOUND}(a)$
5.
 - a. $\text{LS}(\text{John})$
 - b. $\text{HAVE}(\text{John},b)$
 - c. $\text{MOUSE}(b)$
6. [1, 4(b)] $\text{CAT}(a) \vee \text{HOWL}(a)$
7. [2, 5(c)] $\neg\text{HAVE}(x,y) \vee \neg\text{CAT}(y) \vee \neg\text{HAVE}(x,b)$
8. [7, 5(b)] $\neg\text{HAVE}(\text{John},y) \vee \neg\text{CAT}(y)$
9. [6, 8] $\neg\text{HAVE}(\text{John},a) \vee \text{HOWL}(a)$

Exercise 9.4

- Now we proceed to prove the conclusion by resolution using the above clauses. Each result clause is numbered; the numbers of its parent clauses are shown to its left.

1. $\neg\text{HOUND}(x) \vee \text{HOWL}(x)$
2. $\neg\text{HAVE}(x,y) \vee \neg\text{CAT}(y) \vee \neg\text{HAVE}(x,z) \vee \neg\text{MOUSE}(z)$
3. $\neg\text{LS}(x) \vee \neg\text{HAVE}(x,y) \vee \neg\text{HOWL}(y)$
4.
 - a. $\text{HAVE}(\text{John},a)$
 - b. $\text{CAT}(a) \vee \text{HOUND}(a)$
5.
 - a. $\text{LS}(\text{John})$
 - b. $\text{HAVE}(\text{John},b)$
 - c. $\text{MOUSE}(b)$
6. [1, 4(b)] $\text{CAT}(a) \vee \text{HOWL}(a)$
7. [2, 5(c)] $\neg\text{HAVE}(x,y) \vee \neg\text{CAT}(y) \vee \neg\text{HAVE}(x,b)$
8. [7, 5(b)] $\neg\text{HAVE}(\text{John},y) \vee \neg\text{CAT}(y)$
9. [6, 8] $\neg\text{HAVE}(\text{John},a) \vee \text{HOWL}(a)$
10. [4(a), 9] $\text{HOWL}(a)$

Exercise 9.4

- Now we proceed to prove the conclusion by resolution using the above clauses. Each result clause is numbered; the numbers of its parent clauses are shown to its left.

1. $\neg\text{HOUND}(x) \vee \text{HOWL}(x)$
2. $\neg\text{HAVE}(x,y) \vee \neg\text{CAT}(y) \vee \neg\text{HAVE}(x,z) \vee \neg\text{MOUSE}(z)$
3. $\neg\text{LS}(x) \vee \neg\text{HAVE}(x,y) \vee \neg\text{HOWL}(y)$
4.
 - a. $\text{HAVE}(\text{John},a)$
 - b. $\text{CAT}(a) \vee \text{HOUND}(a)$
5.
 - a. $\text{LS}(\text{John})$
 - b. $\text{HAVE}(\text{John},b)$
 - c. $\text{MOUSE}(b)$
6. [1, 4(b)] $\text{CAT}(a) \vee \text{HOWL}(a)$
7. [2, 5(c)] $\neg\text{HAVE}(x,y) \vee \neg\text{CAT}(y) \vee \neg\text{HAVE}(x,b)$
8. [7, 5(b)] $\neg\text{HAVE}(\text{John},y) \vee \neg\text{CAT}(y)$
9. [6, 8] $\neg\text{HAVE}(\text{John},a) \vee \text{HOWL}(a)$
10. [4(a), 9] $\text{HOWL}(a)$
11. [3, 10] $\neg\text{LS}(x) \vee \neg\text{HAVE}(x,a)$

Exercise 9.4

- Now we proceed to prove the conclusion by resolution using the above clauses. Each result clause is numbered; the numbers of its parent clauses are shown to its left.

1. $\neg\text{HOUND}(x) \vee \text{HOWL}(x)$
2. $\neg\text{HAVE}(x,y) \vee \neg\text{CAT}(y) \vee \neg\text{HAVE}(x,z) \vee \neg\text{MOUSE}(z)$
3. $\neg\text{LS}(x) \vee \neg\text{HAVE}(x,y) \vee \neg\text{HOWL}(y)$
4.
 - a. $\text{HAVE}(\text{John},a)$
 - b. $\text{CAT}(a) \vee \text{HOUND}(a)$
5.
 - a. $\text{LS}(\text{John})$
 - b. $\text{HAVE}(\text{John},b)$
 - c. $\text{MOUSE}(b)$
6. [1, 4(b)] $\text{CAT}(a) \vee \text{HOWL}(a)$
7. [2, 5(c)] $\neg\text{HAVE}(x,y) \vee \neg\text{CAT}(y) \vee \neg\text{HAVE}(x,b)$
8. [7, 5(b)] $\neg\text{HAVE}(\text{John},y) \vee \neg\text{CAT}(y)$
9. [6, 8] $\neg\text{HAVE}(\text{John},a) \vee \text{HOWL}(a)$
10. [4(a), 9] $\text{HOWL}(a)$
11. [3, 10] $\neg\text{LS}(x) \vee \neg\text{HAVE}(x,a)$
12. [4(a), 11] $\neg\text{LS}(\text{John})$

Exercise 9.4

- Now we proceed to prove the conclusion by resolution using the above clauses. Each result clause is numbered; the numbers of its parent clauses are shown to its left.

1. $\neg\text{HOUND}(x) \vee \text{HOWL}(x)$
2. $\neg\text{HAVE}(x,y) \vee \neg\text{CAT}(y) \vee \neg\text{HAVE}(x,z) \vee \neg\text{MOUSE}(z)$
3. $\neg\text{LS}(x) \vee \neg\text{HAVE}(x,y) \vee \neg\text{HOWL}(y)$
4.
 - a. $\text{HAVE}(\text{John},a)$
 - b. $\text{CAT}(a) \vee \text{HOUND}(a)$
5.
 - a. $\text{LS}(\text{John})$
 - b. $\text{HAVE}(\text{John},b)$
 - c. $\text{MOUSE}(b)$
6. [1, 4(b)] $\text{CAT}(a) \vee \text{HOWL}(a)$
7. [2, 5(c)] $\neg\text{HAVE}(x,y) \vee \neg\text{CAT}(y) \vee \neg\text{HAVE}(x,b)$
8. [7, 5(b)] $\neg\text{HAVE}(\text{John},y) \vee \neg\text{CAT}(y)$
9. [6, 8] $\neg\text{HAVE}(\text{John},a) \vee \text{HOWL}(a)$
- 10.[4(a), 9] $\text{HOWL}(a)$
- 11.[3, 10] $\neg\text{LS}(x) \vee \neg\text{HAVE}(x,a)$
- 12.[4(a), 11] $\neg\text{LS}(\text{John})$
- 13.[5(a), 12] \square

Exercise 9.5

- Convert to First order Logic:

1. Marcus was a man.
2. Marcus was a Roman.
3. All men are people.
4. Caesar was a ruler.
5. All Romans were either loyal to Caesar or hated him (or both).
6. Everyone is loyal to someone.
7. People only try to assassinate rulers they are not loyal to.
8. Marcus tried to assassinate Caesar.

Exercise 9.5

- Convert to First order Logic:

1. Marcus was a man.

$\text{Man}(\text{Marcus})$

2. Marcus was a Roman.

$\text{Roman}(\text{Marcus})$

3. All men are people.

$\forall x \text{ Man}(x) \Rightarrow \text{Person}(x)$

4. Caesar was a ruler.

$\text{Ruler}(\text{Caesar})$

5. All Romans were either loyal to Caesar or hated him (or both).

$\forall x \text{ Roman}(x) \Rightarrow \text{Loyal}(x, \text{Caesar}) \vee \text{Hate}(x, \text{Caesar})$

6. Everyone is loyal to someone.

$\forall x \exists y \text{ Loyal}(x, y)$

7. People only try to assassinate rulers they are not loyal to.

$\forall x \forall y \text{ Person}(x) \wedge \text{Ruler}(y) \wedge \text{Tryassassin}(x, y) \Rightarrow \neg \text{Loyal}(x, y)$

8. Marcus tried to assassinate Caesar.

$\text{Tryassassin}(\text{Marcus}, \text{Caesar})$

Exercise 9.5

- Convert to Clausal Form

1. Marcus was a man.

Man(Marcus)

2. Marcus was a Roman.

Roman(Marcus)

3. All men are people.

$\neg\text{Man}(x) \vee \text{Person}(x)$

4. Caesar was a ruler.

Ruler(Caesar)

5. All Romans were either loyal to Caesar or hated him (or both).

$\neg\text{Roman}(x) \vee \text{Loyal}(x, \text{Caesar}) \vee \text{Hate}(x, \text{Caesar})$

6. Everyone is loyal to someone.

Loyal(x, F(x))

7. People only try to assassinate rulers they are not loyal to.

$\neg\text{Person}(x) \vee \neg\text{Ruler}(y) \vee \neg\text{Tryassassin}(x, y) \vee \neg\text{Loyal}(x, y)$

8. Marcus tried to assassinate Caesar.

Tryassassin(Marcus, Caesar)

Exercise 9.5

1. **Man(Marcus)**
2. **Roman(Marcus)**
3. **$\neg\text{Man}(x) \vee \text{Person}(x)$**
4. **Ruler(Caesar)**
5. **$\neg\text{Roman}(x) \vee \text{Loyal}(x, \text{Caesar}) \vee \text{Hate}(x, \text{Caesar})$**
6. **$\text{Loyal}(x, F(x))$**
7. **$\neg\text{Person}(x) \vee \neg\text{Ruler}(y) \vee \neg\text{Tryassassin}(x, y) \vee \neg\text{Loyal}(x, y)$**
8. **Tryassassin(Marcus, Caesar)**

9. Who hate Caesar
 $\neg\text{Hate}(x, \text{Caesar}) \vee \text{Ans}(x)$

Exercise 9.5

1. **Man(Marcus)**
2. **Roman(Marcus)**
3. **$\neg\text{Man}(x) \vee \text{Person}(x)$**
4. **Ruler(Caesar)**
5. **$\neg\text{Roman}(x) \vee \text{Loyal}(x, \text{Caesar}) \vee \text{Hate}(x, \text{Caesar})$**
6. **$\text{Loyal}(x, F(x))$**
7. **$\neg\text{Person}(x) \vee \neg\text{Ruler}(y) \vee \neg\text{Tryassassin}(x, y) \vee \neg\text{Loyal}(x, y)$**
8. **Tryassassin(Marcus, Caesar)**
9. **$\neg\text{Hate}(x, \text{Caesar}) \vee \text{Ans}(x)$**

10. **$R[9, 5] \quad \neg\text{Roman}(x) \vee \text{Loyal}(x, \text{Caesar}) \vee \text{Ans}(x)$**

Exercise 9.5

1. **Man(Marcus)**
 2. **Roman(Marcus)**
 3. **$\neg\text{Man}(x) \vee \text{Person}(x)$**
 4. **Ruler(Caesar)**
 5. **$\neg\text{Roman}(x) \vee \text{Loyal}(x, \text{Caesar}) \vee \text{Hate}(x, \text{Caesar})$**
 6. **$\text{Loyal}(x, F(x))$**
 7. **$\neg\text{Person}(x) \vee \neg\text{Ruler}(y) \vee \neg\text{Tryassassin}(x, y) \vee \neg\text{Loyal}(x, y)$**
 8. **Tryassassin(Marcus, Caesar)**
 9. **$\neg\text{Hate}(x, \text{Caesar}) \vee \text{Ans}(x)$**
-
10. R[9, 5] $\neg\text{Roman}(x) \vee \text{Loyal}(x, \text{Caesar}) \vee \text{Ans}(x)$

 11. R[10, 2] $\text{Loyal}(\text{Markus}, \text{Caesar}) \vee \text{Ans}(\text{Markus})$ {x/Marcus}

Exercise 9.5

1. **Man(Marcus)**
 2. **Roman(Marcus)**
 3. **$\neg\text{Man}(x) \vee \text{Person}(x)$**
 4. **Ruler(Caesar)**
 5. **$\neg\text{Roman}(x) \vee \text{Loyal}(x, \text{Caesar}) \vee \text{Hate}(x, \text{Caesar})$**
 6. **$\text{Loyal}(x, F(x))$**
 7. **$\neg\text{Person}(x) \vee \neg\text{Ruler}(y) \vee \neg\text{Tryassassin}(x, y) \vee \neg\text{Loyal}(x, y)$**
 8. **Tryassassin(Marcus, Caesar)**
 9. **$\neg\text{Hate}(x, \text{Caesar}) \vee \text{Ans}(x)$**
-
10. R[9, 5] $\neg\text{Roman}(x) \vee \text{Loyal}(x, \text{Caesar}) \vee \text{Ans}(x)$

 11. R[10, 2] $\text{Loyal}(\text{Markus}, \text{Caesar}) \vee \text{Ans}(\text{Markus})$ {x/Marcus}

 12. R[11, 7] $\neg\text{Person}(\text{Markus}) \vee \neg\text{Ruler}(\text{Caesar}) \vee \neg\text{Tryassassin}(\text{Markus}, \text{Caesar}) \vee \text{Ans}(\text{Markus})$
{x/Markus, y/Caesar}

Exercise 9.5

1. **Man(Marcus)**
 2. **Roman(Marcus)**
 3. **$\neg\text{Man}(x) \vee \text{Person}(x)$**
 4. **Ruler(Caesar)**
 5. **$\neg\text{Roman}(x) \vee \text{Loyal}(x, \text{Caesar}) \vee \text{Hate}(x, \text{Caesar})$**
 6. **$\text{Loyal}(x, F(x))$**
 7. **$\neg\text{Person}(x) \vee \neg\text{Ruler}(y) \vee \neg\text{Tryassassin}(x, y) \vee \neg\text{Loyal}(x, y)$**
 8. **Tryassassin(Marcus, Caesar)**
 9. **$\neg\text{Hate}(x, \text{Caesar}) \vee \text{Ans}(x)$**
-
10. R[9, 5] $\neg\text{Roman}(x) \vee \text{Loyal}(x, \text{Caesar}) \vee \text{Ans}(x)$

 11. R[10, 2] $\text{Loyal}(\text{Markus}, \text{Caesar}) \vee \text{Ans}(\text{Markus})$ {x/Marcus}

 12. R[11, 7] $\neg\text{Person}(\text{Markus}) \vee \neg\text{Ruler}(\text{Caesar}) \vee \neg\text{Tryassassin}(\text{Markus}, \text{Caesar}) \vee \text{Ans}(\text{Markus})$
 {x/Markus, y/Caesar}

 13. R[12, 3] $\neg\text{Man}(\text{Markus}) \vee \neg\text{Ruler}(\text{Caesar}) \vee \neg\text{Tryassassin}(\text{Markus}, \text{Caesar}) \vee \text{Ans}(\text{Markus})$
 {x/Markus}

Exercise 9.5

1. **Man(Marcus)**
 2. **Roman(Marcus)**
 3. **$\neg\text{Man}(x) \vee \text{Person}(x)$**
 4. **Ruler(Caesar)**
 5. **$\neg\text{Roman}(x) \vee \text{Loyal}(x, \text{Caesar}) \vee \text{Hate}(x, \text{Caesar})$**
 6. **$\text{Loyal}(x, F(x))$**
 7. **$\neg\text{Person}(x) \vee \neg\text{Ruler}(y) \vee \neg\text{Tryassassin}(x, y) \vee \neg\text{Loyal}(x, y)$**
 8. **Tryassassin(Marcus, Caesar)**
 9. **$\neg\text{Hate}(x, \text{Caesar}) \vee \text{Ans}(x)$**
-
10. R[9, 5] $\neg\text{Roman}(x) \vee \text{Loyal}(x, \text{Caesar}) \vee \text{Ans}(x)$
 11. R[10, 2] $\text{Loyal}(\text{Markus}, \text{Caesar}) \vee \text{Ans}(\text{Markus})$ {x/Marcus}
 12. R[11, 7] $\neg\text{Person}(\text{Markus}) \vee \neg\text{Ruler}(\text{Caesar}) \vee \neg\text{Tryassassin}(\text{Markus}, \text{Caesar}) \vee \text{Ans}(\text{Markus})$
{x/Markus, y/Caesar}
 13. R[12, 3] $\neg\text{Man}(\text{Markus}) \vee \neg\text{Ruler}(\text{Caesar}) \vee \neg\text{Tryassassin}(\text{Markus}, \text{Caesar}) \vee \text{Ans}(\text{Markus})$
{x/Markus}
 14. R[13, 1] $\neg\text{Ruler}(\text{Caesar}) \vee \neg\text{Tryassassin}(\text{Markus}, \text{Caesar}) \vee \text{Ans}(\text{Markus})$

Exercise 9.5

1. **Man(Marcus)**
 2. **Roman(Marcus)**
 3. **$\neg\text{Man}(x) \vee \text{Person}(x)$**
 4. **Ruler(Caesar)**
 5. **$\neg\text{Roman}(x) \vee \text{Loyal}(x, \text{Caesar}) \vee \text{Hate}(x, \text{Caesar})$**
 6. **$\text{Loyal}(x, F(x))$**
 7. **$\neg\text{Person}(x) \vee \neg\text{Ruler}(y) \vee \neg\text{Tryassassin}(x, y) \vee \neg\text{Loyal}(x, y)$**
 8. **Tryassassin(Marcus, Caesar)**
 9. **$\neg\text{Hate}(x, \text{Caesar}) \vee \text{Ans}(x)$**
-
10. R[9, 5] $\neg\text{Roman}(x) \vee \text{Loyal}(x, \text{Caesar}) \vee \text{Ans}(x)$
 11. R[10, 2] $\text{Loyal}(\text{Markus}, \text{Caesar}) \vee \text{Ans}(\text{Markus})$ {x/Marcus}
 12. R[11, 7] $\neg\text{Person}(\text{Markus}) \vee \neg\text{Ruler}(\text{Caesar}) \vee \neg\text{Tryassassin}(\text{Markus}, \text{Caesar}) \vee \text{Ans}(\text{Markus})$
{x/Markus, y/Caesar}
 13. R[12, 3] $\neg\text{Man}(\text{Markus}) \vee \neg\text{Ruler}(\text{Caesar}) \vee \neg\text{Tryassassin}(\text{Markus}, \text{Caesar}) \vee \text{Ans}(\text{Markus})$
{x/Markus}
 14. R[13, 1] $\neg\text{Ruler}(\text{Caesar}) \vee \neg\text{Tryassassin}(\text{Markus}, \text{Caesar}) \vee \text{Ans}(\text{Markus})$
 15. R[14, 4] $\neg\text{Tryassassin}(\text{Markus}, \text{Caesar}) \vee \text{Ans}(\text{Markus})$

Exercise 9.5

1. **Man(Marcus)**
 2. **Roman(Marcus)**
 3. **$\neg\text{Man}(x) \vee \text{Person}(x)$**
 4. **Ruler(Caesar)**
 5. **$\neg\text{Roman}(x) \vee \text{Loyal}(x, \text{Caesar}) \vee \text{Hate}(x, \text{Caesar})$**
 6. **Loyal(x, F(x))**
 7. **$\neg\text{Person}(x) \vee \neg\text{Ruler}(y) \vee \neg\text{Tryassassin}(x, y) \vee \neg\text{Loyal}(x, y)$**
 8. **Tryassassin(Marcus, Caesar)**
 9. **$\neg\text{Hate}(x, \text{Caesar}) \vee \text{Ans}(x)$**
-
10. R[9, 5] $\neg\text{Roman}(x) \vee \text{Loyal}(x, \text{Caesar}) \vee \text{Ans}(x)$
 11. R[10, 2] Loyal(Markus, Caesar) $\vee \text{Ans}(\text{Markus})$ {x/Marcus}
 12. R[11, 7] $\neg\text{Person}(\text{Markus}) \vee \neg\text{Ruler}(\text{Caesar}) \vee \neg\text{Tryassassin}(\text{Markus}, \text{Caesar}) \vee \text{Ans}(\text{Markus})$
{x/Markus, y/Caesar}
 13. R[12, 3] $\neg\text{Man}(\text{Markus}) \vee \neg\text{Ruler}(\text{Caesar}) \vee \neg\text{Tryassassin}(\text{Markus}, \text{Caesar}) \vee \text{Ans}(\text{Markus})$
{x/Markus}
 14. R[13, 1] $\neg\text{Ruler}(\text{Caesar}) \vee \neg\text{Tryassassin}(\text{Markus}, \text{Caesar}) \vee \text{Ans}(\text{Markus})$
 15. R[14, 4] $\neg\text{Tryassassin}(\text{Markus}, \text{Caesar}) \vee \text{Ans}(\text{Markus})$
 16. R[15a, 8] **Ans(Markus)**