Fundamentals of Artificial Intelligence
Chapter 04: **Beyond Classical Search**

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M.S. Course "Artificial Intelligence Systems", academic year 2021-2022

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Generalities

- So far we addresses a single category of problems:
  1. observable,
  2. deterministic,
  3. with known environment,
  4. s.t. the solution is a sequence of actions.
- What happens when these assumptions are relaxed?
- In order we will:
  - release condition 4 $\Longrightarrow$ local search
  - release condition 2 $\Longrightarrow$ search with non-deterministic actions
  - release condition 1 $\Longrightarrow$ search with no observability or with partial observability
  - release condition 3 $\Longrightarrow$ online search

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General Ideas

- Search techniques: systematic exploration of search space
  - solution to problem: the path to the goal state
  - ex: 8-puzzle
- With many problems, the path to goal is irrelevant
  - goals expressed as conditions, not as explicit list of goal states
  - solution to problem: only the goal state itself
  - ex: N-queens
  - many important applications:
    integrated-circuit design, factory-floor layout, job-shop scheduling, automatic programming, telecommunications network optimization, vehicle routing, portfolio management...
- The state space is a set of “complete” configurations
  - decision problems: find goal configuration satisfying constraints/rules (ex: N-queens)
  - optimization problems: find optimal configurations
    (ex: Travelling Salesperson Problem, TSP)
- If so, we can use iterative-improvement algorithms (in particular local search algorithms):
  - keep a single “current” state, try to improve it

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Local Search

- Idea: use single current state and move to "neighbouring" states
  - operate using a single current node
  - the paths followed by the search are not retained
- Two key advantages:
  - use very little memory (usually constant)
  - can often find reasonable solutions in large or infinite (continuous) state spaces, for which systematic algorithms are unsuitable
- Also useful for pure optimization problems
  - find the best state according to an objective function
  - often do not fit the "standard" search model of previous chapter
  - ex: Darwinian survival of the fittest: metaphor for optimization, but no "goal test" and no "path cost"
- A complete local search algorithm: guaranteed to always find a solution (if exists)
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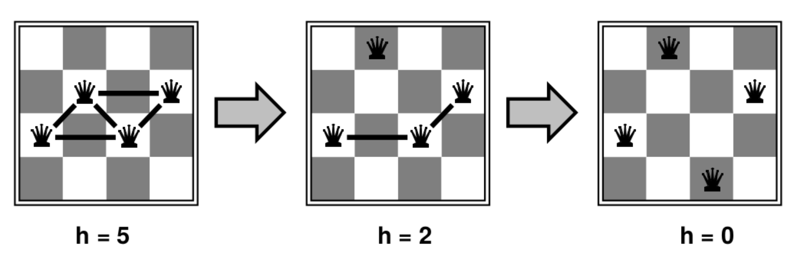
Local Search Example: N-Queens

- One queen per column (incremental representation)
- Cost (h): # of queen pairs on the same row, column, or diagonal
- Goal: h=0
- Step: move a queen vertically to reduce number of conflicts



Almost always solves N-queens problems almost instantaneously for very large N (e.g., N=1million)

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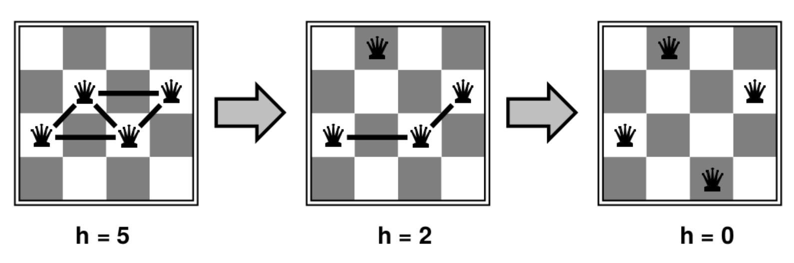

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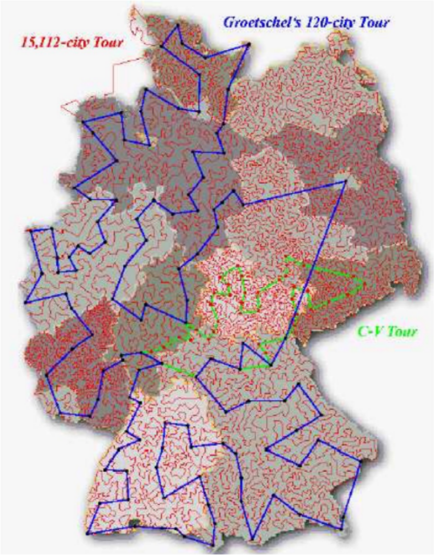
Optimization Local Search Example: TSP

Travelling Salesperson Problem (TSP)

Given an undirected graph, with n nodes and each arc associated with a positive value, find the Hamiltonian tour with the minimum total cost.

Very hard for classic search!



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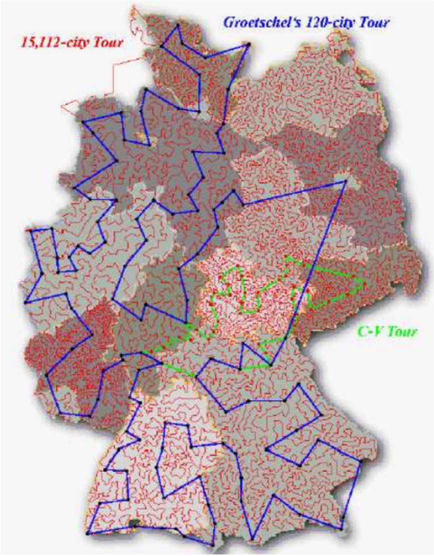

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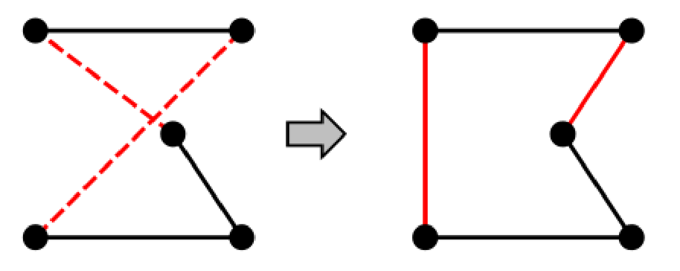
- State represented as a permutation of numbers $(1, 2, ..., n)$
- Cost (h): total cycle length
- Start with any complete tour
- Step: (2-swap) perform pairwise exchange



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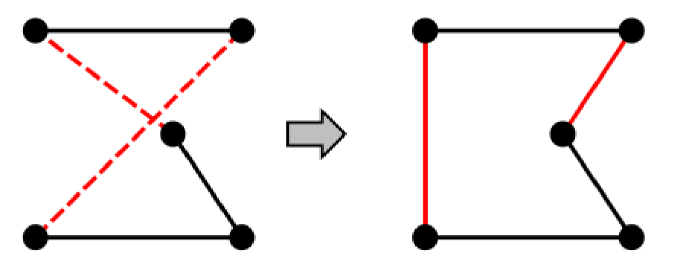

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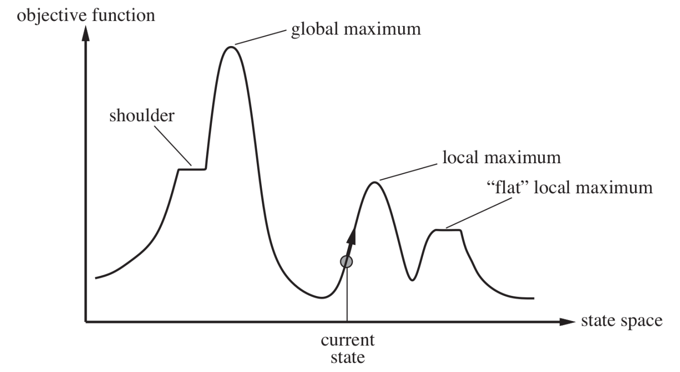
Local Search: State-Space Landscape

State-space landscape (Maximization)

- Local search algorithms explore state-space landscape
  - state space n-dimensional (and typically discrete)
  - move to “nearby” states (neighbours)
- NP-Hard problems may have exponentially-many local optima



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Outline

Hill-Climbing Search (aka Greedy Local Search)

Hill-Climbing

- Very-basic local search algorithm
- Idea: a move is performed only if the solution it produces is better than the current solution
  - (steepest-ascent version): selects the neighbour with best score improvement (select randomly among best neighbours if ≥ 1)
  - does not look ahead of immediate neighbors of the current state
  - stops as soon as it finds a (possibly local) minimum
- Several variants (Stochastic H.C., Random-Restart H.C., ...)
- Often used as part of more complex local-search algorithms

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum

  current ← MAKE-NODE(*problem*.INITIAL-STATE)
  loop do
    neighbor ← a highest-valued successor of *current*
    if neighbor.VALUE ≤ current.VALUE **then return** *current*.STATE
    current ← *neighbor*

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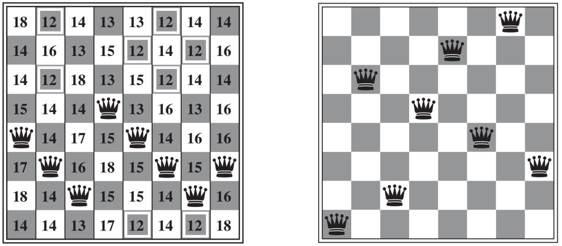
Hill-Climbing Search: Example

8-queen puzzle (minimization)

- Neighbour states: generated by moving one queen vertically
  - Cost (h): # of queen pairs on the same row, column, or diagonal
  - Goal: h=0
- Two scenarios ((a) $\Longrightarrow$ (b) in 5 steps) :
  - (a) 8-queens state with heuristic cost estimate h = 17 (12d, 5h)
  - (b) local minimum: h=1, but all neighbours have higher costs



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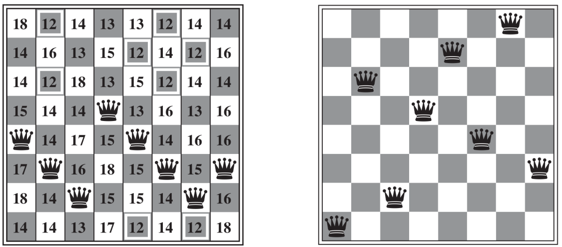

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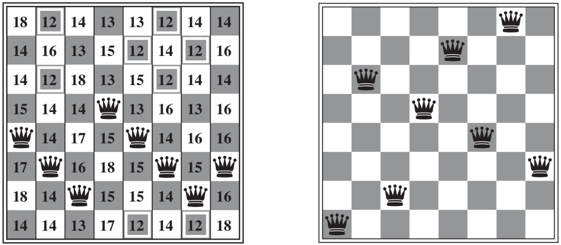

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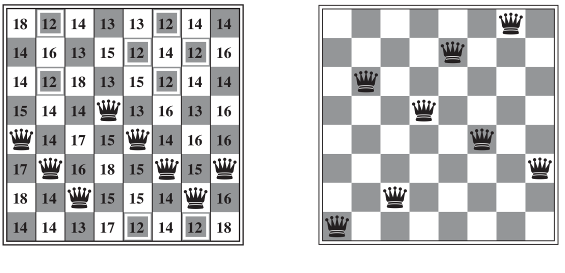

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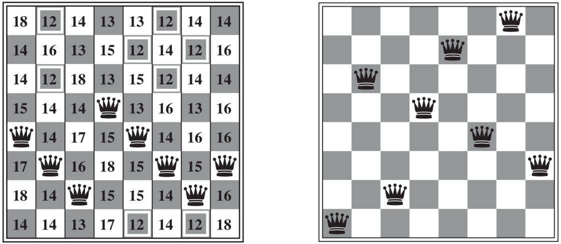

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Hill-Climbing Search: Drawbacks

- Incomplete: gets stuck in local optima, flat local optima & shoulders (aka plateaux), ridges (sequences of local optima)
  - Ex: with 8-queens, gets stuck 86% of the time, fast when succeed

  note: converges very fast till (local) minima or plateaux
- Possible idea: allow 0-progress moves (aka sideways moves)
  - pros: may allow getting out of shoulders
  - cons: may cause infinite loops with flat local optima

  $\Longrightarrow$ set a limit to consecutive sideways moves (e.g. 100)
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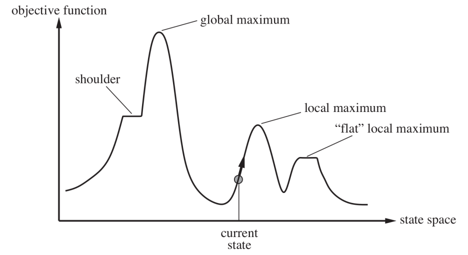

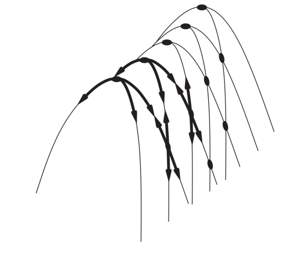

Hill-Climbing Search: Drawbacks

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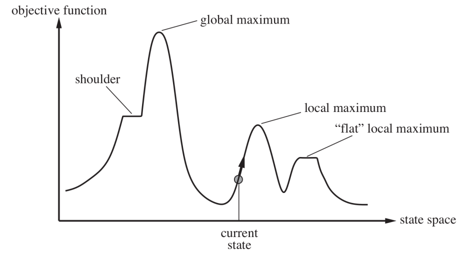

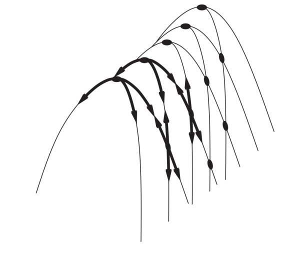

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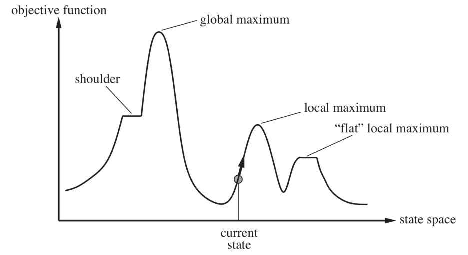

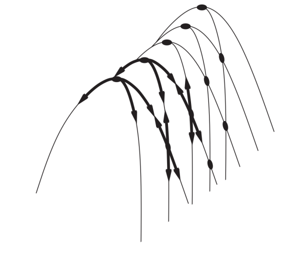

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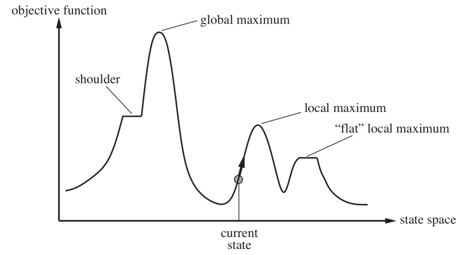

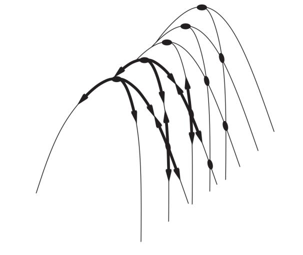

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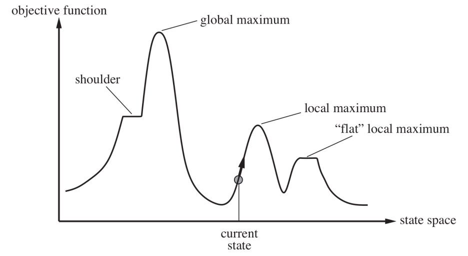

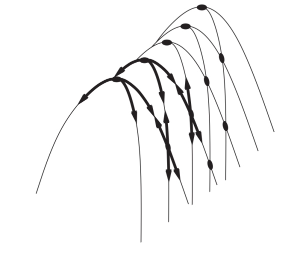

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Hill-climbing: Variations

- Stochastic hill-climbing
  - random selection among the uphill moves
  - selection probability can vary with the steepness of uphill move
  - sometimes slower, but often finds better solutions
- First-choice hill-climbing
  - cfr. stochastic h.c., generates successors randomly until a better one is found
  - good when there are large amounts of successors
- Random-restart hill-climbing
  - conducts a series of hill-climbing searches from randomly generated initial states
  - tries to avoid getting stuck in local maxima

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Outline

Simulated Annealing

- Inspired to statistical-mechanics analysis of metallurgical annealing (Boltzmann's state distributions)
- Idea: Escape local maxima by allowing "bad" moves...
  - "bad move": move toward states with worse value
  - typically pick a move taken at random ("random walk")
- ... but gradually decrease their size and frequency.
  - sideways moves progressively less likely
- Analogy: get a ball into the deepest crevice in a bumpy surface
  - initially shaking hard ("high temperature")
  - progressively shaking less hard ("decrease the temperature")

Widely used in large-scale optimization tasks (e.g. VSLI layout problems, factory scheduling,...)

Simulated Annealing [cont.]

Simulated Annealing (maximization)

- A "temperature" parameter T slowly decreases with steps ("schedule")
- The probability of picking a "bad move":
  - decreases exponentially with the "badness" of the move $|\Delta E|$
  - decreases as the "temperature" T goes down
- If schedule lowers T slowly enough,
  then the algorithm will find a global optimum with probability approaching 1

function SIMULATED-ANNEALING($problem$, $schedule$) **returns** a solution state
  inputs: $problem$, a problem
    $schedule$, a mapping from time to "temperature"

  $current \leftarrow$ MAKE-NODE($problem$.INITIAL-STATE)
  for t = 1 **to** ∞ **do**
    $T \leftarrow schedule(t)$
    if T = 0 **then return** $current$
    $next \leftarrow$ a randomly selected successor of $current$
    $\Delta E \leftarrow next$.VALUE – $current$.VALUE
    if $\Delta E > 0$ **then** $current \leftarrow next$
    else $current \leftarrow next$ only with probability $e^{\Delta E/T}$

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Outline

Local Beam Search

Local Beam Search

- Idea: keep track of k states instead of one
- Initially: k random states
- Step:
  1. determine all successors of k states
  2. if any of successors is goal $\Longrightarrow$ finished
  3. else select k best from successors
- Different from k searches run in parallel:
  - searches that find good states recruit other searches to join them $\Longrightarrow$ information is shared among k search threads
- Lack of diversity: quite often, all k states end up same local hill

$\Longrightarrow$ Stochastic Local Beam: choose k successors randomly, with probability proportional to state success.

Resembles natural selection with asexual reproduction:
the successors (offspring) of a state (organism) populate the next generation according to its value (fitness), with a random component.

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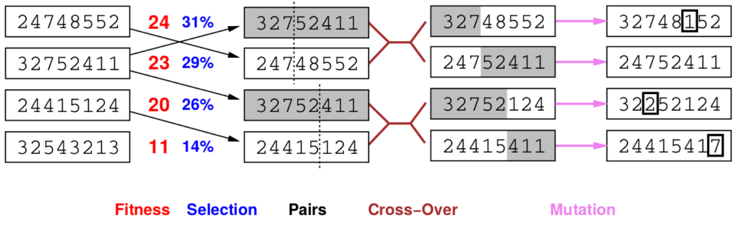
```
function GENETIC-ALGORITHM(population, FITNESS-FN) returns an individual
  inputs: population, a set of individuals
          FITNESS-FN, a function that measures the fitness of an individual

  repeat
     new_population ← empty set
     for i = 1 to SIZE(population) do
        x ← RANDOM-SELECTION(population, FITNESS-FN)
        y ← RANDOM-SELECTION(population, FITNESS-FN)
        child ← REPRODUCE(x, y)
        if (small random probability) then child ← MUTATE(child)
        add child to new_population
     population ← new_population
  until some individual is fit enough, or enough time has elapsed
  return the best individual in population, according to FITNESS-FN
```

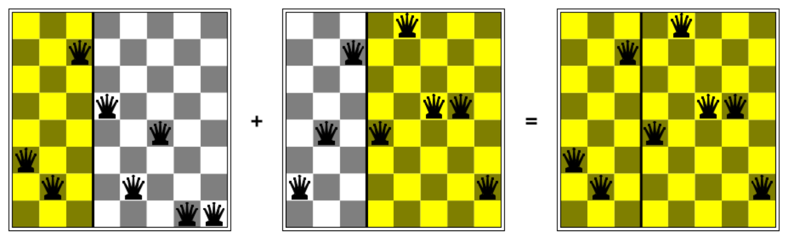
```
function REPRODUCE(x, y) returns an individual
  inputs: x, y, parent individuals

  n ← LENGTH(x); c ← random number from 1 to n
  return APPEND(SUBSTRING(x, 1, c), SUBSTRING(y, c + 1, n))
```

Genetic Algorithms: Example

Example: 8-Queens

state[*i*]: (upward) position of the queen in *i*th column





327|52411 + 247|48552 = 327|48552

Genetic Algorithms: Intuitions, Pros & Cons

Intuitions

- Selection drives the population toward high fitness
- Crossover combines good parts from good solutions (but it might achieve the opposite effect)
- Mutation introduces diversity

Pros & Cons

- Pros:
  - extremely simple
  - general purpose
  - tractable theoretical models
- Cons:
  - not completely understood
  - good coding is crucial (e.g., Gray codes for numbers)
  - too simple genetic operators

Widespread impact on optimization problems, i.e. circuit layout and job-shop scheduling

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Outline

Generalities

- Assumptions so far (see ch. 2 and 3):
  - the environment is deterministic
  - the environment is fully observable
  - the agent knows the effects of each action

$\Longrightarrow$ The agent does not need perception:
  - can calculate which state results from any sequence of actions
  - always knows which state it is in
- If one of the above does not hold, then percepts are useful
  - the future percepts cannot be determined in advance
  - the agent's future actions will depend on future percepts
- Solution: not a sequence but a contingency plan (aka conditional plan, strategy)
  - specifies the actions depending on what percepts are received
- We analyze first the case of nondeterministic environments

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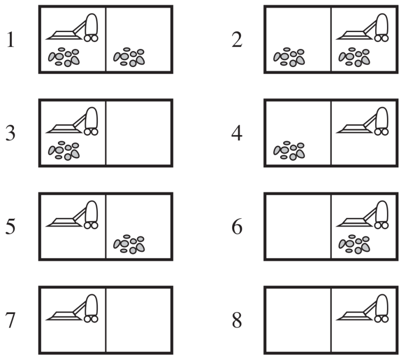
Example: The Erratic Vacuum Cleaner

Erratic Vacuum-Cleaner Example

- actions: Left, Right, Suck
- goal: A and B cleaned (states 7, 8)
- if environment is observable, deterministic, and completely known $\Longrightarrow$ solvable by search algos
- ex: if initially in 1, then [suck,right,suck] leads to 8: [1,5,6,8]



(© S. Russell & P. Norwig, AIMA)

- Nondeterministic version (erratic vacuum cleaner):
  - if dirty square: cleans the square, sometimes cleans also the other square. Ex: $1 \stackrel{suck}{\Longrightarrow} \{5,7\}$
  - if clean square: sometimes deposits dirt on the carpet
    Ex: $5 \stackrel{suck}{\Longrightarrow} \{1,5\}$

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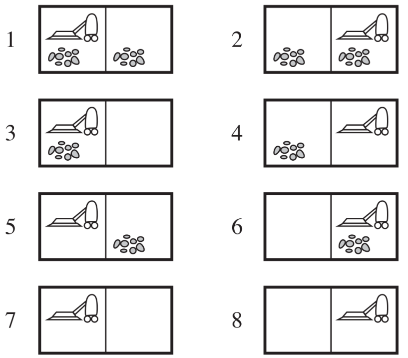

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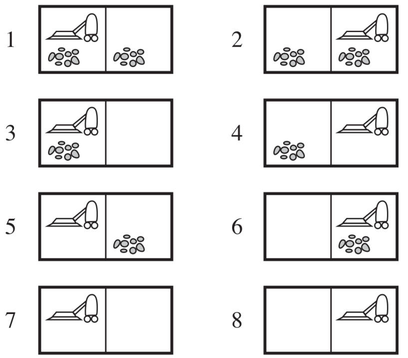

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Searching with Nondeterministic Actions

Generalized notion of transition model

- RESULTS(S,A) returns a set of possible outcomes states
  - Ex: RESULTS(1,SUCK)={5, 7}, RESULTS(5,SUCK)={1, 5}, ...
- A solution is a contingency plan (aka conditional plan, strategy)
  - contains nested conditions on future percepts (if-then-else, case-switch, ...)
  - Ex: from state 1 we can act the following contingency plan:
    [SUCK, IF STATE = 5 THEN [RIGHT, SUCK] ELSE [ ]]
- Can cause loops (see later)

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Searching with Nondeterministic Actions [cont.]

And-Or Search Trees

- In a deterministic environment, branching on agent's choices
  $\Longrightarrow$ OR nodes, hence OR search trees
  - OR nodes correspond to states
- In a nondeterministic environment, branching also on environment's choice of outcome for each action
  - the agent has to handle all such outcomes

  $\Longrightarrow$ AND nodes, hence AND-OR search trees
  - AND nodes correspond to actions
  - leaf nodes are goal, dead-end or loop OR nodes
- A solution for an AND-OR search problem is a subtree s.t.:
  - has a goal node at every leaf
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OR tree: AND-OR tree with 1 outcome each AND node (determinism)

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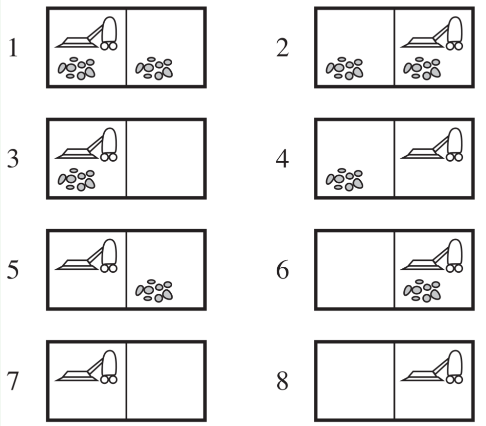
And-Or Search Trees: Example

(Part of) And-Or Search Tree for Erratic Vacuum Cleaner Example.

Problem: Init: 1, Goal: 7,8.

Solution: [SUCK, IF STATE = 5 THEN [RIGHT, SUCK] ELSE [ ]] (solid arcs)



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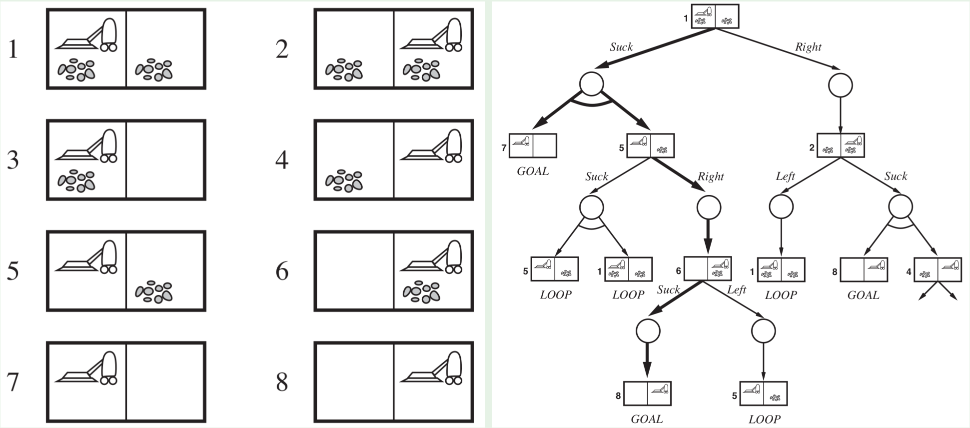

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AND-OR Search

Recursive Depth-First (Tree-based) AND-OR Search

function AND-OR-GRAPH-SEARCH(*problem*) **returns** *a conditional plan, or failure*
  OR-SEARCH(*problem*.INITIAL-STATE, *problem*, [ ])

---

function OR-SEARCH(*state*, *problem*, *path*) **returns** *a conditional plan, or failure*
  if *problem*.GOAL-TEST(*state*) **then return** the empty plan
  if *state* is on *path* **then return** *failure*
  for each *action* **in** *problem*.ACTIONS(*state*) **do**
    plan ← AND-SEARCH(RESULTS(*state*, *action*), *problem*, [*state* | *path*])
    if *plan* ≠ *failure* **then return** [*action* | *plan*]
  return *failure*

---

function AND-SEARCH(*states*, *problem*, *path*) **returns** *a conditional plan, or failure*
  for each s_i **in** *states* **do**
    $plan_i$ ← OR-SEARCH(s_i, *problem*, *path*)
    if $plan_i$ = *failure* **then return** *failure*
  return [**if** s_1 **then** $plan_1$ **else if** s_2 **then** $plan_2$ **else** . . . **if** s_{n-1} **then** $plan_{n-1}$ **else** $plan_n$]

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Note: nested if-then-else can be rewritten as case-switch

AND-OR Search [cont.]

Recursive Depth-First (Tree-based) AND-OR Search

- Cycles: if the current state already occurs in the path $\Longrightarrow$ failure
  - cycle detection like with ordinary DFS
  - does not mean "no solution"
  - means "if there is a non-cyclic solution,
    it must be reachable from the earlier incarnation of the current state"

$\Longrightarrow$ Complete (if state space finite): every path must reach a goal, a dead-end or loop state

- Can be augmented with "explored" data structure for avoiding redundant branches (graph-based search)
- Can also be explored by breadth-first or best-first method
  - e.g. A^* variant for AND-OR search available (see AIMA book)

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AND-OR Search: Cyclic Solutions

- Some problems have no acyclic solutions
- A cyclic plan may be considered a cyclic solution provided that:
  - every leaf is a goal state (loop states not considered leaves), and
  - a leaf is reachable from every point in the plan
- Can be expressed by means of introducing
  - labels, and backward goto's to labels
  - loop syntax (e.g., while-do)

$\Longrightarrow$ Executing a cyclic solution eventually reaches a goal, provided that each outcome of a nondeterministic action eventually occurs

- Is this assumption reasonable?
- Yes, provided we distinguish: $\langle$*nondeterministic, observable*$\rangle$ $\neq$ $\langle$*deterministic, partially-observable*$\rangle$
- Ex: device may not always work $\neq$ device is broken (but we don't know it)

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- Yes, provided we distinguish:
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AND-OR Search: Cyclic Solutions

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Example: Slippery Vacuum Cleaner

- Movement actions may fail: e.g., $\mathit{Results}(1, \mathit{Right}) = \{1, 2\}$
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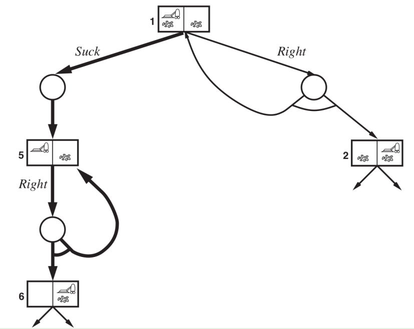

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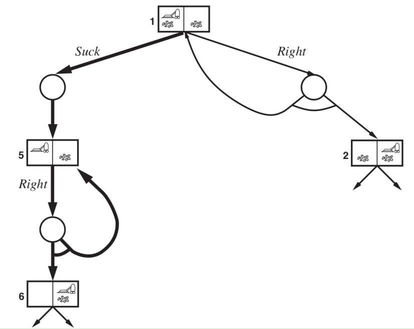

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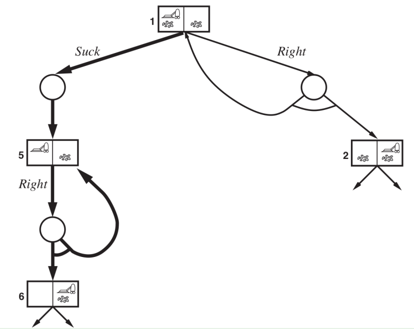

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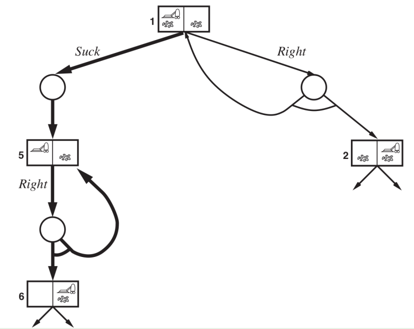

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Outline

Generalities

Partial Observability

- Partial observability: percepts do not capture the whole state
  - partial state corresponds to a set of possible physical states
- If the agent is in one of several possible physical states, then an action may lead to one of several possible outcomes, even if the environment is deterministic

Belief States

- Belief state: the agent's current belief about the possible physical states it might be in, given the previous sequence of actions and percepts
  - is a set of physical states: (but does not know in which one)
  - contains the actual physical state the agent is in
  - ex: {1,2}: the agent is either in state 1 or in state 2 (but it does not know in which one)
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- 2^n possible belief states out of n possible physical states!

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Outline

Search with No Observation

Search with No Observation (aka Sensorless Search or Conformant Search)

- Idea: To solve sensorless problems, the agent searches in the space of belief states rather than in that of physical states
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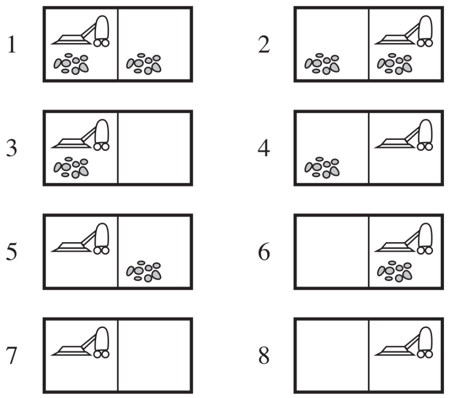
Search with No Observation: Example

Example: Sensorless Vacuum Cleaner

- the vacuum cleaner knows the geography of its world, but it doesn't know its location or the distribution of dirt
  - initial state: $\{1, 2, 3, 4, 5, 6, 7, 8\}$
  - after action RIGHT, state is $\{2, 4, 6, 8\}$
  - after action sequence [RIGHT,SUCK], state is $\{4, 8\}$
  - after action sequence [RIGHT,SUCK,LEFT,SUCK], state is $\{7\}$



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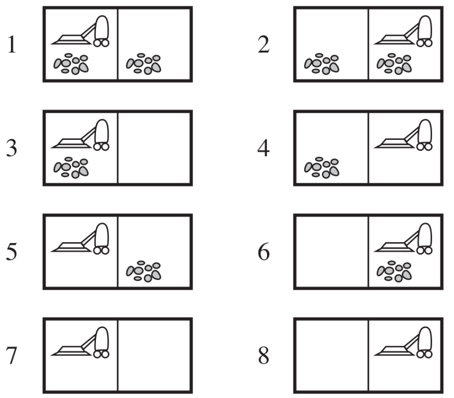

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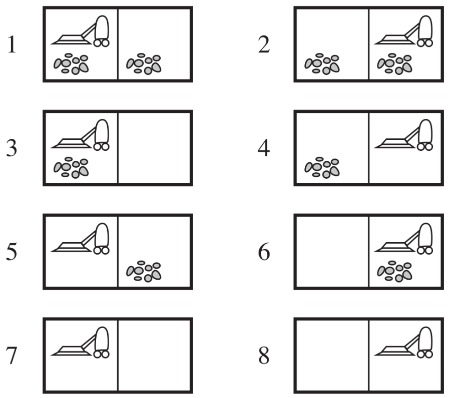

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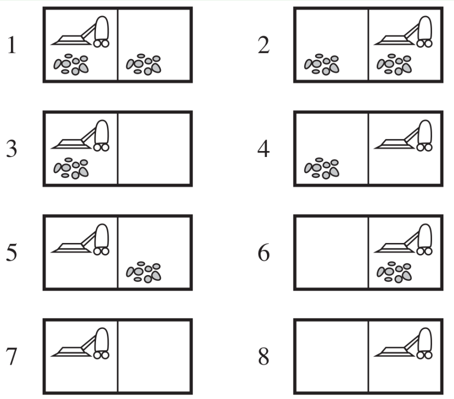

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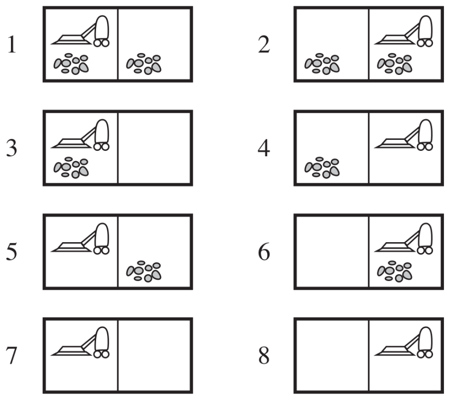

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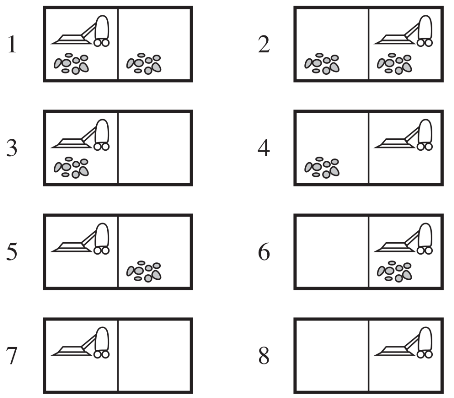

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Belief-State Problem Formulation

- Belief states: subsets of physical states
  - If P has N states, then the sensorless problem has up to 2^N states
- Initial state: typically the set of all physical states in P
- Actions: (assumption: illegal actions have no effects)
  - $Actions(b) \stackrel{\text{def}}{=} \bigcup_{s \in b} Actions_P(s)$
- Transition model:
  - for deterministic actions: $b' = Result(b, a) \stackrel{\text{def}}{=} \{s' \mid s' = Result_P(s, a) \text{ and } s \in b\}$
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  - This step is called Prediction: $b' \stackrel{\text{def}}{=} Predict(b, a)$
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- Path cost: (assumption: cost of an action the same in all states)
  - $StepCost(a, b) \stackrel{\text{def}}{=} StepCost_P(a, s)$, $\forall s \in b$

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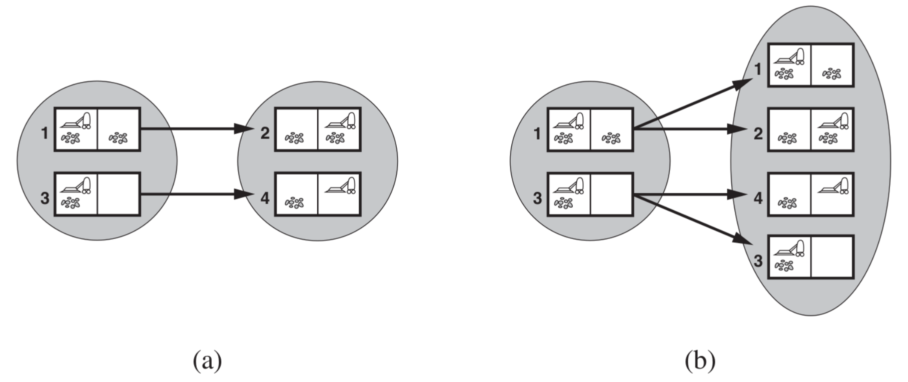
Belief-State Problem Formulation [cont.]

Example: Sensorless Vacuum Cleaner, plain and slippery versions

Prediction: $\mathit{Result}(\{1,3\}, \mathit{Right})$, deterministic (a) and nondeterministic action (b)



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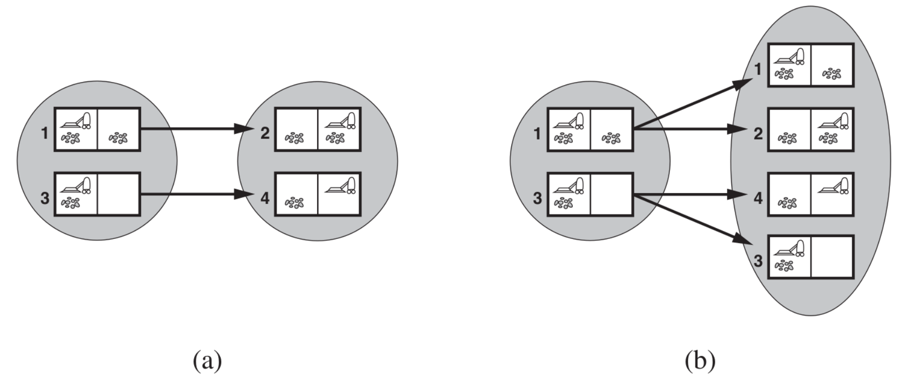

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Example: Sensorless Vacuum Cleaner: Belief State Space

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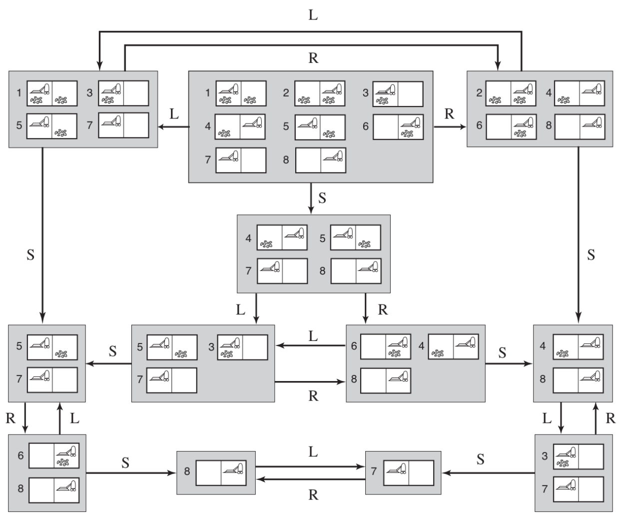

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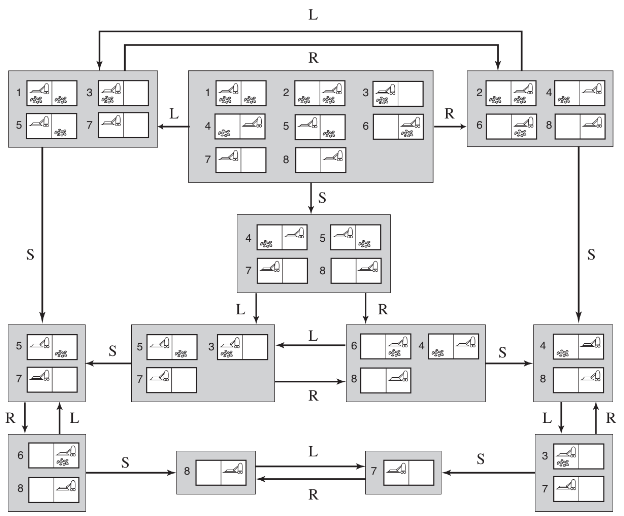
Example: Sensorless Vacuum Cleaner: Belief State Space

(self-loops are omitted)



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Exercises

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Draw the Belief State Space in case of:

- Erratic vacuum cleaner
- Slippery vacuum cleaner

Belief-State Problem Formulation [cont.]

Remarks

- if $b \subseteq b'$, then $Result(b, a) \subseteq Result(b', a)$
- If a is deterministic, then $|Result(b, a)| \leq |b|$
- The agent might achieve the goal earlier than $GoalTest(b)$ holds, but it does not know it

Properties

- An action sequence is a solution for b iff it leads b to to a goal
- If an action sequence is a solution for a belief state b, then it is also a solution for any belief state b' s.t. $b' \subseteq b$
  - if $b \xrightarrow{a_1} \dots \xrightarrow{a_k} g$, then $b' \xrightarrow{a_1} \dots \xrightarrow{a_k} g$

We can apply to the Belief-State space any search algorithm.

- we can discard a path reaching a belief state b if $b' \subseteq b$ has already been generated and discarded
- if a solution for b has been found, then any $b' \subseteq b$ is solvable

$\Longrightarrow$ Dramatically improves efficiency

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Outline

Search with Observations

Perception and Belief-State Problem Formulation

- $Percept(s)$ returns the percept received in state s
  (if sensing is nondeterministic, a function $Percepts(s)$ returns a set of possible percepts)
  - ex: local-sensing vacuum cleaner, can perceive dirty/clean only on the current position: $Percept(1) = [A, Dirty]$
  - with fully observable problems: $Percept(s) = s, \forall s$
  - with sensorless problems: $Percept(s) = null, \forall s$
- Partial observations: many states can produce the same percept
  - ex: $Percept(1) = Percept(3) = [A, Dirty]$

  $\Longrightarrow$ $Percepts(s)$ may correspond to many different candidate states
- $Actions()$, $StepCost()$, $GoalTest()$: as with sensorless case

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Transition Model with Perceptions

The Prediction-Observation-Update process

- Three steps:
  1. Prediction: (same as for sensorless):
     $\hat{b} = Predict(b, a) \stackrel{\text{def}}{=} Result_{(sensorless)}(b, a) = \{s' \mid s' = Result_P(s, a) \text{ and } s \in b\}$
  2. Observation prediction: determines the set of percepts that could be observed in the predicted belief state: $PossiblePercepts(\hat{b}) \stackrel{\text{def}}{=} \{o \mid o = Percept(s) \text{ and } s \in \hat{b}\}$
  3. Update: for each percept o, determine the belief state b_o, i.e., the subset of states in $\hat{b}$ that could have produced the percept o:
     - $b_o = Update(\hat{b}, o) \stackrel{\text{def}}{=} \{s \mid s \in \hat{b} \text{ and } o = Percept(s)\}$

$$\Longrightarrow Result(b, a) = \left\{ b_o \;\middle|\; \begin{array}{rl} b_o = & Update(Predict(b, a), o) \;\text{ and} \\ o \in & PossiblePercepts(Predict(b, a)) \end{array} \right\}$$

- set (not union!) of belief states, one for each possible percepts o
- $b_o \subseteq \hat{b},\ \forall o \Longrightarrow$ sensing reduces uncertainty!
- (if sensing is deterministic) the b_o's are all disjoint (each s belongs to b_o s.t. $o = Percept(s)$)
  $\Longrightarrow \hat{b}$ partitioned into smaller belief states, one for each possible next percept

$\Longrightarrow$ Non-deterministic belief-state problem

- due to the inability to predict exactly the next percept

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  $\Longrightarrow \hat{b}$ partitioned into smaller belief states, one for each possible next percept

$\Longrightarrow$ Non-deterministic belief-state problem

- due to the inability to predict exactly the next percept

Transition Model with Perceptions

The Prediction-Observation-Update process

- Three steps:
  1. Prediction: (same as for sensorless):
     $\hat{b} = Predict(b,a) \stackrel{\text{def}}{=} Result_{(sensorless)}(b,a) = \{s' \mid s' = Result_P(s,a) \text{ and } s \in b\}$
  2. Observation prediction: determines the set of percepts that could be observed in the predicted belief state: $PossiblePercepts(\hat{b}) \stackrel{\text{def}}{=} \{o \mid o = Percept(s) \text{ and } s \in \hat{b}\}$
  3. Update: for each percept o, determine the belief state b_o, i.e., the subset of states in $\hat{b}$ that could have produced the percept o:
     - $b_o = Update(\hat{b}, o) \stackrel{\text{def}}{=} \{s \mid s \in \hat{b} \text{ and } o = Percept(s)\}$

$$\Longrightarrow Result(b,a) = \left\{ b_o \;\middle|\; \begin{array}{rl} b_o = & Update(Predict(b,a),o) \;\text{ and} \\ o \in & PossiblePercepts(Predict(b,a)) \end{array} \right\}$$

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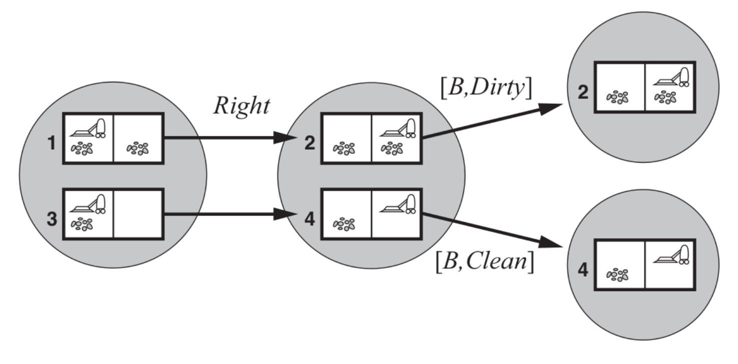
Transition Model with Perceptions: Example

Deterministic actions: Local-sensing vacuum cleaner

- $\hat{b} = Predict(\{1, 3\}, Right) = \{2, 4\}$
- $PossiblePercepts(\hat{b}) = \{[B, Dirty], [B, Clean]\}$
- $Result(\{1, 3\}, Right) = \{\{2\}, \{4\}\}$



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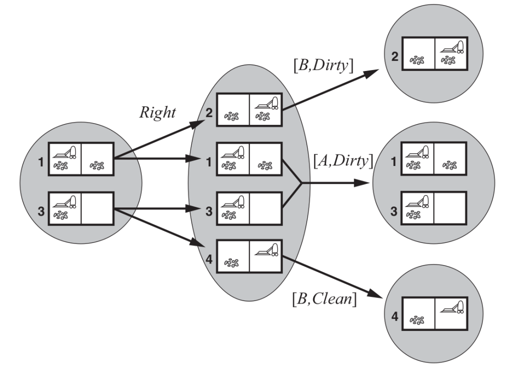
Transition Model with Perceptions: Example

Nondeterministic actions: Slippery local-sensing vacuum cleaner

- $\hat{b} = Predict(\{1,3\}, Right) = \{1,2,3,4\}$
- $PossiblePercepts(\hat{b}) = \{[B, Dirty], [A, Dirty], [B, Clean]\}$
- $Result(\{1,3\}, Right) = \{\{2\}, \{1,3\}, \{4\}\}$



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Solving Partially-Observable Problems

- Formulation as a nondeterministic belief-state search problem
  - non-determinism due to different possible percepts

$\Longrightarrow$ The AND-OR search algorithms can be applied

$\Longrightarrow$ The solution is a conditional plan

Solution for initial percept [A, Dirty] (deterministic): [Suck, Right, if Bstate = {6} then Suck else [ ]]

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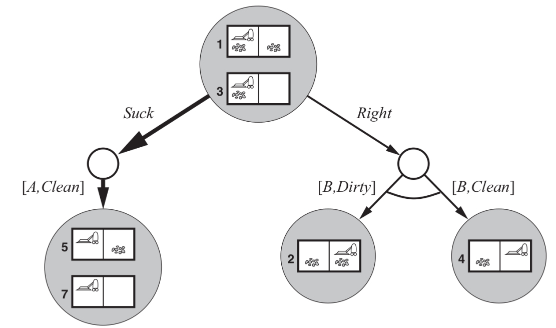

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An Agent for Partially-Observable Environments

- Agent quite similar to the simple problem-solving agent [Ch.3]:
  - formulates a problem (as a belief-state search)
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  - executes the solution
- Two main differences:
  - the solution is a conditional plan, not an action sequence
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The computation has to happen as fast as percepts are coming in
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Example: Belief-State Maintenance

Example: Kindergarden Vacuum-Cleaner

- local sensing $\Longrightarrow$ partially observable
- any square may become dirty at any time unless the agent is actively cleaning it at that moment $\Longrightarrow$ nondeterministic
- Ex: $Update(\overbrace{Predict(\{1,3\}, Suck)}^{\{5,7\}}, [A, Clean]) = \{5,7\}$
- Ex: $Update(\overbrace{Predict(\{5,7\}, Right)}^{\{2,4,6,8\}}, [B, Dirty]) = \{2,6\}$

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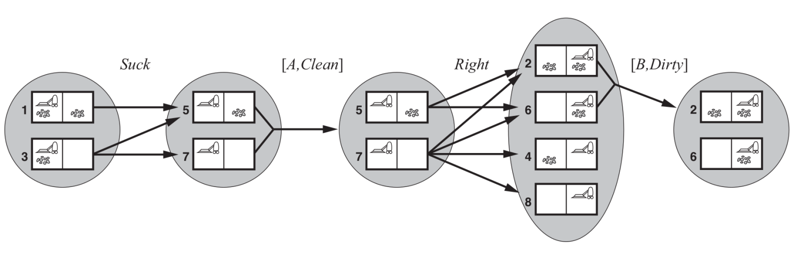

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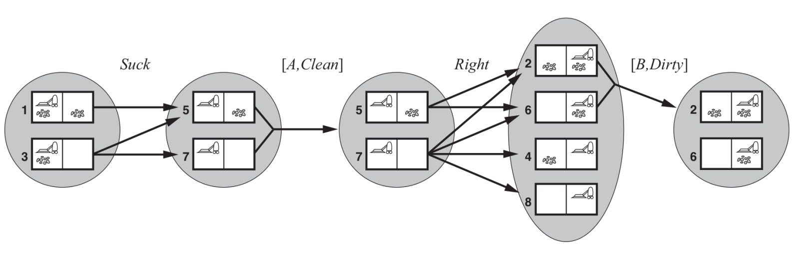

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Example:

- Knows the map, senses walls in the four directions (NESW)
  - localization broken: does not know where it is
  - navigation broken: does not know the direction is moving to $\Longrightarrow$ move is nondeterministic
  - goal: localization (know where it is)
- $b = \{all\ locations\}$, $o = NSW$
  1. $b_o = Update(b, NSW) = (a)$
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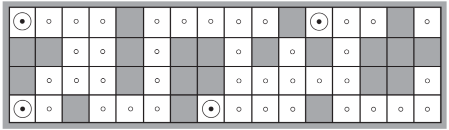
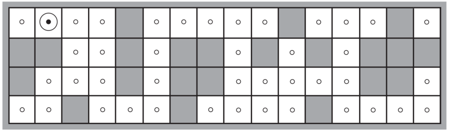


(a) Possible locations of robot after $E_1 = NSW$



(b) Possible locations of robot After $E_1 = NSW, E_2 = NS$

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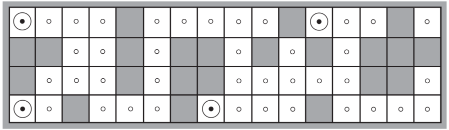

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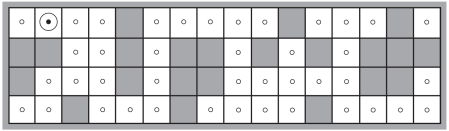

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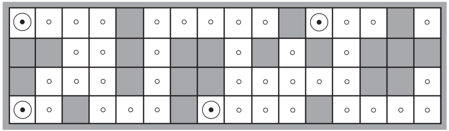

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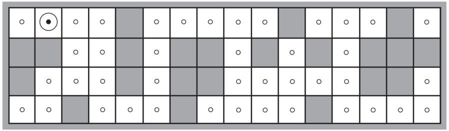

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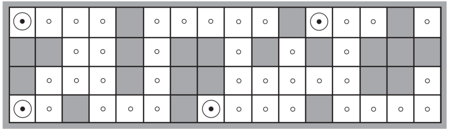

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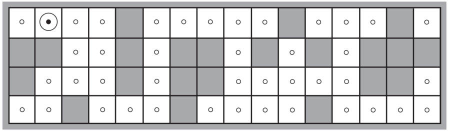

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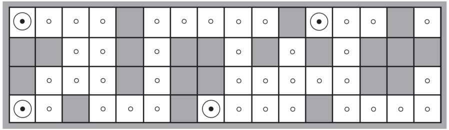

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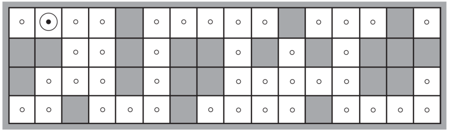

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Outline

Generalities

Online vs. offline search

- So far: Offline search
  - it computes a complete solutions before executing it
- Online search: agent interleaves computation and action
  - it takes an action,
  - then it observes the environment and computes the next action
  - (repeat)
- Necessary in dynamic domains or unknown domains
  - cannot know the states and consequences of actions
  - faces an exploration problem: must use actions as experiments in order to learn enough
  - ex: a robot placed in a new building $\Longrightarrow$ must explore it to build a map for getting from A to B
  - ex: newborn baby $\Longrightarrow$ acts to learn the outcome of his/her actions
- Useful in nondeterministic domains
  - prevents search blowup

Must be solved by executing actions, rather than by pure computation

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  - (repeat)
- Necessary in dynamic domains or unknown domains
  - cannot know the states and consequences of actions
  - faces an exploration problem: must use actions as experiments in order to learn enough
  - ex: a robot placed in a new building $\Longrightarrow$ must explore it to build a map for getting from A to B
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Online Search

Working Hypotheses

- Assumption: a deterministic and fully observable environment
- The agent knows only
  - $Actions(s)$, which returns the list of actions allowed in s
  - the step-cost function $c(s, a, s')$ (cannot be used until s' is known)
  - $GoalTest(s)$
- Remark: The agent cannot determine $Result(s, a)$
  - except by actually being in s and doing a
- The agent knows an admissible heuristic function h(s), that estimates the distance from the current state to a goal state
- Objective: reach goal with minimal cost
  - Cost: total cost of traveled path
  - Competitive ratio: ratio of cost over cost of the solution path if search space is known ($+\infty$ if agent in a deadend)

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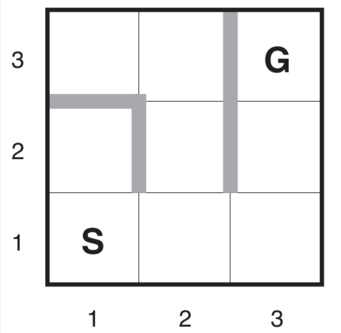
Online Search: Example

Example: a simple maze problem

- the agent does not know that going Up from (1,1) leads to (1,2)
- having done that, it does not know that going Down leads to (1,1)
- the agent might know the location of the goal
- it may be able to use the Manhattan-distance heuristic



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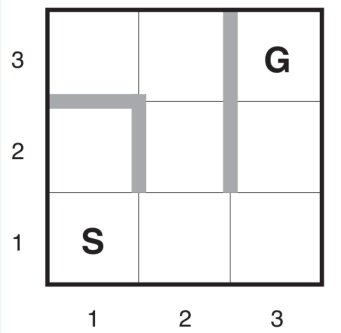

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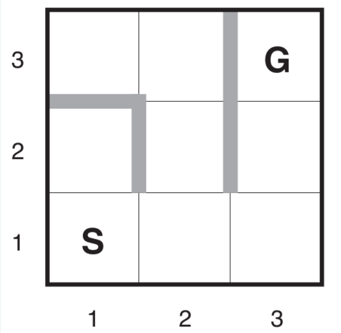

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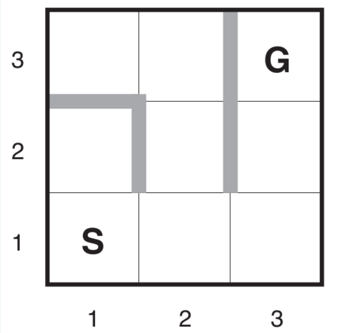

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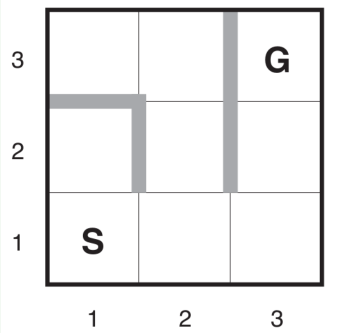

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Online Search: Deadends

Inevitability of Deadends

- Online search may face deadends (e.g., with irreversible actions)
- No algorithm can avoid dead ends in all state spaces
- Adversary argument: for each algo, an adversary can construct the state space while the agent explores it
  - If states S and A visit. What next?

  $\Longrightarrow$ if algo goes right, adversary builds (top), otherwise builds (bot)

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- Assumption the state space is safely explorable: some goal state is reachable from every reachable state (ex: reversible actions)

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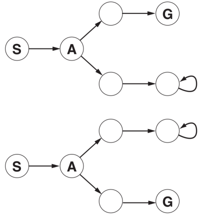

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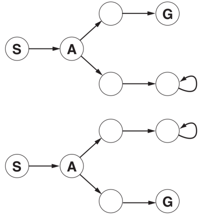

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Online Search Agents

Online Search Agents: Basic Ideas

- Idea: The agent creates & maintains a map of the environment ($result[s, a]$)
  - map is updated based on percept input after every action
  - map is used to decide next action
- Difference wrt. offline algorithms (ex A^*, BFS)
  - Can only expand the node it is physically in
    - $\Longrightarrow$ expand nodes in local order
    - $\Longrightarrow$ DFS natural candidate for an online version
  - Needs to backtrack physically
    - DFS: go back to the state from which the agent most recently entered the current state
    - must keep a table with the predecessor states of each state to which the agent has not yet backtracked ($unbacktracked[s]$)
    - $\Longrightarrow$ backtrack physically (find an action reversing the generation of s)
  - Works only if actions are always reversible
  - Worst case: each node is visited twice
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    - an online iterative deepening approach solves this problem

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Online DFS Search Agents

function ONLINE-DFS-AGENT(s') **returns** an action
  inputs: s', a percept that identifies the current state
  persistent: $result$, a table indexed by state and action, initially empty
    $untried$, a table that lists, for each state, the actions not yet tried
    $unbacktracked$, a table that lists, for each state, the backtracks not yet tried
    s, a, the previous state and action, initially null

  if GOAL-TEST(s') **then return** $stop$
  if s' is a new state (not in $untried$) **then** $untried[s'] \leftarrow$ ACTIONS(s')
  if s is not null **then**
    $result[s, a] \leftarrow s'$
    add s to the front of $unbacktracked[s']$
  if $untried[s']$ is empty **then**
    if $unbacktracked[s']$ is empty **then return** $stop$
    else $a \leftarrow$ an action b such that $result[s', b]$ = POP($unbacktracked[s']$)
  else $a \leftarrow$ POP($untried[s']$)
  $s \leftarrow s'$
  return a

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Online Local Search

- Hill Climbing natural candidate for online search
  - locality of search
  - only one state is stored
  - unfortunately, stuck in local minima
  - random restarts not possible
- Possible solution: Random Walk
  - selects randomly one available actions from the current state
  - preference can be given to actions that have not yet been tried
  - eventually finds a goal or complete its exploration if space is finite
  - unfortunately, very slow

Random Walk: example

- random walk takes exponentially many steps to find a goal (backward progress is twice as likely as forward progress)

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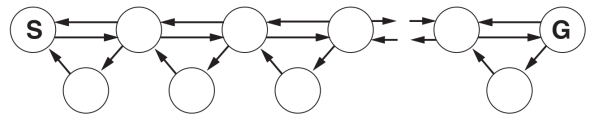

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Online A^*: $LRTA^*$

$LRTA^*$: General ideas

- Better possible solution: add memory to hill climbing
- Idea: store a "current best estimate" H(s) of the cost to reach the goal from each state that has been visited
  - initially $h(s)$
  - updated as the agent gains experience in the state space

  (recall that $h(s)$ is in general "too optimistic")

$\Longrightarrow$ Learning Real-Time A^* ($LRTA^*$)

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- chooses the "apparently best" move a according to current $H()$
- updates the cost estimate $H(s)$ for the state s it has just left, using the cost estimate of the target state s'
  - $H(s) := c(s, a, s') + H(s')$
- "optimism under uncertainty": untried actions in s are assumed to lead immediately to the goal with the least possible cost $h(s)$

$\Longrightarrow$ encourages the agent to explore new, possibly promising paths

An $LRTA^*$ agent is guaranteed to find a goal in any finite, safely explorable environment.

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  (recall that $h(s)$ is in general “too optimistic”)

$\Longrightarrow$ Learning Real-Time A^* ($LRTA^*$)

- builds a map of the environment in the result[s,a] table
- chooses the “apparently best” move a according to current $H()$
- updates the cost estimate $H(s)$ for the state s it has just left, using the cost estimate of the target state s'
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- ”optimism under uncertainty”: untried actions in s are assumed to lead immediately to the goal with the least possible cost $h(s)$

$\Longrightarrow$ encourages the agent to explore new, possibly promising paths

An $LRTA^*$ agent is guaranteed to find a goal in any finite, safely explorable environment.

Online A^*: $LRTA^*$

$LRTA^*$: General ideas

- Better possible solution: add memory to hill climbing
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```
function LRTA*-AGENT(s') returns an action
   inputs: s', a percept that identifies the current state
   persistent: result, a table, indexed by state and action, initially empty
               H, a table of cost estimates indexed by state, initially empty
               s, a, the previous state and action, initially null

   if GOAL-TEST(s') then return stop
   if s' is a new state (not in H) then H[s'] ← h(s')
   if s is not null
       result[s, a] ← s'
       H[s] ←       min        LRTA*-COST(s, b, result[s, b], H)
              b ∈ ACTIONS(s)
   a ← an action b in ACTIONS(s') that minimizes LRTA*-COST(s', b, result[s', b], H)
   s ← s'
   return a

function LRTA*-COST(s, a, s', H) returns a cost estimate
   if s' is undefined then return h(s)
   else return c(s, a, s') + H[s']
```

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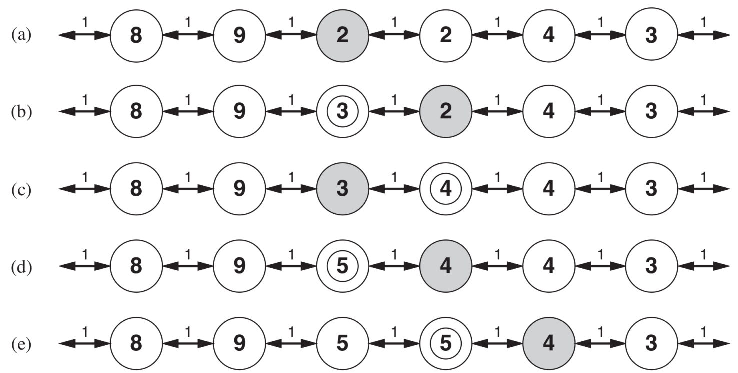
Example: *LRTA**

Five iterations of *LRTA** on a one-dimensional state space

- states labeled with current H(s), arcs labeled with step cost
- shaded state marks the location of the agent,
- updated cost estimates a each iteration are circled



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