# Course "**Fundamentals of Artificial Intelligence**"
# EXAM TEXT

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[COPY WITH SOLUTIONS]

# 1

Consider propositional logic (PL); let $A$, $B$, $C$, $D$, $E$, $F$, $G$ be atomic propositions. For each of the following statements, say if it is true or false.

($a$) $((A \wedge B) \rightarrow C)$ is equivalent to $(A \rightarrow (B \rightarrow C))$
[ Solution: True ]

($b$) $((A \leftrightarrow \neg B) \leftrightarrow \neg C)$ is valid if and only if $((\neg A \leftrightarrow (B \leftrightarrow C))$ is unsatisfiable
[ Solution: True ]

($c$) $(A \wedge \neg B) \vee (C \wedge \neg D)$ is equivalent to $(E \vee F) \wedge (E \leftrightarrow (A \wedge \neg B)) \wedge (F \leftrightarrow (C \wedge \neg D))$
[ Solution: False ]

($d$) $(A \wedge \neg B) \models \neg C$ if and only if $A \models \neg C$ and $\neg B \models \neg C$
[ Solution: False ]

# 2

Consider first-order-logic (FOL); let $P, Q, R$ be predicates; let $x, y$ be variables.
For each of the following statements, say if it is true or false.

$(a)$ $\forall x.(P(x) \lor Q(x))$ is equivalent to $(\forall x.P(x)) \lor (\forall y.Q(y))$
[ Solution: false ]

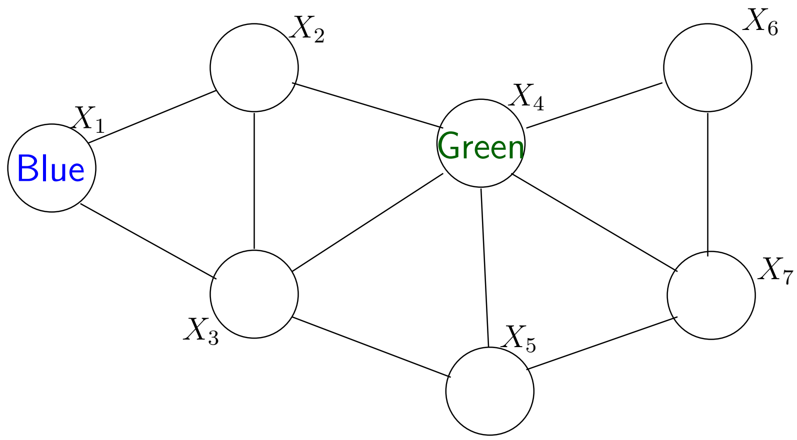
$(b)$ $\exists y.\forall x.R(x, y) \models \forall x.\exists y.R(x, y)$
[ Solution: true ]

$(c)$ $\exists x.P(x) \lor \exists x.Q(x) \models \exists x.(P(x) \lor Q(x))$
[ Solution: true ]

$(d)$ $\forall x \exists y.(P(x) \to Q(x, y))$ is equivalent to $\neg P(x) \lor Q(x, F_1(x))$ for some Skolem function $F_1$
[ Solution: false ]

# 3

Consider the following constraint graph of a map coloring problem, with domain $D \stackrel{\text{def}}{=} \{\mathsf{Green}, \mathsf{Red}, \mathsf{Blue}\}$, and consider the partial value assignment induced by the following unary constraints: $\{X_1 = \mathsf{Blue}, X_4 = \mathsf{Green}\}$.




For each of the following facts, say if it is true or false

($a$) $X_1 \neq \mathsf{Red}$ can be inferred by one node-consistency propagation step
[ Solution: true ]

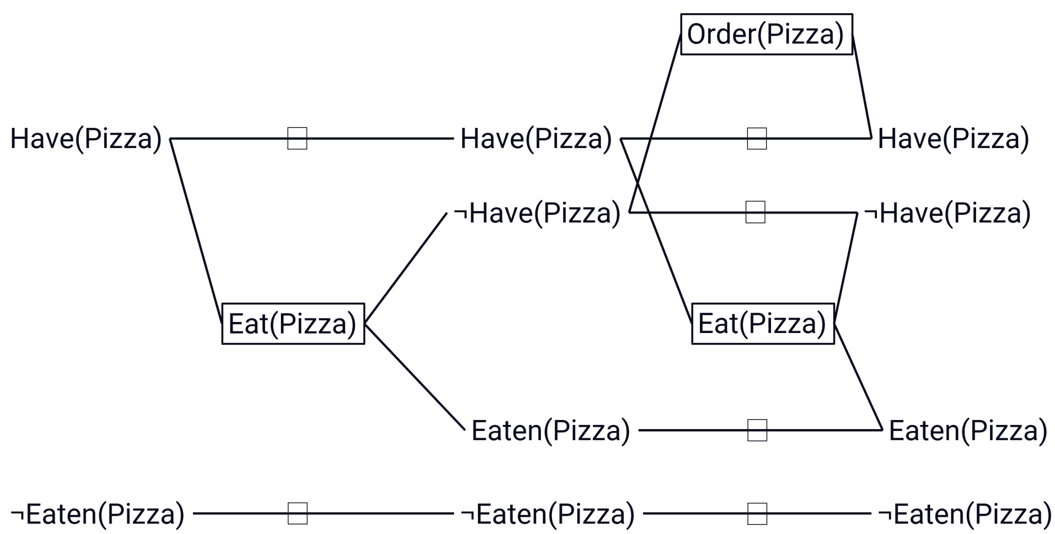
($b$) $X_2 \neq \mathsf{Green}$ can be inferred by forward checking
[ Solution: true ]

($c$) Forward checking allows for detecting an inconsistency.
[ Solution: false ]

($d$) AC-3 allows for detecting an inconsistency.
[ Solution: true ]

# 4

Consider the graph shown below.




For each of the following facts, say if it is true or false.

(*a*) At level $A_0$ there is a mutex between Eat and the persistence of Have
[ Solution: true ]

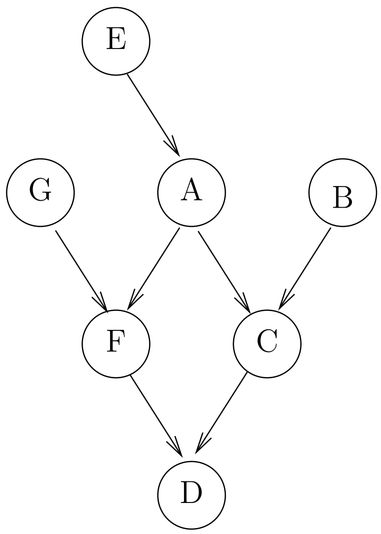
(*b*) At level $A_1$ the mutex between Eat and Order is a competing needs.
[ Solution: true ]

(*c*) At level $S_1$ there is a mutex between ¬ Have and Eaten
[ Solution: false ]

(*d*) At level $A_1$ the mutex between Eat and the persistence of Have is an interference
[ Solution: true ]

# 5

Consider the following DAG of a Bayesian network.




For each of the following facts, say if it is true or false.

($a$) $\mathbf{P}(F|GACB) = \mathbf{P}(F|GA)$
[ Solution: true, due to local semantics ]

($b$) $\mathbf{P}(F|GAD) = \mathbf{P}(F|GA)$
[ Solution: false ]

($c$) $\mathbf{P}(F|GAE) = \mathbf{P}(F|GA)$
[ Solution: true, due to local semantics ]

($d$) $\mathbf{P}(F|ABCDEG) = \mathbf{P}(F|ACDG)$
[ Solution: true, due to Markov blanket rule ]

# 6

Given the following set of propositional clauses $\Gamma$:

$$\begin{array}{l}
(\ \ A \lor \ \ D \lor \neg F) \\
(\neg A \lor \neg B \lor \ \ C) \\
(\ \ A) \\
(\neg A \lor \ \ B) \\
(\neg G) \\
(\ \ B \lor \ \ E \lor \neg G) \\
(\neg C \lor \ \ E) \\
(\neg B \lor \neg C \lor \ \ D) \\
(\neg E \lor \ \ F) \\
(\ \ C \lor \neg E \lor \ \ G) \\
(\neg B \lor \neg F \lor \ \ G)
\end{array}$$

Produce a PL-resolution proof that $\Gamma$ is unsatisfiable.
Such proof must be written as a sequence of resolution steps in the form:

$[Clause_{11},\ Clause_{12}] \Longrightarrow Resolvent\_Clause_1;$
$[Clause_{21},\ Clause_{22}] \Longrightarrow Resolvent\_Clause_2;$
...;
$[Clause_{k1},\ Clause_{k2}] \Longrightarrow Resolvent\_Clause_k;$

s.t. $Resolvent\_Clause_k$ is the empty clause, and each $Clause_{ij}$ is either in $\Gamma$ or is a resolvent clause $Resolvent\_Clause_m$ resulting from previous steps, i.e. s.t. $m < i$.

[ Solution:

$[(\ \ A),\ (\neg A \lor \neg B \lor \ \ C)] \ \Longrightarrow (\neg B \lor \ \ C);$
$[(\ \ A),\ (\neg A \lor \ \ B)] \ \Longrightarrow (\ \ B);$
$[(\ \ B),\ (\neg B \lor \ \ C)] \ \Longrightarrow (\ \ C);$
$[(\ \ B),\ (\neg B \lor \neg F \lor \ \ G)] \ \Longrightarrow (\neg F \lor \ \ G);$
$[(\ \ C),\ (\neg C \lor \ \ E)] \ \Longrightarrow (\ \ E);$
$[(\ \ E),\ (\neg E \lor \ \ F)] \ \Longrightarrow (\ \ F);$
$[(\ \ F),\ (\neg F \lor \ \ G)] \ \Longrightarrow (\ \ G);$
$[(\neg G),\ (\ \ G)] \ \Longrightarrow ();$

]

# 7

For each of the following FOL formulas, compute its CNF-Ization.
Use symbols $C_1, C_2, C_3, ...$ for Skolem constants and symbols $F_1, F_2, F_3, ...$ for Skolem functions.

$(a)$ $\forall x.(\exists y.P(x,y) \rightarrow \forall z.Q(x,z))$
[ Solution:
$\forall x.(\exists y.P(x,y) \rightarrow \forall z.Q(x,z))$
$\forall x.(\neg\exists y.P(x,y) \vee \forall z.Q(x,z))$
$\forall x.(\forall y.\neg P(x,y) \vee \forall z.Q(x,z))$
$\neg P(x,y) \vee Q(x,z)$ ]

$(b)$ $\forall x.(\forall y.P(x,y) \rightarrow \exists z.Q(x,z))$
[ Solution:
$\forall x.(\forall y.P(x,y) \rightarrow \exists z.Q(x,z))$
$\forall x.(\neg\forall y.P(x,y) \vee \exists z.Q(x,z))$
$\forall x.(\exists y.\neg P(x,y) \vee \exists z.Q(x,z))$
$\neg P(x,F_1(x)) \vee Q(x,F_2(x))$ ]

$(c)$ $\exists x.\forall y.\exists z.P(x,y,z)$
[ Solution:
$P(C_1, y, F_1(x)$ ]

$(d)$ $(\exists x.\forall y.\exists z.P(x,y,z)) \rightarrow (\exists x.\forall y.\exists z.Q(x,y,z))$
[ Solution:
$(\exists x.\forall y.\exists z.P(x,y,z)) \rightarrow (\exists x.\forall y.\exists z.Q(x,y,z))$
$(\neg\exists x.\forall y.\exists z.P(x,y,z)) \vee (\exists x.\forall y.\exists z.Q(x,y,z))$
$(\forall x.\exists y.\forall z.\neg P(x,y,z)) \vee (\exists x.\forall y.\exists z.Q(x,y,z))$
$\neg P(x, F_1(x), z) \;\vee\; Q(C_2, y, F_2(y)$ ]

# 8

Given:

- a set of basic concepts: $\{\mathsf{Person}, \mathsf{Dog}, \mathsf{Cat}, \mathsf{Female}, \mathsf{Male}\}$
- a set of relations: $\{\mathsf{hasChild}, \mathsf{hasPet}\}$

with their standard meaning ("hasChild" refers also to animals).
Write a $\mathcal{T}$-box in $\mathcal{ALCN}$ description logic defining the following concepts

(*a*) DogLover: a person with at least two dogs
[ Solution:
$\mathsf{DogLover} \equiv \mathsf{Person} \sqcap (\geq 2)\mathsf{hasPet}.\mathsf{Dog}$
or any $\mathcal{T}$-box which is logically equivalent to it ]

(*b*) ChildlessFemaleCat: childless female cat
[ Solution:
$\mathsf{ChildlessFemaleCat} \equiv \mathsf{Cat} \sqcap \mathsf{Female} \sqcap \neg\exists.\mathsf{hasChild}.\mathsf{Cat}$
or any $\mathcal{T}$-box which is logically equivalent to it ]

(*c*) PersonWithMaleDogs: a person with male dogs
[ Solution:
$\mathsf{PersonWithMaleDogs} \equiv \mathsf{Person} \sqcap \exists\mathsf{hasPet}(\mathsf{Dog} \sqcap \mathsf{Male})$
or any $\mathcal{T}$-box which is logically equivalent to it ]

(*d*) ManWithDogsOrCats: man whose pets are all dogs or [1] cats
[ Solution:
$\mathsf{ManWithDogsOrCats} \equiv \mathsf{Person} \sqcap \mathsf{Male} \sqcap \forall\mathsf{hasPet}(\mathsf{Dog} \sqcup \mathsf{Cat})$
or any $\mathcal{T}$-box which is logically equivalent to it ]

[1] non-exclusive or.

# 9

Given the random propositional variables $A, B, C$ and their joint probability distribution $\mathbf{P}(\mathbf{A}, \mathbf{B}, \mathbf{C})$ described as follows:

| $A$ | $B$ | $C$ | $\mathbf{P}(\mathbf{A}, \mathbf{B}, \mathbf{C})$ |
|---|---|---|---|
| $T$ | $T$ | $T$ | 0.056 |
| $T$ | $T$ | $F$ | 0.024 |
| $T$ | $F$ | $T$ | 0.192 |
| $T$ | $F$ | $F$ | 0.048 |
| $F$ | $T$ | $T$ | 0.024 |
| $F$ | $T$ | $F$ | 0.096 |
| $F$ | $F$ | $T$ | 0.128 |
| $F$ | $F$ | $F$ | 0.432 |

$(a)$ Using marginalization, compute the probability $P(a)$

[ Solution:

$P(a)$

$= P(a, b, c) + P(a, b, \neg c) + P(a, \neg b, c) + P(a, \neg b, \neg c) =$

$= 0.056 + 0.024 + 0.192 + 0.048 = 0.32$ ]

$(b)$ Using normalization, compute the probability $P(a|b, c)$

[ Solution:

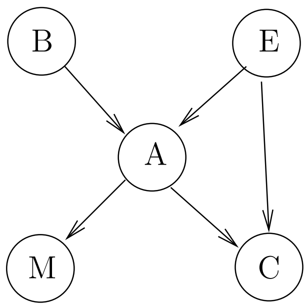
$$P(a|b, c) = \alpha \cdot P(a, b, c) = \overbrace{\frac{1}{P(a, b, c) + P(\neg a, b, c)}}^{\alpha} \cdot P(a, b, c) =$$

$1/(0.056 + 0.024) * 0.056 = 0.7$ ]

Notation: uppercase letters are used for propositional variables representing random events, whereas the corresponding lowercase letters represent truth assignments to such propositional variables.
Ex: $a \stackrel{\text{def}}{=} (A = \mathsf{true})$, $\neg a \stackrel{\text{def}}{=} (A = \mathsf{false})$.

# 10

Consider the following simple Bayesian network, where $\mathbf{P}(\mathbf{B})$, $\mathbf{P}(\mathbf{E})$, $\mathbf{P}(\mathbf{A}|\mathbf{B},\mathbf{E})$, $\mathbf{P}(\mathbf{M}|\mathbf{A})$, $\mathbf{P}(\mathbf{C}|\mathbf{A},\mathbf{E})$, are defined in the following annex file (columns O-P, reported also in columns F-J):
2021.06.15-25390314-Bayes.xlsx




($a$) Compute the full distribution $\mathbf{P}(\mathbf{A},\mathbf{B},\mathbf{E},\mathbf{M})$ and write it into column K of the above-mentioned file.

($b$) Can you say something about $\mathbf{P}(\mathbf{M}|\mathbf{A},\mathbf{B},\mathbf{E})$ without any further computation?

($c$) Compute by normalization the partial distribution $\mathbf{P}(\mathbf{M}|\mathbf{A},\mathbf{B},\mathbf{E})$ and write it in column L and the normalization values $\alpha$ in column M of the above-mentioned file. Verify that the result complies with your answer of point ($b$)

You can use the arithmetic excel operators $(=,+,-,*,/)$
[ Solution:
Consider the annex file 2021.06.15-25390314-Bayes-sol.xlsx

We first notice that $\mathbf{P}(\mathbf{A},\mathbf{B},\mathbf{E},\mathbf{M})$ and $\mathbf{P}(\mathbf{M}|\mathbf{A},\mathbf{B},\mathbf{E})$ are independent from $\mathbf{C}$ by Global Semantics and Independence Theorem respectively, so that we can ignore $\mathbf{P}(\mathbf{C}|\mathbf{A},\mathbf{E})$.

($a$) The full distribution $\mathbf{P}(\mathbf{A},\mathbf{B},\mathbf{E},\mathbf{M}) \stackrel{\text{def}}{=} \mathbf{P}(\mathbf{B}) \cdot \mathbf{P}(\mathbf{E}) \cdot \mathbf{P}(\mathbf{A}|\mathbf{B},\mathbf{E}) \cdot \mathbf{P}(\mathbf{M}|\mathbf{A})$ is reported in column K of the above-mentioned file.

($b$) By construction of Bayesian networks, $\mathbf{M}$ is independent on $\mathbf{B}$ and $\mathbf{E}$ given $\mathbf{A}$, so that we have that $\mathbf{P}(\mathbf{M}|\mathbf{A},\mathbf{B},\mathbf{E}) = \mathbf{P}(\mathbf{M}|\mathbf{A})$.

($c$) By normalization, the partial distribution $\mathbf{P}(\mathbf{M}|\mathbf{A},\mathbf{B},\mathbf{E}) = \alpha \cdot \mathbf{P}(\mathbf{A},\mathbf{B},\mathbf{E},\mathbf{M})$, where, for each truth value combination of $\langle A,B,E\rangle$, $\alpha \stackrel{\text{def}}{=} 1/(P(M,A,B,E) + P(\neg M,A,B,E))$. The result is reported in column L of the above-mentioned file, and the values of $\alpha$ in column M.
Notice that $\mathbf{P}(\mathbf{M}|\mathbf{A},\mathbf{B},\mathbf{E}) = \mathbf{P}(\mathbf{M}|\mathbf{A})$, –possibly modulo numerical roundings– by comparing column F with column L.

]

# 11

Given the tree provided within the file 2021-06-15-alphabetapruning-25390314.pptx in your Google Drive folder, please do the following tasks:

- Report the alpha, beta, and node values for each MIN and MAX nodes. Each value has to be provided directly within the file by replacing the $\infty$ symbol for the alpha and beta values, and by replacing the dot within each MIN and MAX node.
- Mark the pruned branches. When a pruning operation is performed, each element under the pruned branch (including the pruned branch) has to be colored in red (it is enough to set the border colors in red).

An example is contained within your Google Drive folder.

[ Solution:

The solutions is available within the file 2021-06-15-alphabetapruning-25390314-withsolutions.pptx in your Google Drive folder.

]

# 12

The graph contained within the 2021-06-15-astar-25390314.pptx file in your Google Drive folder represents the states space of a hypothetical search problem where:

- States are denoted by letters.
- Arcs are labeled with the cost of traversing them.
- The estimated cost to a goal (i.e., the $h$ function) is reported inside nodes (so that lower scores are better).

Considering $ST$ as the initial state and $EN$ the goal state, please apply the A* search algorithm and report each step of the resolution process. Then, explain if the heuristic adopted is admissible or not. The solution format has to be provided as shown in the example contained in the file 2021-01-12-a-star-example.pdf The table to fill is already contained in the 2021-06-15-astar-25390314.pptxfile.

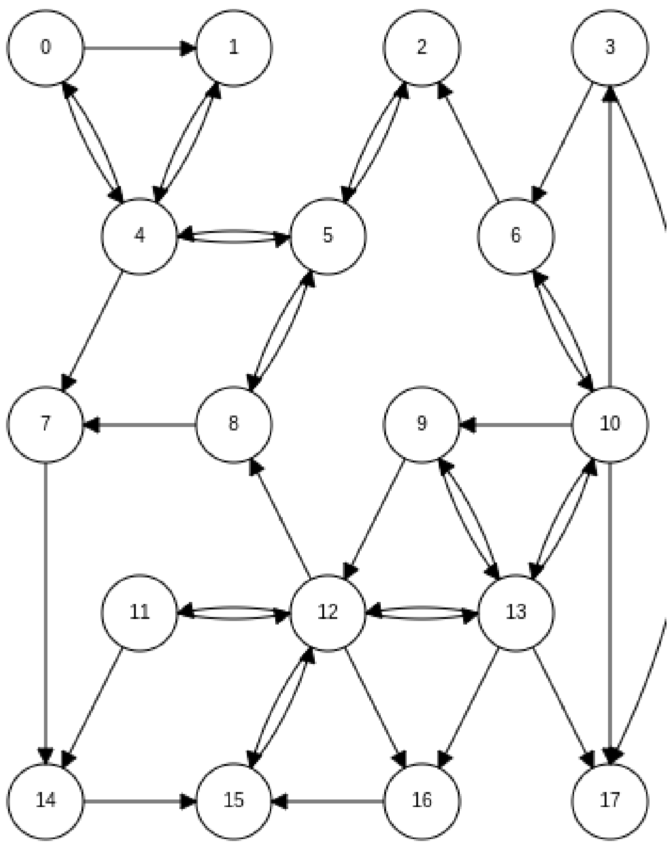
[ Solution:

The solutions is available within the file 2021-06-15-astar-25390314-withsolutions.pptx in your Google Drive folder.

]

# 13

Consider the graph shown below.




Please complete the following tasks.

- Provide the list of nodes explored by the BFS algorithm performing the goal test before the generation of a new node.
- Provide the list of nodes explored by the DFS algorithm.

For both BFS and DFS algorithm, the starting node is 0 and the goal state is 17. Nodes are explored in numerical ascending order.

[ Solution:

Exploration of the BFS algorithm:
0, 1, 4, 5, 7, 2, 8, 14, 15, 12, 11, 13, 16, 9, 10, 17

Exploration of the DFS algorithm:
0, 1, 4, 5, 2, 8, 7, 14, 15, 12, 11, 13, 9, 10, 3, 6, 17
]

# 14

TEXT

Consider the following constraint network.

Variables: $X_1, X_2, X_3, X_4, X_5$

Domains: $D_1 = \{3,4,5,7,9\}, D_2 = \{2,4,6,8,9\}, D_3 = \{1,2,6,8,9\}, D_4 = \{1,2,3,8,9\}, D_5 = \{2,5,6,7,8\}$ Constraints:

$$X_1 < X_2 \text{ } or \text{ } X_2 - X_1 = 2$$

$$X_2 > X_3$$

$$X_2 < X_4 \text{ } or \text{ } X_2 - X_4 = 1$$

$$X_3 < X_5$$

Please complete the following tasks.

($a$) Is the network arc-consistent? If not, compute the arc-consistent network.

($b$) If the consistency holds, provide the first admissible solution by exploring the domains from $D_1$ to $D_5$ and the values in ascending order.

[ Solution:

Enforce arc consistency between $X_1$ and $X_2$ led to $D_1 = \{3,4,5,7\}$ and $D_2 = \{4,6,8,9\}$
Enforce arc consistency between $X_2$ and $X_3$ led to $D_3 = \{1,2,6,8\}$
Enforce arc consistency between $X_2$ and $X_4$ led to $D_4 = \{3,8,9\}$
Enforce arc consistency between $X_3$ and $X_5$ led to $D_3 = \{1,2,6\}$

One possible solution: $X_1 = 3,\ X_2 = 4,\ X_3 = 1,\ X_4 = 3,\ X_5 = 2$ ]

# 15

Given the image provided within the file 2021-06-15-intervals-25390314.png in your Google Drive folder, please state the interval-algebra relations that hold between the provided pairs related to the described real-world event:

Pairs:
$\langle FirstHalf, AwayTeamAhead\rangle$
$\langle OnanaWarned, OpendaScored\rangle$
$\langle BoussaidPlayed, OnanaPlayed\rangle$
$\langle SovetWarned, AwayTeamAhead\rangle$
$\langle SardellaScored, SecondHalf\rangle$
$\langle OnanaScored, KenessovPlayed\rangle$
$\langle FirstHalf, SiquetPlayed\rangle$
$\langle OnanaWarned, TieResult\rangle$

Notice and notations:

- events like goals, have not to be intended as instantaneous events, but like events during a certain (small) amount of time;
- a player is intended to be warned from the moment in which he received the yellow card, until the end of the match or until the moment in which he is substituted;
- you have to assume that the halftime break exists.
- the list of relations have to be provided by using the format: $Relation(Event1, Event2)$

[ Solution:

$Finishes(FirstHalf, AwayTeamAhead)$
$During(OpendaScored, OnanaWarned)$
$Overlap(BoussaidPlayed, OnanaPlayed)$
$Finishes(SovetWarned, AwayTeamAhead)$
$Before(SardellaScored, SecondHalf)$
$Before(OnanaScored, KenessovPlayed)$
$Starts(FirstHalf, SiquetPlayed)$
$After(TieResult, OnanaWarned)$

]