Course "**Fundamentals of Artificial Intelligence**"
EXAM TEXT

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2021.02.17

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[COPY WITH SOLUTIONS]

1

Let φ be a generic Boolean formula, and let $\varphi_1 \stackrel{\text{def}}{=} CNF(\varphi)$, s.t. $CNF()$ is the "classic" CNF conversion (i.e., the one using DeMorgan's rules). Let $|\varphi|$ and $|\varphi_1|$ denote the size of φ and φ_1 respectively.

For each of the following sentences, say if it is true or false.

(a) $|\varphi_1|$ is in worst-case polynomial in size wrt. $|\varphi|$. [Solution: False.]

(b) φ_1 has the same number of distinct Boolean variables as φ has. [Solution: True.]

(c) A model for φ_1 (if any) is also a model for φ, and vice versa. [Solution: True.]

(d) φ_1 is valid if and only if φ is valid. [Solution: True.]

2

Consider first-order-logic (FOL); let P, Q, R be predicates; let x, z be variables.
For each of the following statements, say if it is true or false.

(a) $\forall x.(P(x) \land Q(x))$ is equivalent to $(\forall x.P(x)) \land (\forall z.Q(z))$
[Solution: true]

(b) $\forall x.\exists z.R(x, z) \models \exists z.\forall x.R(x, z)$
[Solution: false]

(c) $(\exists x.P(x)) \land (\exists x.Q(x)) \models \exists x.(P(x) \land Q(x))$
[Solution: false]

(d) $\forall x.(P(x) \to Q(x))$ is equivalent to $\neg\exists x.(P(x) \land \neg Q(x))$
[Solution: true]

3

Given a generical search problem, assume time and space complexity are measured in terms of

b : maximum branching factor of the search tree

m : maximum depth of the state space (assume m is finite)

s : depth of the shallowest solution

Assume also that all steps cost are 1.

For each of the following facts, say if it is true or false

(a) Breadth-First Search requires $O(bm)$ memory to find a solution.
[Solution: false: it requires $O(b^s)$ memory]

(b) Depth-First Search with loop-prevention requires $O(b^s)$ time to find a solution.
[Solution: false: it requires $O(b^m)$ time]

(c) Iterative-Deepening-Search is optimal
[Solution: true]

(d) Iterative-Deepening-Search requires $O(bs)$ memory to find a solution.
[Solution: true]

4

Consider the following time-interval definitions in Allen's interval algebra:

(i) $VolleyMatch \stackrel{\text{def}}{=} (3.00pm, 4.45pm)$

(ii) $Commercials \stackrel{\text{def}}{=} (3.45pm, 4.00pm)$

(iii) $Movie \stackrel{\text{def}}{=} (4.00pm, 6.10pm)$

.

For each of the following facts, say if it is true or false

(a) $During(Commercials, VolleyMatch)$
[Solution: true]

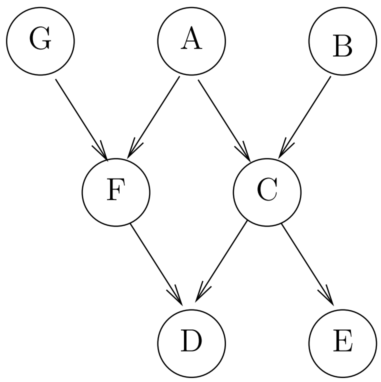
(b) $Meet(Commercials, Movie)$
[Solution: true]

(c) $Before(Commercials, Movie)$
[Solution: false]

(d) $Overlap(Movie, VolleyMatch)$
[Solution: false]

5

Consider the following DAG of a Bayesian network.



For each of the following facts, say if it is true or false.

(a) $\mathbf{P}(C|ABF) = \mathbf{P}(C|AB)$
[Solution: true, due to local semantics]

(b) $\mathbf{P}(C|ABD) = \mathbf{P}(C|AB)$
[Solution: false]

(c) $\mathbf{P}(C|DEF) = \mathbf{P}(C|DE)$
[Solution: false]

(d) $\mathbf{P}(C|ABDEFG) = \mathbf{P}(C|ABDEF)$
[Solution: true, due to Markov blanket rule]

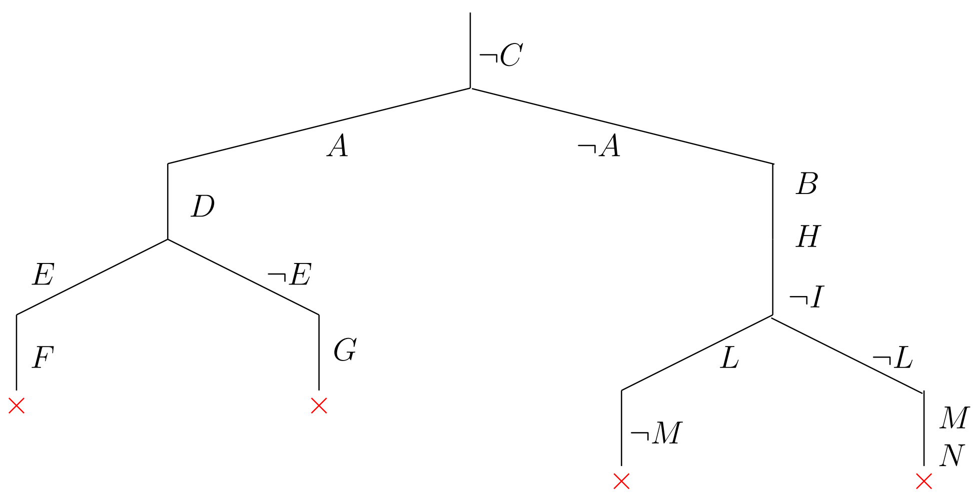
6

Consider the following CNF formula in PL:

$$
\begin{array}{l}
(\neg C\ ) \wedge \\
(\ B \vee A \vee C) \wedge \\
(\neg A \vee D\ ) \wedge \\
(\neg E \vee \neg A \vee F) \wedge \\
(\neg E \vee \neg F \vee \neg A) \wedge \\
(\ G \vee \neg A \vee E) \wedge \\
(\ E \vee \neg G \vee \neg A) \wedge \\
(\ A \vee H \vee C) \wedge \\
(\neg H \vee \neg I \vee A) \wedge \\
(\ I \vee L \vee M) \wedge \\
(\neg L \vee C \vee \neg M) \wedge \\
(\ A \vee \neg L \vee M) \wedge \\
(\ L \vee N \vee \neg H) \wedge \\
(\ I \vee L \vee \neg N)
\end{array}
$$

Draw the search tree obtained by applying to the above formula the DPLL algorithm without the pure-symbol rule. Variables should be chosen according to **alphabetic order**, and assigned **true** first.

[Solution:



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7

Consider the following FOL KB:

1. $\forall x.\{[\forall y.(\mathsf{Child}(y) \to \mathsf{Loves}(x,y))] \to [\exists y.\mathsf{Loves}(y,x)]\}$
2. $\forall x.[\mathsf{Child}(x) \to \mathsf{Loves}(\mathsf{Mark},x)]$
3. $\mathsf{Beats}(\mathsf{Mark},\mathsf{Paul}) \lor \mathsf{Beats}(\mathsf{John},\mathsf{Paul})$
4. $\mathsf{Child}(\mathsf{Paul})$
5. $\forall x.\{[\exists z.(\mathsf{Child}(z) \land \mathsf{Beats}(x,z))] \to [\forall y.\neg\mathsf{Loves}(y,x)]\}$

(a) Compute the CNF-ization of the KB, Skolemize & standardize variables

(b) Write a FOL-resolution inference of the query $\mathsf{Beats}(\mathsf{John},\mathsf{Paul})$ from the CNF-ized KB

[Solution:

(a) Compute the CNF-ization of the KB, Skolemize & standardize variables:

1. $\forall x.\{[\forall y.(\mathsf{Child}(y) \to \mathsf{Loves}(x,y))] \to [\exists y.\mathsf{Loves}(y,x)]\} \Longrightarrow$ (rewrite "$\to$")
   $\forall x.\{[\neg\forall y.(\mathsf{Child}(y) \to \mathsf{Loves}(x,y))] \lor [\exists y.\mathsf{Loves}(y,x)]\} \Longrightarrow$ (rewrite "$\neg\forall$","$\neg \to$")
   $\forall x.\{[\exists y.(\mathsf{Child}(y) \land \neg\mathsf{Loves}(x,y))] \lor [\exists y.\mathsf{Loves}(y,x)]\} \Longrightarrow$ (drop quantifiers & skolemize)
   $\{[(\mathsf{Child}(F(x)) \land \neg\mathsf{Loves}(x,F(x)))] \lor [\mathsf{Loves}(G(x),x)]\} \Longrightarrow$ (apply DeMorgan rule, standarize)
   1. $\mathsf{Child}(F(x)) \lor \mathsf{Loves}(G(x),x)$
   2. $\neg\mathsf{Loves}(y,F(y)) \lor \mathsf{Loves}(G(y),y)$
2. $\neg\mathsf{Child}(z) \lor \mathsf{Loves}(\mathsf{Mark},z)$
3. $\mathsf{Beats}(\mathsf{Mark},\mathsf{Paul}) \lor \mathsf{Beats}(\mathsf{John},\mathsf{Paul})$
4. $\mathsf{Child}(\mathsf{Paul})$
5. $\forall x.\{[\exists z.(\mathsf{Child}(z) \land \mathsf{Beats}(x,z))] \to [\forall y.\neg\mathsf{Loves}(y,x)]\} \Longrightarrow$ (rewrite "$\to$")
   $\forall x.\{[\neg\exists z.(\mathsf{Child}(z) \land \mathsf{Beats}(x,z))] \lor [\forall y.\neg\mathsf{Loves}(y,x)]\} \Longrightarrow$ (rewrite "$\neg\exists$","$\neg\land$")
   $\forall x.\{[\forall z.(\neg\mathsf{Child}(z) \lor \neg\mathsf{Beats}(x,z))] \lor [\forall y.\neg\mathsf{Loves}(y,x)]\} \Longrightarrow$ (drop quantifiers, standarize)
   $\neg\mathsf{Child}(z_2) \lor \neg\mathsf{Beats}(x_2,z_2)\neg \lor \mathsf{Loves}(y_2,x_2)$

where $F(), G()$ are Skolem unary functions.

(b) Write a FOL-resolution inference of the query $\mathsf{Beats}(\mathsf{John},\mathsf{Paul})$ from the CNF-ized KB:

6. $[1.2,\ 2.] \Longrightarrow \neg\mathsf{Child}(F(\mathsf{Mark})) \lor \mathsf{Loves}(G(\mathsf{Mark}),\mathsf{Mark})$;
7. $[1.1,\ 6.] \Longrightarrow \mathsf{Loves}(G(\mathsf{Mark}),\mathsf{Mark})$;
8. $[4,\ 5.] \Longrightarrow \neg\mathsf{Beats}(x_2,\mathsf{Paul}) \lor \neg\mathsf{Loves}(y_2,x_2)$;
9. $[7,\ 8.] \Longrightarrow \neg\mathsf{Beats}(\mathsf{Mark},\mathsf{Paul})$;
10. $[3,\ 9.] \Longrightarrow \mathsf{Beats}(\mathsf{John},\mathsf{Paul})$;

]

8

Given:

- a set of basic concepts: $\{\mathsf{Person}, \mathsf{Male}, \mathsf{Doctor}, \mathsf{Engineer}\}$
- a set of relations: $\{\mathsf{hasChild}\}$

with their obvious meaning.
Write a $\mathcal{T}$-box in $\mathcal{ALCN}$ description logic defining the following concepts

(a) Female, Man, Woman (with their standard meaning)
[Solution:
$$\begin{array}{lcl} \mathsf{Female} & \equiv & \neg\mathsf{Male} \\ \mathsf{Man} & \equiv & \mathsf{Person} \sqcap \mathsf{Male} \\ \mathsf{Woman} & \equiv & \mathsf{Person} \sqcap \mathsf{Female} \end{array}$$
or any $\mathcal{T}$-box which is logically equivalent to it]

(b) femaleDoctorWithoutChildren: female doctor with no children
[Solution:
$$\mathsf{femaleDoctorWithoutChildren} \equiv \mathsf{Woman} \sqcap \mathsf{Doctor} \sqcap \neg\exists\mathsf{hasChild}.\mathsf{Person}$$
or any $\mathcal{T}$-box which is logically equivalent to it]

(c) fatherOfFemaleDoctor: father of at least two female doctors
[Solution:
$$\mathsf{fatherOfFemaleDoctor} \equiv \mathsf{Man} \sqcap (\geq 2)\mathsf{hasChild}(\mathsf{Female} \sqcap \mathsf{Doctor})$$
or any $\mathcal{T}$-box which is logically equivalent to it]

(d) motherOfDoctorsOrEngineers: woman whose children are all engineers or [1] doctors
[Solution:
$$\mathsf{motherOfDoctorsOrEngineers} \equiv \mathsf{Woman} \sqcap \forall\mathsf{hasChild}(\mathsf{Engineer} \sqcup \mathsf{Doctor})$$
or any $\mathcal{T}$-box which is logically equivalent to it]

[1] non-exclusive or.

9

Assume the following facts are known from medicine literature: [2]

- 4 persons over 1000 suffer of malaria
- 5 % of persons have high temperature
- one person with malaria has high temperature with probability 0.9

Given that a person has high temperature, compute the probability of having malaria.

[Solution:
From the data, we have that:

$$\begin{array}{lll} P(malaria) & = \frac{4}{1000} & = 0.004 \\ P(hightemperature) & = \frac{5}{100} & = 0.05 \\ P(hightemperature|malaria) & & = 0.9 \end{array}$$

Using Bayes' rule:

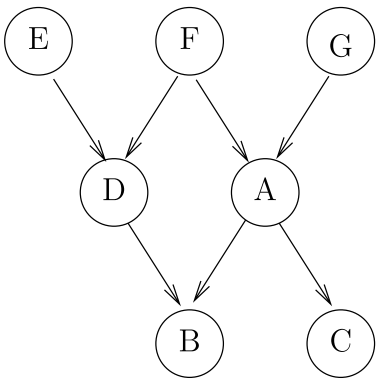
$$P(malaria|hightemperature) = \frac{P(hightemperature|malaria) \cdot P(malaria)}{P(hightemperature)} = \frac{0.9 \cdot 0.004}{0.05} = 0.072$$

]

[2] These data are pure fantasy and have no correspondence with real-world medicine.

10

Consider the following Bayesian network.



$\mathbf{P}$(F) = 0.4 $\mathbf{P}$(G)=0.3 $\mathbf{P}$(E)=0.2

E	$\mathbf{P}(D\|E)$
T	0.7
F	0.1

A	$\mathbf{P}(C\|A)$
T	0.4
F	0.7

F	G	$\mathbf{P}(A\|F,G)$
T	T	0.6
T	F	0.3
F	T	0.7
F	F	0.5

E	F	$\mathbf{P}(D\|E,F)$
T	T	0.5
T	F	0.3
F	T	0.1
F	F	0.4

D	A	$\mathbf{P}(B\|D,A)$
T	T	0.1
T	F	0.2
F	T	0.7
F	F	0.3

Compute $\mathbf{P}(F|a)$.

Notation: lowercase letters are used for literals representing truth assignment to Boolean variables.
Ex: $f \stackrel{\text{def}}{=} (F = \mathsf{true})$, $\neg a \stackrel{\text{def}}{=} (A = \mathsf{false})$.

[Solution:

We recall the independence theorem (slide 35 in Ch 14):
For query X and evidence E, Y is irrelevant unless $Y \in Ancestors(X \cup E)$.
Thus B, C, D, E are irrelevant:

$$\mathbf{P}(F|A) = \alpha\mathbf{P}(F,A) \stackrel{\text{def}}{=} \alpha \sum_{G,D,E,D,G} \mathbf{P}(F,A,G,B,C,D,G) = \alpha \sum_{G} \mathbf{P}(F,A,G) = \alpha \sum_{G} \mathbf{P}(A|F,G)\mathbf{P}(F)\mathbf{P}(G)$$

Thus:

$$\begin{array}{ll}
P(f|a) & = \\
\alpha \cdot [P(a|f,g) \cdot P(f) \cdot P(g) + P(a|f,\neg g) \cdot P(f) \cdot P(\neg g)] & = \\
\alpha \cdot P(f) \cdot [P(a|f,g) \cdot P(g) + P(a|f,\neg g) \cdot P(\neg g)] & = \\
\alpha \cdot 0.4 \cdot [0.6 \cdot 0.3 + 0.3 \cdot 0.7] & = \alpha \cdot 0.156 \\
P(\neg f|a) & = \\
(...) & = \\
\alpha \cdot P(\neg f) \cdot [P(a|\neg f,g) \cdot P(g) + P(a|\neg f,\neg g) \cdot P(\neg g)] & = \\
\alpha \cdot 0.6 \cdot [0.7 \cdot 0.3 + 0.5 \cdot 0.7] & = \alpha \cdot 0.336
\end{array}$$

$\alpha = 1/(0.156 + 0.336) = 2.0325$ thus $\mathbf{P}(F|a) = \alpha\langle 0.156, 0.336\rangle = \langle 0.317, 0.683\rangle$
]

11

Given the tree provided within the file 2020-02-18-alphabetapruning-v1.pptx in your Google Drive folder, please do the following tasks:

- Report the alpha, beta, and node values for each MIN and MAX nodes. Each value has to be provided directly within the file by replacing the ∞ symbol for the alpha and beta values, and by replacing the dot within each MIN and MAX node.
- Mark the pruned branches. When a pruning operation is performed, each element under the pruned branch (including the pruned branch) has to be colored in red (it is enough to set the border colors in red).

An example is contained within your Google Drive folder.

[Solution:

The solutions is available within the file 2020-02-18-alphabetapruning-v1-withsolutions.pptx in your Google Drive folder.

]

12

The graph contained within the 2020-02-18-astar-v1.pptx file in your Google Drive folder represents the states space of a hypothetical search problem where:

- States are denoted by letters.
- Arcs are labeled with the cost of traversing them.
- The estimated cost to a goal (i.e., the h function) is reported inside nodes (so that lower scores are better).

Considering ST as the initial state and EN the goal state, please apply the A* search algorithm and report each step of the resolution process. Then, explain if the heuristic adopted is admissible or not. The solution format has to be provided as shown in the example contained in the file 2021-01-12-a-star-example.pdf The table to fill is already contained in the 2020-02-18-astar-v1.pptxfile.

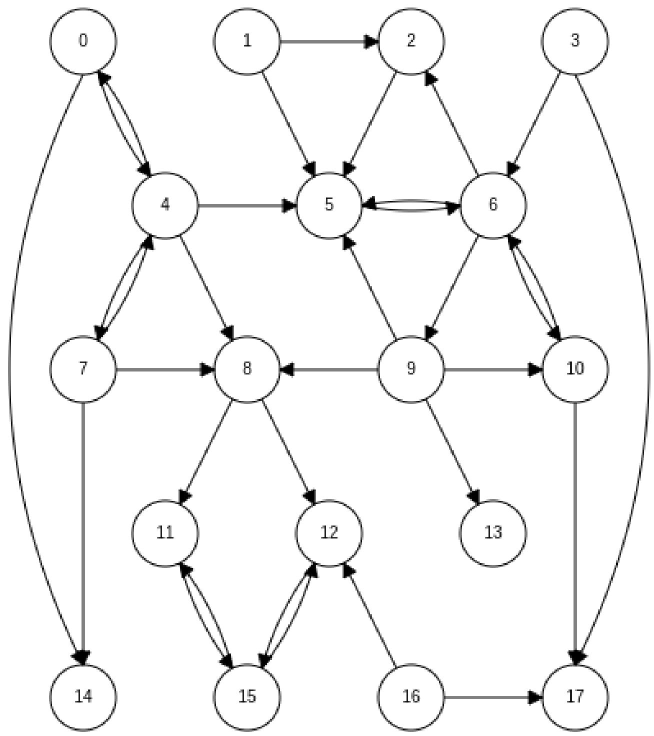
[Solution:

The solutions is available within the file 2020-02-18-astar-v1-withsolutions.pptx in your Google Drive folder.

]

13

Consider the graph shown below.



Please complete the following tasks.

- Provide the list of nodes explored by the BFS algorithm performing the goal test before the generation of a new node.
- Provide the list of nodes explored by the DFS algorithm.

For both BFS and DFS algorithm, the starting node is 0 and the goal state is 17. Nodes are explored in numerical ascending order.

[Solution:

Exploration of the BFS algorithm:
0, 4, 14, 5, 7, 8, 6, 11, 12, 2, 9, 10, 15, 13, 17

Exploration of the DFS algorithm:
0, 4, 5, 6, 2, 9, 8, 11, 15, 12, 10, 17
]

14

TEXT

Consider the following constraint network.

Variables: X_1, X_2, X_3, X_4, X_5

Domains: $D_1 = \{3,4,5,7,9\}, D_2 = \{2,4,6,8,9\}, D_3 = \{1,2,6,8,9\}, D_4 = \{1,2,3,8,9\}, D_5 = \{2,5,6,7,8\}$ Constraints:

$$X_1 > X_2 \ or \ X_2 - X_1 = 1$$

$$X_2 < X_3$$

$$X_2 > X_4 \ or \ X_2 - X_4 = 2$$

$$X_3 > X_5$$

Please complete the following tasks.

(a) Is the network arc-consistent? If not, compute the arc-consistent network.

(b) If the consistency holds, provide the first admissible solution by exploring the domains from D_1 to D_5 and the values in ascending order.

[Solution:

Enforce arc consistency between X_1 and X_2 led to $D_2 = \{2,4,6,8\}$

Enforce arc consistency between X_2 and X_3 led to $D_3 = \{6,8,9\}$

Enforce arc consistency between X_2 and X_4 led to $D_4 = \{1,2,3\}$

One possible solution: $X_1 = 3,\ X_2 = 2,\ X_3 = 6,\ X_4 = 1,\ X_5 = 2$]

15

Given the plan described below:

Init(Stocks(Wood))

Goal(Stocks(Wood) ∧ Burned(Wood))

Action(Burn(Wood)) PRECOND: Stocks(Wood) EFFECT: ¬Stocks(Wood) ∧ Burned(Wood))

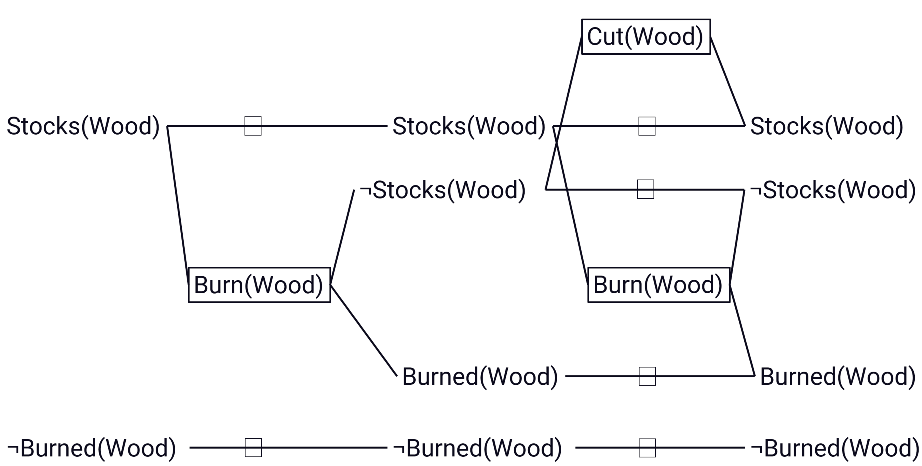
Action(Cut(Wood)) PRECOND: ¬Stocks(Wood) EFFECT: Stocks(Wood))

Draw the planning graph by using the following notation:

- Rectangles indicate actions.
- Small squares persistent actions (no-ops).
- Straight lines indicate preconditions and effects.
- Arcs indicate mutex links.

Within the planning graph, please enumerate the added mutex links and provide a description of the kind of mutex relation.

[Solution:



Inconsistent effect:
Persistence of Stocks(Wood), Burn(Wood)
¬Burned(Wood), Burn(Wood)
Cut(Wood), Burn(Wood)

Interference:
Persistence of Stocks(Wood), Burn(Wood)
Persistence of ¬Stocks(Wood), Cut(Wood)
Persistence of Stocks(Wood), Persistence of ¬Stocks(Wood)
Persistence of Burned(Wood), Persistence of ¬Burned(Wood)

Competing needs:
Cut(Wood), Burn(Wood)
Persistence of Stocks(Wood), Persistence of ¬Stocks(Wood)
Persistence of Burned(Wood), Persistence of ¬Burned(Wood)

Inconsistent support:
Stocks(Wood), ¬Stocks(Wood)
Burned(Wood), ¬Burned(Wood)

All ways of achieving them are pairwise mutex:
Stocks(Wood), Burned(Wood)
¬Stocks(Wood), ¬Burned(Wood)
]