# Course "**Fundamentals of Artificial Intelligence**"
# EXAM TEXT

Prof. Roberto Sebastiani
DISI, Università di Trento, Italy

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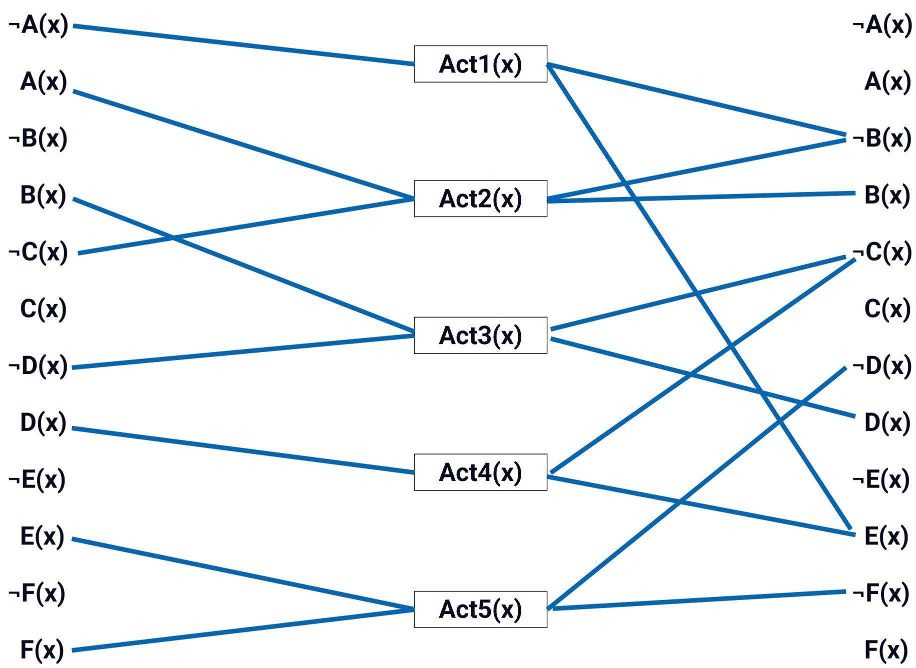
13276934

[COPY WITH SOLUTIONS]

# 1

Consider the graph shown below.




For each of the following facts, say if it is true or false.

(*a*) The mutex between Act4 and Act5 is an interference.
[ Solution: true ]

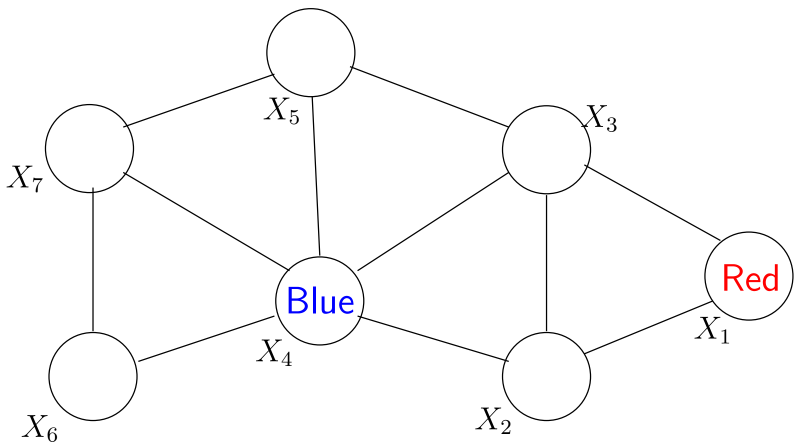
(*b*) There is at least one mutex between Act3 and Act5.
[ Solution: false ]

(*c*) The mutex between the persistence of C(x) and Act5 is an inconsistent effect.
[ Solution: false ]

(*d*) There are no mutex between C(x) and ¬C(x).
[ Solution: false ]

# 2

Consider the following constraint graph of a map coloring problem, with domain $D \stackrel{\text{def}}{=} \{\text{Blue}, \text{Green}, \text{Red}\}$, and consider the partial value assignment induced by the following unary constraints: $\{X_1 = \text{Red}, X_4 = \text{Blue}\}$ (see figure).




For each of the following facts, say if it is true or false

(a) $X_2 = \text{Green}$ can be inferred by one node-consistency propagation step
[ Solution: false ]

(b) $X_1 \neq \text{Blue}$ can be inferred by one node-consistency propagation step
[ Solution: true ]

(c) Forward checking allows for detecting an inconsistency.
[ Solution: false ]

(d) $X_2 = \text{Green}$ can be inferred by forward checking
[ Solution: true ]

# 3

Consider first-order-logic (FOL); let $Q, R, P$ be predicates; let $y, z$ be variables.
For each of the following statements, say if it is true or false.

$(a)$ $\exists z.\forall y.P(y,z) \models \forall y.\exists z.P(y,z)$
[ Solution: true ]

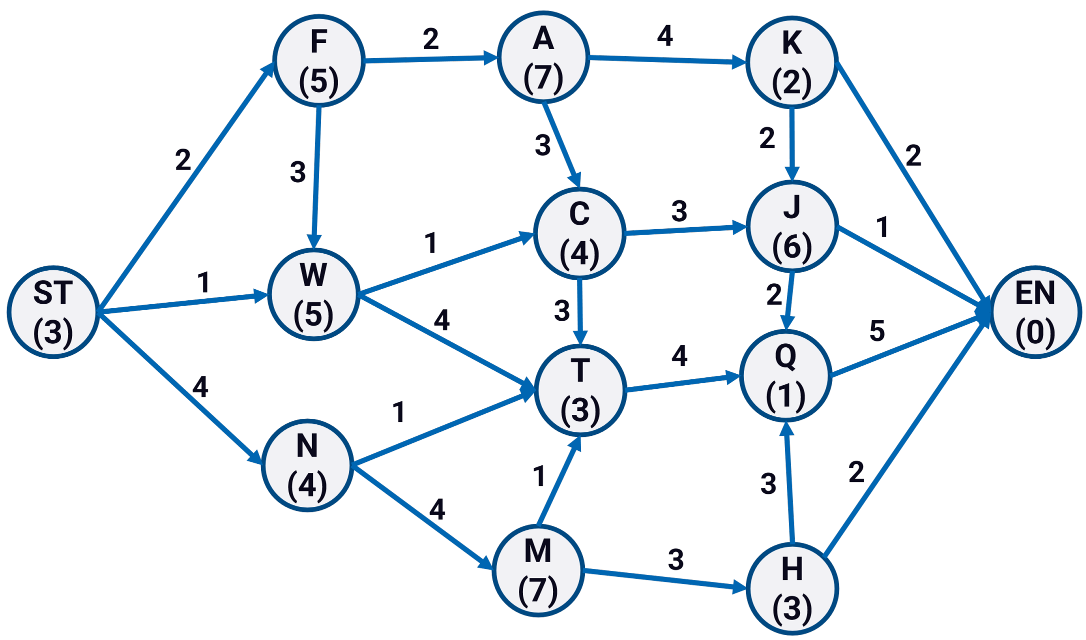
$(b)$ $(\exists y.Q(y)) \vee (\exists z.R(z))$ is equivalent to $(\exists y.Q(y)) \vee (\exists y.R(y))$
[ Solution: True ]

$(c)$ $\forall y \exists z \forall x.(Q(y,z) \rightarrow R(x,z))$ is equivalent to $\neg Q(y, F_1(y)) \vee R(x, F_1(y))$ for some Skolem function $F_1$
[ Solution: False (it is only equivalently satisfiable to it) ]

$(d)$ $\forall y.Q(y) \vee \forall z.R(z) \models \forall y.(Q(y) \vee R(y))$
[ Solution: True ]

# 4

Consider the graph shown below.




For each of the following facts, say if it is true or false.

(*a*) The path ST-N is explored.
[ Solution: true ]

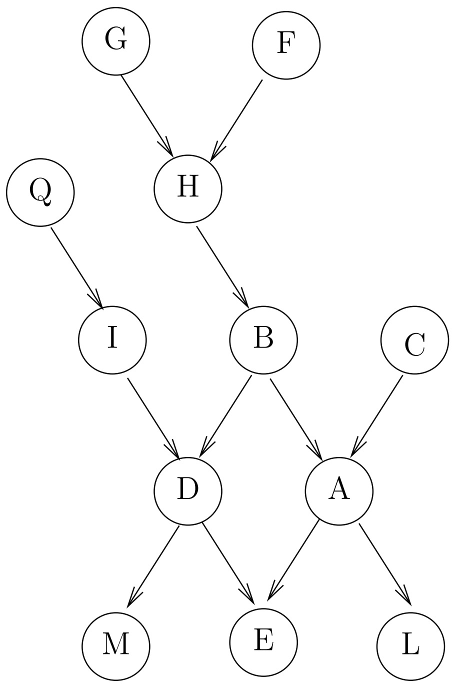
(*b*) The A* algorithm always finds the optimal path.
[ Solution: false ]

(*c*) The path ST-N-T-M is explored.
[ Solution: false ]

(*d*) The path ST-W-C-T is discarded.
[ Solution: true ]

# 5

Consider the following DAG of a Bayesian network.




For each of the following facts, say if it is true or false.

($a$) For query $\mathbf{P}(G|A)$, $L$ is irrelevant.
[ Solution: true, because it does not belong to $Ancestors(A, G)$ ]

($b$) For query $\mathbf{P}(A|G)$, $F$ is irrelevant.
[ Solution: false, because it belongs to $Ancestors(A, G)$ ]

($c$) $\mathbf{P}(D|IB) = \mathbf{P}(D|IBQ)$
[ Solution: true, due to local semantics (Q is a nondescendant of D) ]

($d$) $\mathbf{P}(A|BCDELI) = \mathbf{P}(A|BCDEL)$
[ Solution: true, due to Markov blanket rule ]

## 6

Let $P()$, $Q()$, $R()$, $S()$, $T()$, $U()$ denote predicates, $a$, $b$, $c$, $d$, $e$ denote constants.

Consider the following set of clauses:

$$
\begin{array}{l}
R(a) \lor Q(c) \lor P(b) \\
\neg T(a) \lor P(b) \\
\neg R(a) \lor P(b) \lor S(e) \\
\neg Q(c) \lor S(e) \lor \neg U(d) \\
\neg P(b) \lor T(a) \\
U(d) \lor S(e) \\
\neg S(e) \lor P(b) \\
\neg P(b)
\end{array}
$$

Build a refutation using the Hyper-Resolution strategy.

[ Solution: The Hyper-Resolution strategy works by always resolving one electron (a clause with positive literals only) with a nucleus (a clause with at least one negative literal). Thus one possible refutation is:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{R(a) \lor Q(c) \lor P(b) \quad \neg R(a) \lor P(b) \lor S(e)}{Q(c) \lor P(b) \lor S(e)} \quad \neg Q(c) \lor S(e) \lor \neg U(d)}{P(b) \lor S(e) \lor \neg U(d)} \quad U(d) \lor S(e)}{P(b) \lor S(e)} \quad \neg S(e) \lor P(b)}{P(b)} \quad \neg P(b)}{\bot}$$

This can be merged into one single step:

$$\dfrac{R(a) \lor Q(c) \lor P(b) \quad \neg R(a) \lor P(b) \lor S(e) \quad \neg Q(c) \lor S(e) \lor \neg U(d) \quad U(d) \lor S(e) \quad \neg S(e) \lor P(b) \quad \neg P(b)}{\bot}$$

]

# 7

($a$) Describe as Pseudo-Code the Uniform-Cost Search (UCS) stategy (graph version).

[ Solution:

```
function UNIFORM-COST-SEARCH(problem) returns a solution, or failure
  node ← a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
  frontier ← a priority queue ordered by PATH-COST, with node as the only element
  explored ← an empty set
  loop do
      if EMPTY?(frontier) then return failure
      node ← POP(frontier)   /* chooses the lowest-cost node in frontier */
      if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
      add node.STATE to explored
      for each action in problem.ACTIONS(node.STATE) do
          child ← CHILD-NODE(problem, node, action)
          if child.STATE is not in explored or frontier then
              frontier ← INSERT(child, frontier)
          else if child.STATE is in frontier with higher PATH-COST then
              replace that frontier node with child
```

or any schema equivalent to the above one (from AIMA book). ]

($b$) When is the goal test applied to a node?

1 when the node is selected for expansion

2 when the node is first generated

(Say if 1. or 2.)

[ Solution: 1. ]

# 8

Consider the following propositional formula $\varphi$:

$$((\ A_3 \wedge \neg A_4) \vee (\neg A_5 \wedge \ \ A_2))$$

1. Using the $CNF_{label}$ conversion, produce the CNF formula $CNF_{label}(\varphi)$.

   [ Solution: We introduce fresh Boolean variables naming the subformulas of $\varphi$:

   $$\overbrace{(\overbrace{(\ A_3 \wedge \neg A_4)}^{B_1} \vee \overbrace{(\neg A_5 \wedge \ \ A_2)}^{B_2})}^{B}$$

   from which we obtain:

   $$\begin{array}{ll} & (\ \ B) \\ \wedge & (\neg B \vee \ \ B_1 \vee \ \ B_2) \\ \wedge & (\ \ B \vee \neg B_1) \\ \wedge & (\ \ B \vee \neg B_2) \\ \wedge & (\neg B_1 \vee \ \ A_3) \wedge (\neg B_1 \vee \neg A_4) \\ \wedge & (\ \ B_1 \vee \neg A_3 \vee \ \ A_4) \\ \wedge & (\neg B_2 \vee \neg A_5) \wedge (\neg B_2 \vee \ \ A_2) \\ \wedge & (\ \ B_2 \vee \ \ A_5 \vee \neg A_2) \end{array}.$$

   ]

2. For each of the following sentences, only one is true. Say which one.

   (i) $\varphi$ and $CNF_{label}(\varphi)$ are equivalent. [ Solution: False ]

   (ii) There is no relation between the satisfiablity of $\varphi$ and that of $CNF_{label}(\varphi)$. [ Solution: False ]

   (iii) $\varphi$ and $CNF_{label}(\varphi)$ are not necessarily equivalent. $CNF_{label}(\varphi)$ has a model if and only $\varphi$ has a model. [ Solution: true ]

# 9

Consider the following CNF formula in PL:

$$\begin{array}{l}
( \ E \ \vee\neg B \ \vee \ N) \wedge \\
( \ I \ \vee \ L \ \vee \ M) \wedge \\
( \ A \ \vee \ H \ \vee \ C) \wedge \\
(\neg H \ \vee \ I \ \vee \ A) \wedge \\
(\neg L \ \vee \ C \ \vee\neg M) \wedge \\
( \ E \ \vee\neg F \ \vee\neg A) \wedge \\
(\neg G \ \vee\neg A \ \vee\neg E) \wedge \\
( \ E \ \vee\neg G \ \vee \ A) \wedge \\
( \ N \ \vee \ L \ \vee \ M)
\end{array}$$

Consider the WalkSAT algorithm, with probability parameter $p = 0.1$. Suppose at a given step the current assignment is

$$\{ \ A, \ B, \ C, \ D, \neg E, \neg F, \ G, \neg H, \ I, \neg L, \neg M, \neg N\}.$$

Assuming the most-likely event happens, describe what the assignment is after the next step. [ Solution:

The current assignments makes only the first and last clauses unsatisfied. Since $p = 0.1$, the most likely event is that the algorithm flips the symbol in the first clause or last clause which maximizes the number of satisfied clause at next step.

We notice that flipping $\neg N$ would cause both clauses to become true and no other clause to become unsatisfied, so that the formula will be satisfied, whereas flipping either of $\neg E, \ B, \neg L, \neg M$ would cause other clauses to remain or become unsatisfied. Thus WalkSAT flips $\neg N$, obtaining the assignment

$$\{ \ A, \ B, \ C, \ D, \neg E, \neg F, \ G, \neg H, \ I, \neg L, \neg M, \ N\}.$$

which satisfied all clauses.
]

# 10

Given:

- a set of basic concepts: $\{\mathsf{Person}, \mathsf{Cat}, \mathsf{Dog}, \mathsf{Female}, \mathsf{Male}\}$
- a set of relations: $\{\mathsf{hasChild}, \mathsf{hasPet}\}$

with their standard meaning ("hasChild" refers also to animals).
Write a $\mathcal{T}$-box in $\mathcal{ALCN}$ description logic defining the following concepts

(*a*) CatLover: a person with at least 3 cats
[ Solution:
$\mathsf{CatLover} \equiv \mathsf{Person} \sqcap (\geq 3)\mathsf{hasPet.Cat}$
or any $\mathcal{T}$-box which is logically equivalent to it ]

(*b*) ChildlessFemaleFish: childless female dog
[ Solution:
$\mathsf{ChildlessFemaleFish} \equiv \mathsf{Dog} \sqcap \mathsf{Female} \sqcap \neg\exists.\mathsf{hasChild.Dog}$
or any $\mathcal{T}$-box which is logically equivalent to it ]

(*c*) PersonWithMaleSnakes: a person with male cats
[ Solution:
$\mathsf{PersonWithMaleSnakes} \equiv \mathsf{Person} \sqcap \exists\mathsf{hasPet}(\mathsf{Cat} \sqcap \mathsf{Male})$
or any $\mathcal{T}$-box which is logically equivalent to it ]

(*d*) ManWithSnakesOrFishs: man whose pets are all cats or [1] dogs
[ Solution:
$\mathsf{ManWithSnakesOrFishs} \equiv \mathsf{Person} \sqcap \mathsf{Male} \sqcap \forall\mathsf{hasPet}(\mathsf{Cat} \sqcup \mathsf{Dog})$
or any $\mathcal{T}$-box which is logically equivalent to it ]

[1] non-exclusive or.

# 11

Consider the following problem statements:
Variables: $V_1$, $V_2$, $V_3$, $V_4$, $V_5$
and the respective domain $D_i$ for each variable $V_i$:
$D_1 = \{R, W, S, D, F\}$,
$D_2 = \{R, W, S, D, F\}$,
$D_3 = \{S, D, G\}$,
$D_4 = \{F, H, J\}$,
$D_5 = \{S, K\}$

Constraints:

$C_1$: only one among S, D, and K can be assigned to the five variables.

$C_2$: only one among R and H can be assigned to the five variables.

$C_3$: only one among F and J can be assigned to the five variables.

$C_4$: No double counting says if a literal is assigned to one variable it can't be assigned to another variable

Please draw the exploration graph in order to find the first admissible solution by exploring the domains from $D_1$ to $D_5$ and their values from left to right.

[ Solution:

]

# 12

Given the tree provided within the Annex (marked as Alpha-Beta Pruning exercise), please do the following tasks:

- Report the value of each MIN, MAX, and CHANCE nodes. Each value has to be provided directly within each node in the paper provided.
- Mark the pruned branches. When a pruning operation is performed, each element under the pruned branch (including the pruned branch) has to be marked with the X. When pruning operations are performed, please report also the values of $\alpha$ and $\beta$.

[ Solution:

The solutions is available within the file 2021-09-07-alphabetapruning-v3-withsolutions.pptx in your Google Drive folder.
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# 13

Given the plan described below:

Init(H(x) ∧ L(x) ∧ D(x))

Goal(¬H(x) ∧ S(x) ∧ N(x))

Action(ACT1(x), PRECOND: L(x) EFFECT: S(x))

Action(ACT3(x), PRECOND: D(x) EFFECT: N(x))

Action(ACT6(x), PRECOND: H(x) EFFECT: ¬H(x) ∧ ¬L(x))

Action(ACT8(x), PRECOND: H(x) EFFECT: ¬H(x) ∧ ¬D(x))

Draw the planning graph by using the following notation:

- Rectangles indicate actions.
- Small squares persistent actions (no-ops).
- Straight lines indicate preconditions and effects.
- Arcs indicate mutex links.

Considering the mutex, please draw only one mutex for each type, if any.
[ Solution:

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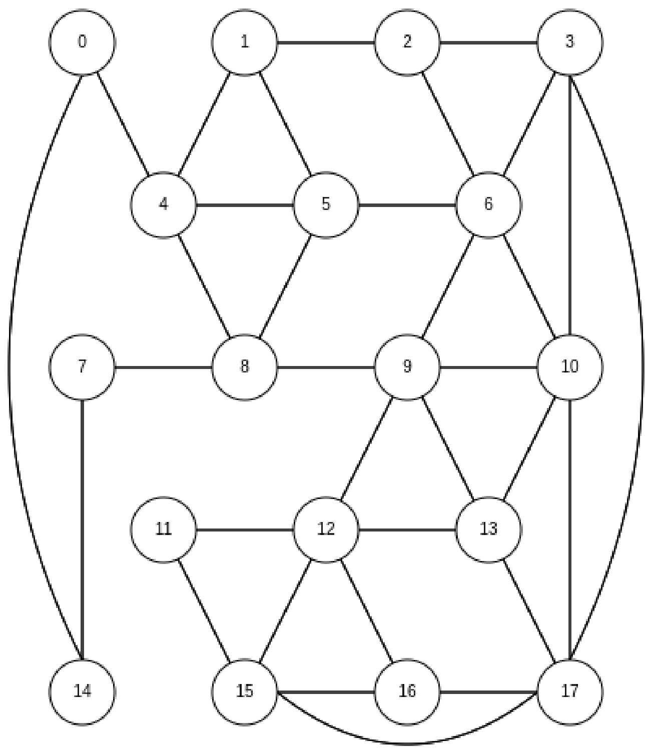
# 14

Consider the graph shown below and the following statements:

- The BFS algorithm is executed by performing an early goal test.
- The DFS algorithm is executed by performing a late goal test.




Please shows which is the first algorithm reaches the goal node 17.
[ Solution:

Exploration of the BFS algorithm:
0, 14, 7, 1, 11, 4, 2, 12, 8, 15, 9, 17, 10, 16
Exploration of the DFS algorithm:
0, 14, 11, 12, 15, 17, 16
]

# 15

The maze contained within the included paper represents the states space of a hypothetical search problem where:

- The cell labeled with the letter **S** is the starting point.
- The cell labeled with the letter **G** is the goal.

By following the instructions included in the Annex, please perform the first six steps of the **LRTA*** algorithm. A "step" consists in: (i) to move the agent to the next cell and (ii) to update the heuristic value of the cell the agent left. At each step, please mark the cell where the agent is positioned with a **X** in the bottom right corner of the cell.

[ Solution:

The solutions is available within the file 2021-09-07-astar-v3-withsolutions.pptx in your Google Drive folder.

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