Course "**Fundamentals of Artificial Intelligence**"
EXAM TEXT

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[COPY WITH SOLUTIONS]

1

Consider propositional logic (PL); let C, D, E, F, G, A, B be atomic propositions
We adopt the set notation for resolution rules, s.t. Γ denotes a set of clauses.

For each of the following statements, say if it is true or false.

(a) The following is a correct application of the PL unit-resolution rule:

$$\frac{\Gamma, \quad (\neg C), \quad (\ C \vee \neg D \vee \ E)}{\Gamma, \quad (\neg C), \quad (\neg D \vee \ E)}$$

[Solution: true]

(b) The following is a correct application of the PL clause-subsumption rule:

$$\frac{\Gamma, \quad (\ C \vee \neg D), \quad (\ C \vee \neg D \vee \ E)}{\Gamma, \quad (\ C \vee \neg D \vee \ E)}$$

[Solution: false]

(c) The following is a correct application of the PL general resolution rule:

$$\frac{\Gamma, \quad (\neg C \vee \ D \vee \ F), \quad (\neg D \vee \neg E \vee \neg F)}{\Gamma, \quad (\neg C \vee \ F \vee \neg E \vee \neg F)}$$

[Solution: true]

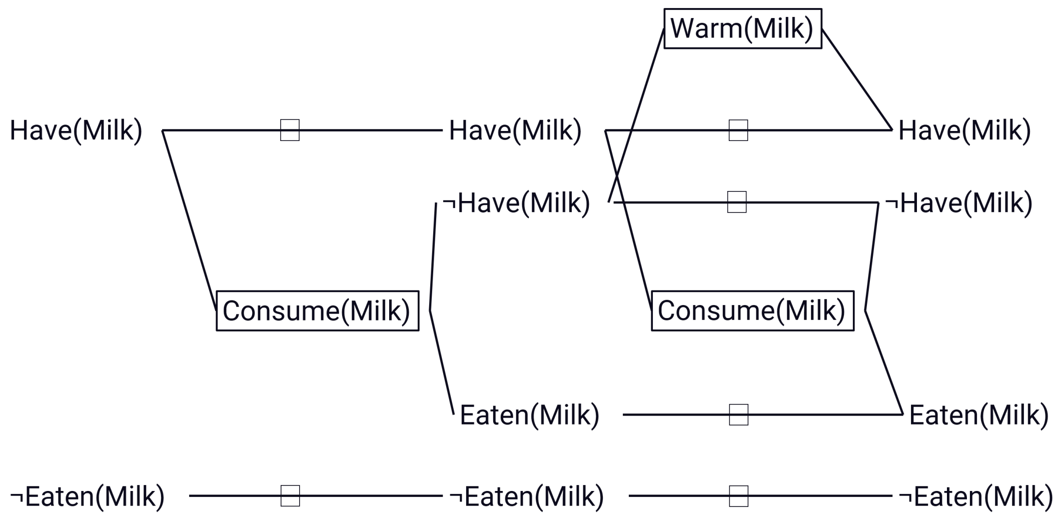
(d) The following is a correct application of the PL general resolution rule:

$$\frac{\Gamma, \quad (\ C \vee \neg D \vee \ E), \quad (\ D \vee \neg E \vee \ F)}{\Gamma, \quad (\ C \vee \ F)}$$

[Solution: False]

2

Consider the graph shown below.



For each of the following facts, say if it is true or false.

(*a*) At level A_1 the mutex between Consume and the persistence of Have is an interference
[Solution: true]

(*b*) At level S_1 there are no mutex between $\neg$ Have and Eaten
[Solution: true]

(*c*) At level A_1 the mutex between Consume and Warm is a competing needs.
[Solution: true]

(*d*) At level A_0 there are no mutex between Consume and the persistence of Have
[Solution: false]

3

In the following FOL formulas, let $R()$, $Q()$, $P()$ denote predicates, $h()$, $g()$, $f()$, $F_1()$, $F_2()$, $F_3()$ denote functions, x, y, z, x_1, x_2, x_3 denote variables, A, B, C, C_1, C_2, C_3 denote constants.

For each of the following facts, say if it is true or false.

(a) $\forall x_1.((\forall x_2.R(x_1,x_2)) \rightarrow (\exists x_3.P(x_1,x_3)))$
can be CNF-ized into
$\neg R(x_1,x_2) \vee P(x_1,F_1(x_1))$
[Solution: false]

(b) $(\exists x_2.\forall x_1.R(x_1,x_2)) \rightarrow (\forall x_1.\exists x_2.Q(x_1,x_2))$
can be CNF-ized into
$\neg R(F_1(x_2),x_2) \vee Q(x_1,F_2(x_1))$
[Solution: true]

(c) $(\forall x_1.\exists x_2.R(x_1,x_2)) \rightarrow (\exists x_2.\forall x_1.Q(x_1,x_2))$
can be CNF-ized into
$\neg R(x_1,F_1(x_1)) \vee Q(C_1,x_2)$
[Solution: false]

(d) $(\forall x_1.R(x_1)) \rightarrow (\forall x_2.Q(x_2))$
can be CNF-ized into
$\neg R(C_1) \vee Q(x_2)$
[Solution: true]

4

Let us consider the PEAS description of the task environment for an automated taxi. For each of the following facts, say if it is true or false.

(a) Traffic is a Sensors
[Solution: false]

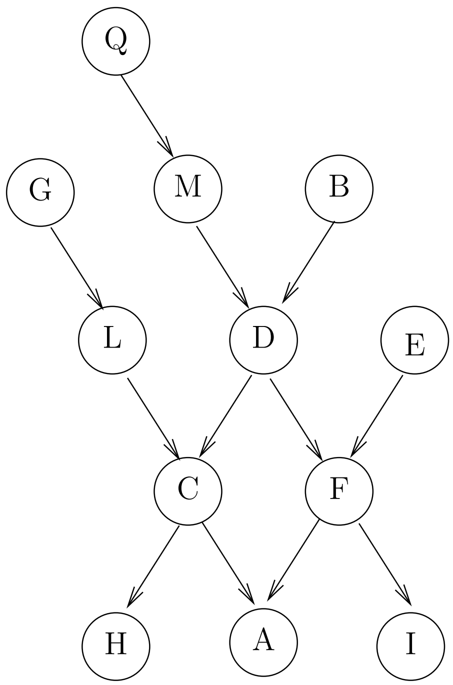
(b) Touchscreen is a Sensor
[Solution: true]

(c) GPS is a Sensor
[Solution: true]

(d) Profit is a Environment
[Solution: false]

5

Consider the following DAG of a Bayesian network.



For each of the following facts, say if it is true or false.

(*a*) $\mathbf{P}(D|MBLECFAG) = \mathbf{P}(D|MBLECF)$
[Solution: true, due to Markov blanket rule]

(*b*) $\mathbf{P}(C|LDE) = \mathbf{P}(C|LD)$
[Solution: true, due to local semantics (E is a nondescendant of C)]

(*c*) $\mathbf{P}(D|MBF) = \mathbf{P}(D|MB)$
[Solution: False (F is not a nondescendant of D)]

(*d*) $\mathbf{P}(D|MBQ) = \mathbf{P}(D|MB)$
[Solution: true, due to local semantics (Q is a nondescendant of D)]

6

(*a*) Describe as Pseudo-Code the Breadth-First Search (BFS) stategy (graph version).

[Solution:

```
function BREADTH-FIRST-SEARCH(problem) returns a solution, or failure
  node ← a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
  if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
  frontier ← a FIFO queue with node as the only element
  explored ← an empty set
  loop do
     if EMPTY?(frontier) then return failure
     node ← POP(frontier)  /* chooses the shallowest node in frontier */
     add node.STATE to explored
     for each action in problem.ACTIONS(node.STATE) do
        child ← CHILD-NODE(problem, node, action)
        if child.STATE is not in explored or frontier then
           if problem.GOAL-TEST(child.STATE) then return SOLUTION(child)
           frontier ← INSERT(child, frontier)
```

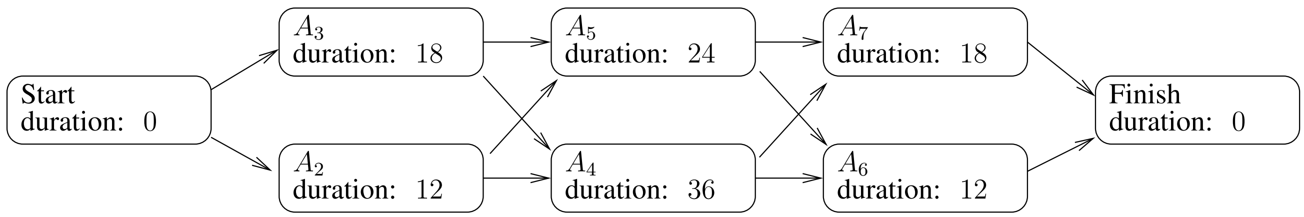
or any schema equivalent to the above one (from AIMA book).]

(*b*) calling B the branching factor and S the depth of the shallowest solution,

- what is the time complexity of the procedure? [Solution: $O(B^S)$]
- what is the memory complexity of the procedure? [Solution: $O(B^S)$]

7

Consider the following partial-order plan, with respective predicted duration of each action.



(a) Find the critical path in the form $\{Start, A_{i_1}, ..., A_{i_k}, Finish\}$ and its duration.
[Solution: Critical path: $\{Start, A_3, A_4, A_7, Finish\}$
Duration: 18+36+18=72 time units]

Suppose that, due to some technical problem, A_2 takes 18 time units longer than expected. If so:

(b) How much the start of A_4 is delayed of?
[Solution: 12+18-18=12 time units]

(c) what would be critical path and the global duration then?
[Solution: Critical path: $\{Start, A_2, A_4, A_7, Finish\}$
Duration: (12+18)+36+18=84 time units]

8

Consider the following Horn formula in PL:

$$
\begin{array}{llll}
(\neg B & \vee\neg C & \vee\ \ D) & \wedge \\
(\ \ G & \vee\neg D & \vee\neg B) & \wedge \\
(\neg L & \vee\ \ N & \vee\neg H) & \wedge \\
(\ \ D &) & & \\
(\neg D & \vee\ \ A &) & \wedge \\
(\neg I & \vee\ \ L & \vee\neg N) & \wedge \\
(\ \ B & \vee\neg D & \vee\neg A) & \wedge \\
(\neg B & \vee\neg D & \vee\ \ C) & \wedge \\
(\neg D & \vee\ \ H & \vee\neg F) & \wedge \\
(\neg H & \vee\neg I & \vee\ \ D) & \wedge \\
(\ \ I & \vee\neg L & \vee\neg M) & \wedge \\
(\neg L & \vee\ \ F & \vee\neg M) & \wedge \\
(\ \ B & \vee\neg G & \vee\neg D) & \wedge \\
(\neg D & \vee\neg L & \vee\ \ M) & \wedge
\end{array}
$$

Using the simple polynomial procedure for Horn formulas,

(a) decide if the formula is satisfiable or not

(b) if satisfiable, return the satisfying truth assignment;
if unsatisfiable, return the falsified clause.

[Solution:

(a) (i) run unit propagation: D, A, B, G, C. The resulting formula is:

$$
\begin{array}{llll}
(& \vee & \vee\ \ D) & \wedge \\
(\ \ G & \vee & \vee) & \wedge \\
(\neg L & \vee\ \ N & \vee\neg H) & \wedge \\
(\ \ D &) & & \\
(& \vee\ \ A &) & \wedge \\
(\neg I & \vee\ \ L & \vee\neg N) & \wedge \\
(\ \ B & \vee & \vee) & \wedge \\
(& \vee & \vee\ \ C) & \wedge \\
(& \vee\ \ H & \vee\neg F) & \wedge \\
(\neg H & \vee\neg I & \vee\ \ D) & \wedge \\
(\ \ I & \vee\neg L & \vee\neg M) & \wedge \\
(\neg L & \vee\ \ F & \vee\neg M) & \wedge \\
(\ \ B & \vee & \vee) & \wedge \\
(& \vee\neg L & \vee\ \ M) & \wedge
\end{array}
$$

(ii) the resulting formula contains no empty clause, thus the original formula is satisfiable

(b) the satisfying truth assignment is $\{\ D,\ A,\ B,\ G,\ C, \neg F, \neg H, \neg I, \neg L, \neg M, \neg N\}$.

]

9

Let $R()$, $Q()$, $P()$ denote predicates, $h()$, $g()$, $f()$ denote functions, x, y, z denote variables, A, B, denote constants, each possibly with suffixes.

For each of the following pair of clauses C_1, C_2:

- compute the most-general unifier (mgu) θ of their resolvent, or say "NULL" if none exists (represent the unifier as a list $\{variable_1/term_1, variable_2/term_2, ...\}$)
- compute the clause C resulting from resolving C_1 and C_2 with mgu θ, or say "NO SOLUTION" if resolution is not applicable

(a) $C_1 \stackrel{\text{def}}{=} P(h(x), z) \vee R(f(x), B)$
$C_2 \stackrel{\text{def}}{=} \neg P(h(g(B)), y) \vee \neg R(A, y)$
[Solution:
$\theta = \{x/g(B), y/z\}$ // or $\theta = \{x/g(B), z/y\}$
$C = R(f(g(B)), B) \vee \neg R(A, z)$ // or $C = R(f(g(B)), B) \vee \neg R(A, y)$]

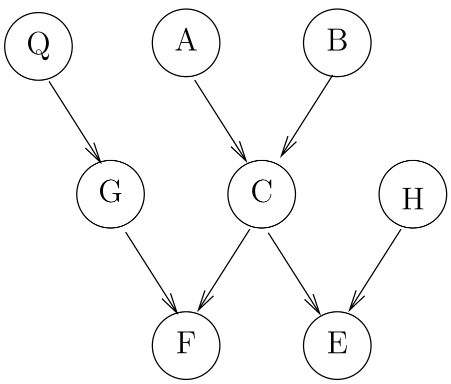
(b) $C_1 \stackrel{\text{def}}{=} R(x, f(x)) \vee Q(x, h(z))$
$C_2 \stackrel{\text{def}}{=} \neg Q(g(z), x) \vee P(y, g(A))$
[Solution:
$\theta =$ NULL
$C =$ NO SOLUTION]

(c) $C_1 \stackrel{\text{def}}{=} \neg R(A, h(y)) \vee P(h(A), h(z))$,
$C_2 \stackrel{\text{def}}{=} R(y, z) \vee P(x, f(y))$
[Solution:
$\theta = \{y/A, z/h(y)\}$
$C = P(h(A), h(h(y))) \vee P(x, f(A))$]

(d) $C_1 \stackrel{\text{def}}{=} R(h(x), y) \vee Q(h(y), g(z))$,
$C_2 \stackrel{\text{def}}{=} \neg R(z, g(B)) \vee P(h(A), f(z))$
[Solution:
$\theta = \{z/h(x), y/g(B)\}$
$C = Q(h(g(B)), g(h(x))) \vee P(h(A), f(h(x)))$]

10

Consider the following Bayesian network:



where, for every node X, Y, Z:

- $P(x) \stackrel{\text{def}}{=} 0.4$, if X has no parent
- $P(x|Y) \stackrel{\text{def}}{=}$ (table below) if X has only one parent Y

Y	$P(x\|Y)$
T	0.4
F	0.3

- $P(x|Y,Z) \stackrel{\text{def}}{=}$ (table below)

Y	Z	$P(x\|Y,Z)$
T	T	0.3
F	T	0.6
T	F	0.2
F	F	0.1

if X has one left parent Y and one right parent Z.

Compute the conditional distribution $\mathbf{P}(\mathbf{A}|\mathbf{C})$.

[Solution: Recall the Irrelevant-variable Theorem: For every query X and evidence E, variable Y is irrelevant unless $Y \in Ancestors(X, E)$. Hence, all variables except C, A, B are irrelevant for $\mathbf{P}(\mathbf{A}|\mathbf{C})$. Thus:

$$\mathbf{P}(\mathbf{A}|\mathbf{C}) = \alpha\mathbf{P}(\mathbf{A},\mathbf{C}) = \alpha\sum_B \mathbf{P}(\mathbf{A},\mathbf{C},\mathbf{B}) = \alpha\sum_B \mathbf{P}(\mathbf{C}|\mathbf{A},\mathbf{B}) * \mathbf{P}(\mathbf{A}) * \mathbf{P}(\mathbf{B}) = \alpha\mathbf{P}(\mathbf{A}) * \sum_B \mathbf{P}(\mathbf{C}|\mathbf{A},\mathbf{B}) * \mathbf{P}(\mathbf{B})$$

$$\begin{aligned}
P(a|c) &= \alpha_1 \cdot P(a)\,[P(c|a,b)P(b) + P(c|a,\neg b)P(\neg b)] \\
&= \alpha_1 \cdot 0.4 * (0.3 * 0.4 + 0.2 * 0.6) = \alpha_1 \cdot 0.096 \\
P(\neg a|c) &= \alpha_1 \cdot P(\neg a)\,[P(c|\neg a,b)P(b) + P(c|\neg a,\neg b)P(\neg b)] \\
&= \alpha_1 \cdot 0.6 * (0.6 * 0.4 + 0.1 * 0.6) = \alpha_1 \cdot 0.18 \\
\alpha_1 &= 1/(0.4 * (0.3 * 0.4 + 0.2 * 0.6) + 0.6 * (0.6 * 0.4 + 0.1 * 0.6)) \simeq 3.623 \\
(P(a|c),)P(\neg a|c)) &\simeq (0.347, 0.652) \\
P(a|\neg c) &= \alpha_2 \cdot P(a)\,[P(\neg c|a,b)P(b) + P(\neg c|a,\neg b)P(\neg b)] \\
&= \alpha_2 \cdot 0.4 * (0.7 * 0.4 + 0.8 * 0.6) = \alpha_2 \cdot 0.304 \\
P(\neg a|\neg c) &= \alpha_2 \cdot P(\neg a)\,[P(\neg c|\neg a,b)P(b) + P(\neg c|\neg a,\neg b)P(\neg b)] \\
&= \alpha_2 \cdot 0.6 * (0.4 * 0.4 + 0.9 * 0.6) = \alpha_2 \cdot 0.42 \\
\alpha_2 &= 1/(0.4 * (0.7 * 0.4 + 0.8 * 0.6) + 0.6 * (0.4 * 0.4 + 0.9 * 0.6)) \simeq 1.381 \\
(P(a|\neg c),)P(\neg a|\neg c)) &\simeq (0.419, 0.58)
\end{aligned}$$

]

Notation: uppercase letters are used for propositional variables representing random events, whereas the corresponding lowercase letters represent truth assignments to such propositional variables. Ex: $a \stackrel{\text{def}}{=} (A = \text{true})$, $\neg a \stackrel{\text{def}}{=} (A = \text{false})$.

11

Given the tree provided within the Annex (marked as Alpha-Beta Pruning exercise), please do the following tasks:

- Report the value of each MIN and MAX nodes. Each value has to be provided directly within each MIN and MAX node in the paper provided.
- Mark the pruned branches. When a pruning operation is performed, each element under the pruned branch (including the pruned branch) has to be marked with the X. When pruning operations are performed, please report also the values of α and β.

[Solution:

The solutions is available within the file 2021-09-07-alphabetapruning-v4-withsolutions.pptx in your Google Drive folder.
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12

Consider the following constraint network.
Variables: X_1, X_2, X_3, X_4, X_5
Domains: $D_1 = \{2, 3, 4, 6, 8, 9\}, D_2 = \{1, 3, 4, 5, 6, 9\}, D_3 = \{1, 2, 3, 6, 8, 9\}, D_4 = \{2, 3, 5, 6, 7, 9\}, D_5 = \{3, 4, 5, 7, 8, 9\}$ Constraints:

$$X_2 = X_1$$

$$X_2 > X_3 \text{ or } X_3 + X_2 = 7$$

$$X_3 \neq X_4 \text{ or } X_4 > X_3$$

$$X_5 = X_4$$

Please complete the following tasks.

(a) Is the network arc-consistent? If not, compute the arc-consistent network.

(b) If the consistency holds, provide the first admissible solution by exploring the domains from D_1 to D_5 and the values in descending order.

[Solution:
Enforce arc consistency between X_1 and X_2 led to $D_1 = \{3, 4, 6, 9\}$ and $D_2 = \{3, 4, 6, 9\}$
Enforce arc consistency between X_2 and X_3 led to $D_3 = \{1, 2, 3, 6, 8\}$
Enforce arc consistency between X_5 and X_4 led to $D_5 = \{3, 5, 7, 9\}$ and $D_4 = \{3, 5, 7, 9\}$

One possible solution: $X_1 = 9$, $X_2 = 9$, $X_3 = 8$, $X_4 = 9$, $X_5 = 9$]

13

The graph contained within the Annex represents the states space of a hypothetical search problem where:

- States are denoted by letters.
- Arcs are labeled with the cost of traversing them.
- The estimated cost to a goal (i.e., the h function) is reported inside nodes (so that lower scores are better).

Considering ST as the initial state and EN the goal state, please apply the **A*** search algorithm and report each step of the resolution process. Then, explain if the heuristic adopted is admissible or not. The solution format has to be provided as shown during the laboratory session.

[Solution:

The solutions is available within the file 2021-09-07-astar-v4-withsolutions.pptx in your Google Drive folder.

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14

The empty planning graph contained within the Annex represents a hypothetical list of actions and fluents. Please link fluents with actions in order to satisfy the three requests reported on the top-right corner of the paper.

[Solution:

The solutions is available within the file 2021-09-07-astar-v4-withsolutions.pptx in your Google Drive folder.

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15

The maze contained within the included paper represents the states space of a hypothetical search problem where:

- The cell labeled with the letter **S** is the starting point.
- The cell labeled with the letter **G** is the goal.

By following the instructions included in the Annex, please perform the first six steps of the **LRTA*** algorithm. At each step, please mark the cell where the agent is positioned with a **X** in the bottom right corner of the cell.

[Solution:

The solutions is available within the file 2021-09-07-astar-v4-withsolutions.pptx in your Google Drive folder.

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