Fundamentals of Artificial Intelligence

Laboratory

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Exercise 12.1

- Represent the following seven sentences using and extending the representations developed in the chapter:

  a. Water is a liquid between 0 and 100 degrees.
  b. Water boils at 100 degrees.
  c. The water in John's water bottle is frozen.
  d. Perrier is a kind of water.
  e. John has Perrier in his water bottle.
  f. All liquids have a freezing point.
  g. A liter of water weighs more than a liter of alcohol.

- Remember that we defined substances so that Water is a category whose elements are all those things of which one might say "it's water." One tricky part is that the English language is ambiguous. One sense of the word "water" includes ice ("that's frozen water"), while another sense excludes it: ("that's not water—it's ice"). The sentences here seem to use the first sense, so we will stick with that. It is the sense that is roughly synonymous with H2O.
  The other tricky part is that we are dealing with objects that change (freeze and melt) over time. Thus, it won't do to say $w \in Liquid$, because w (a mass of water) might be a liquid at one time and a solid at another. For simplicity, we will use a situation calculus representation, with sentences such as $T(w \in Liquid, s)$. There are many possible correct answers to each of these. The key thing is to be consistent in the way that information is represented. For example, do not use Liquid as a predicate on objects if Water is used as a substance category.

Exercise 12.1

- Represent the following seven sentences using and extending the representations developed in the chapter:

  a. Water is a liquid between 0 and 100 degrees.

  We will translate this as "For any water and any situation, the water is liquid iff and only if the water's temperature in the situation is between 0 and 100 centigrade."
  $\forall w,s\ w \in Water \Rightarrow (Centigrade(0) < Temperature(w, s) < Centigrade(100)) \Leftrightarrow T(w \in Liquid, s)$

Exercise 12.1

- Represent the following seven sentences using and extending the representations developed in the chapter:

  b. Water boils at 100 degrees.

  It is a good idea here to do some tool-building. On page 243 we used MeltingPoint as a predicate applying to individual instances of a substance. Here, we will define SBoilingPoint to denote the boiling point of all instances of a substance. The basic meaning of boiling is that instances of the substance becomes gaseous above the boiling point:

  $SBoilingPoint(c, bp) \Leftrightarrow \forall x,s\ x \in c \Rightarrow (\forall t\ T(Temperature(x, t), s) \land t > bp \Rightarrow T(x \in Gas, s))$

  Then we need only say SBoilingPoint(Water, Centigrade(100)).

Exercise 12.1

- Represent the following seven sentences using and extending the representations developed in the chapter:

  c. The water in John's water bottle is frozen.

  We will use the constant N ow to represent the situation in which this sentence holds. Note that it is easy to make mistakes in which one asserts that only some of the water in the bottle is frozen.

  $\exists b\ \forall w\ w \in Water \land b \in WaterBottles \land Has(John, b, Now) \land Inside(w, b, Now) \Rightarrow (w \in Solid, Now)$

Exercise 12.1

- Represent the following seven sentences using and extending the representations developed in the chapter:

  d. Perrier is a kind of water.

  Perrier $\subset$ Water

Exercise 12.1

- Represent the following seven sentences using and extending the representations developed in the chapter:

  e. John has Perrier in his water bottle.

$\exists b\ \forall w\ w \in Water \land b \in WaterBottles \land Has(John, b, Now) \land Inside(w, b, Now) \Rightarrow w \in Perrier$

Exercise 12.1

- Represent the following seven sentences using and extending the representations developed in the chapter:

  f. All liquids have a freezing point.

  Presumably what this means is that all substances that are liquid at room temperature have a freezing point. If we use RT LiquidSubstance to denote this class of substances, then we have

  $\forall c\ RTLiquidSubstance(c) \Rightarrow \exists t\ SFreezingPoint(c, t)$

  where SFreezingPoint is defined similarly to SBoilingPoint. Note that this statement is false in the real world: we can invent categories such as “blue liquid” which do not have a unique freezing point. An interesting exercise would be to define a “pure” substance as one all of whose instances have the same chemical composition.

Exercise 12.1

- Represent the following seven sentences using and extending the representations developed in the chapter:

    g. A liter of water weighs more than a liter of alcohol.

$$\forall\, w, a \;\; w \in Water \land a \in Alcohol \land Volume(w) = Liters(1)$$
$$\land Volume(a) = Liters(1) \Rightarrow Mass(w) > Mass(a)$$

Exercise 12.2

- State the interval-algebra relation that holds between every pair of the following real-world events:

  - LK: The life of President Kennedy.
  - IK: The infancy of President Kennedy.
  - PK: The presidency of President Kennedy.
  - LJ: The life of President Johnson.
  - PJ: The presidency of President Johnson.
  - LO: The life of President Obama.

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  - LK: The life of President Kennedy.
  - IK: The infancy of President Kennedy.
  - PK: The presidency of President Kennedy.
  - LJ: The life of President Johnson.
  - PJ: The presidency of President Johnson.
  - LO: The life of President Obama.

Starts(IK, LK).
Finishes(PK, LK).
During(LK, LJ).
Meets(LK, PJ).
Overlap(LK, LC).
Before(IK, PK).
During(IK, LJ).
Before(IK, PJ).
Before(IK, LC).
During(PK, LJ).
Meets(PK, PJ).
During(PK, LC).
During(PJ, LJ).
Overlap(LJ, LC).
During(PJ, LC).

Exercise 12.3

- Construct a semantic network that (partially) describes supermarkets. Use concepts such as Supermarket, Shop, Food, Employee, and property edges such as sells and works-for. Find additional concepts and relationships.

Exercise 12.4

- Construct a semantic network describing a tennis match. Please define concepts and relationships about the events occur during a match.

Exercise 12.5

- State the interval-algebra relation that holds between every pair of the following real-world events occurring during a tennis match:

  - FieldSelect
  - Match
  - FirstGame
  - 7thGame
  - EndFirstGame
  - FirstSet
  - SecondSet
  - EndFirstSet
  - EndMatch
  - AceDuring7thGame
  - ThreeGamesAllFirstSet
  - BreakPointDuring9thGame

Exercise 12.6

- **The Muddy Children puzzle**

1. n children meet their father after playing in the mud. The father notices that k of the children have mud dots on their foreheads.
2. Each child sees everybody else's foreheads, but not his own.
3. The father says: "At least one of you has mud on his forehead."
4. The father then says: "Do any of you know that you have mud on your forehead? If you do, raise your hand now."
5. No one raises his hand.
6. The father repeats the question, and again no one moves.
7. After exactly k repetitions, all children with muddy foreheads raise their hands simultaneously.

Exercise 12.6

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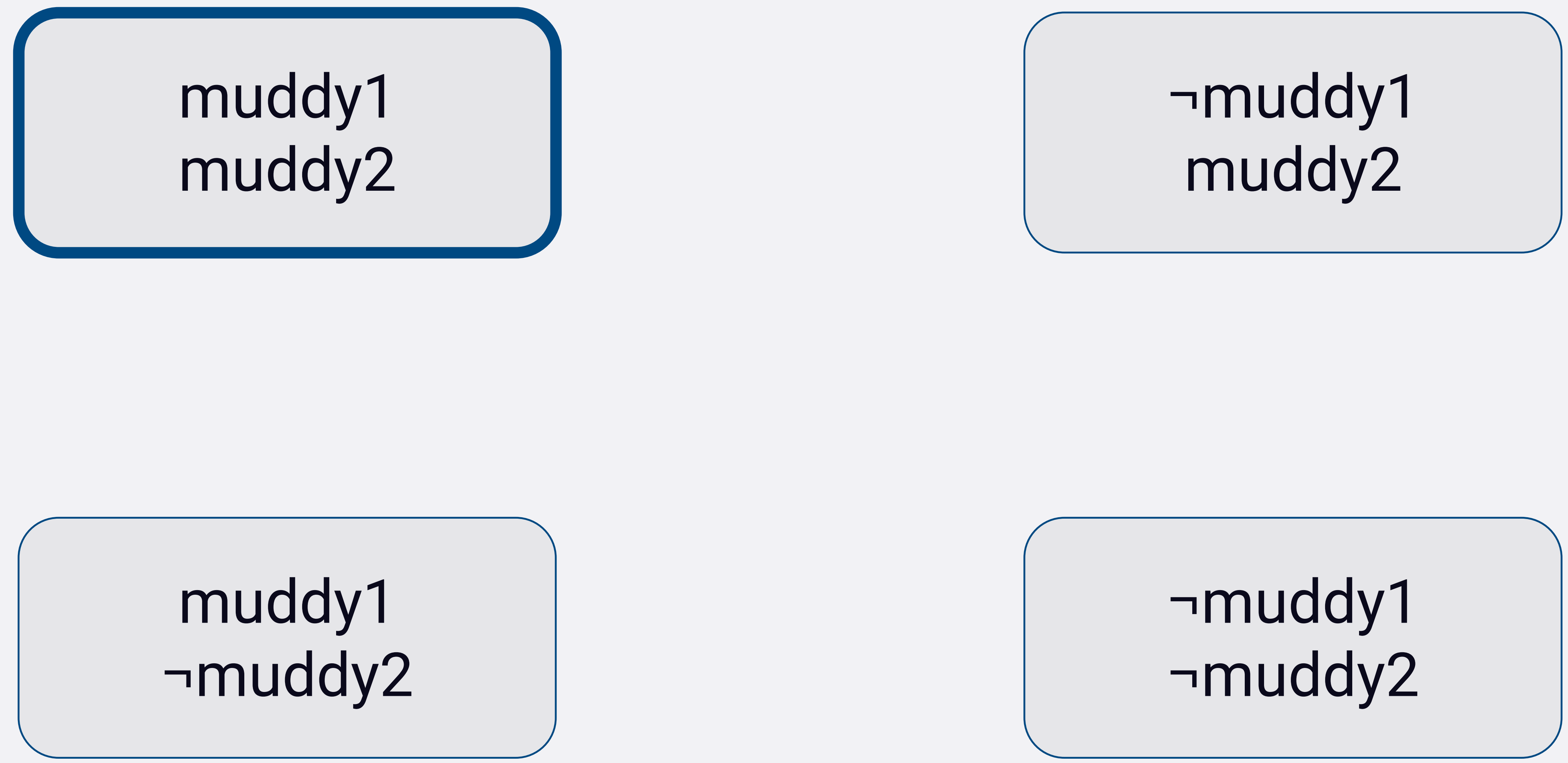
  5. No one raises his hand.
  6. The father repeats the question, and again no one moves.
  7. After exactly k repetitions, all children with muddy foreheads raise their hands simultaneously.

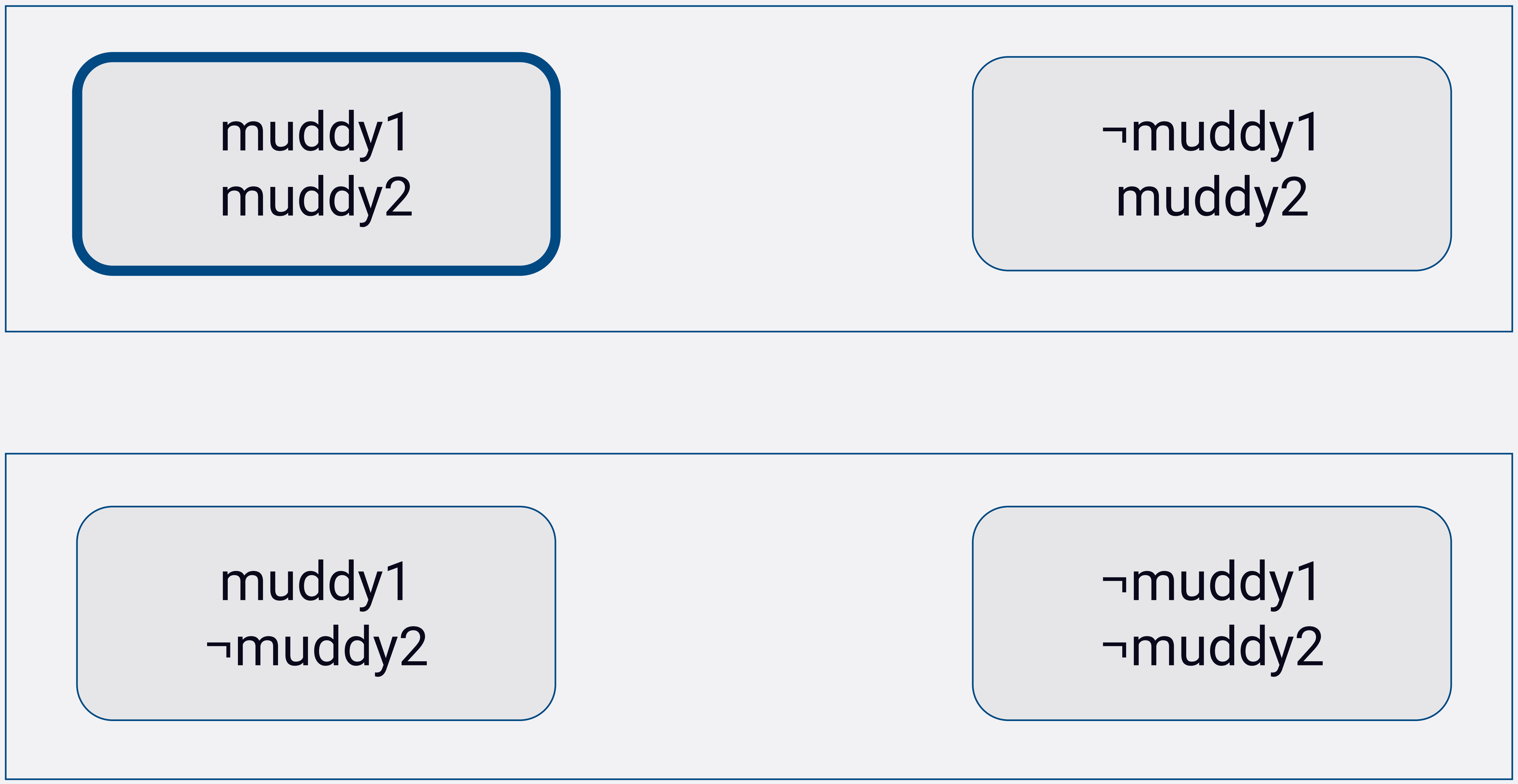
- By supposing to have n=2 and k=2:
  - Design the partitioned world.
  - Design the Kripke model.
  - Show what happen after each father's statement.

Exercise 12.6



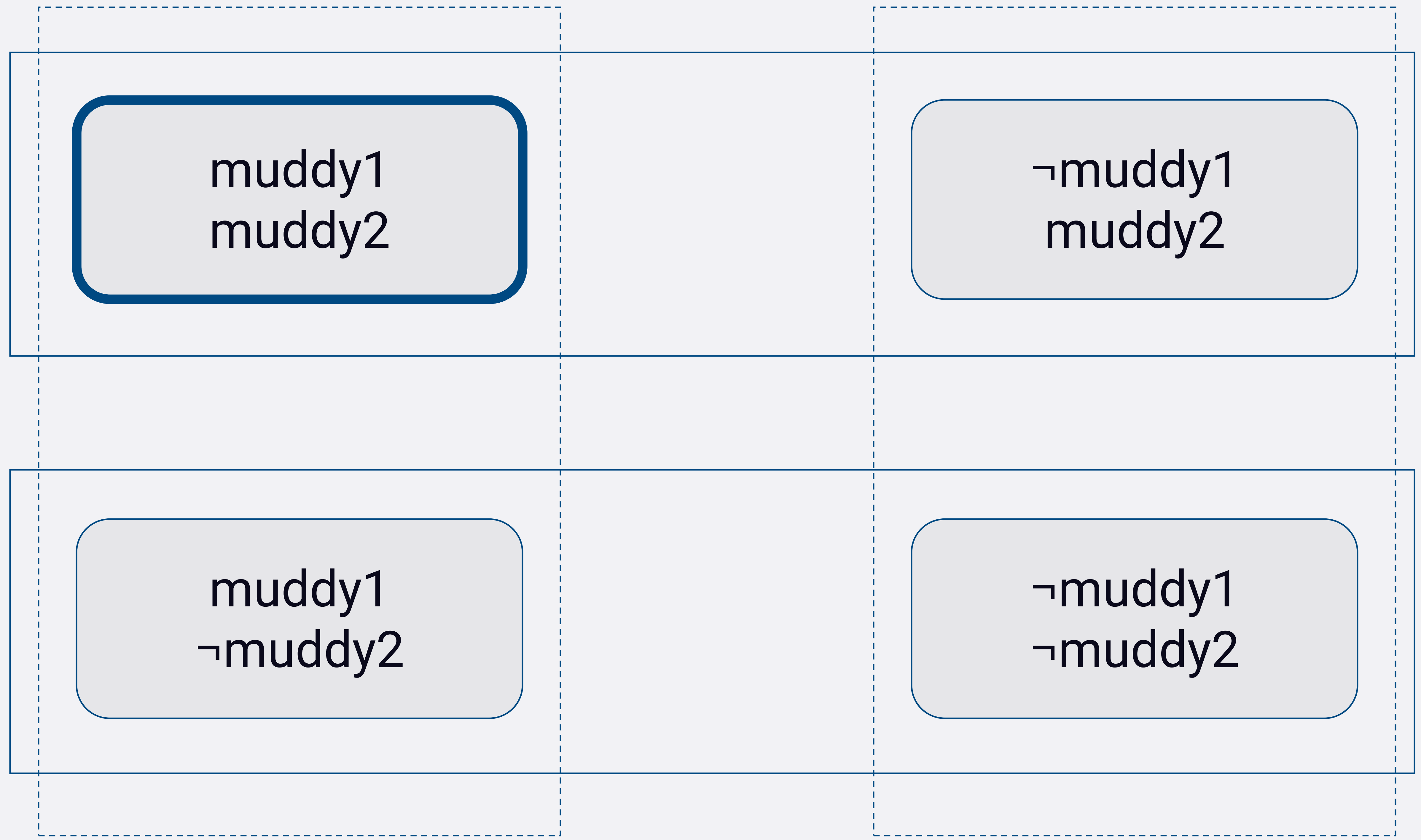
- Bold shape: actual world

Exercise 12.6



- Bold shape: actual world
- Solid boxes: equivalent classes for agent A_1 (partition for agent 1, i.e., what agent 1 knows)

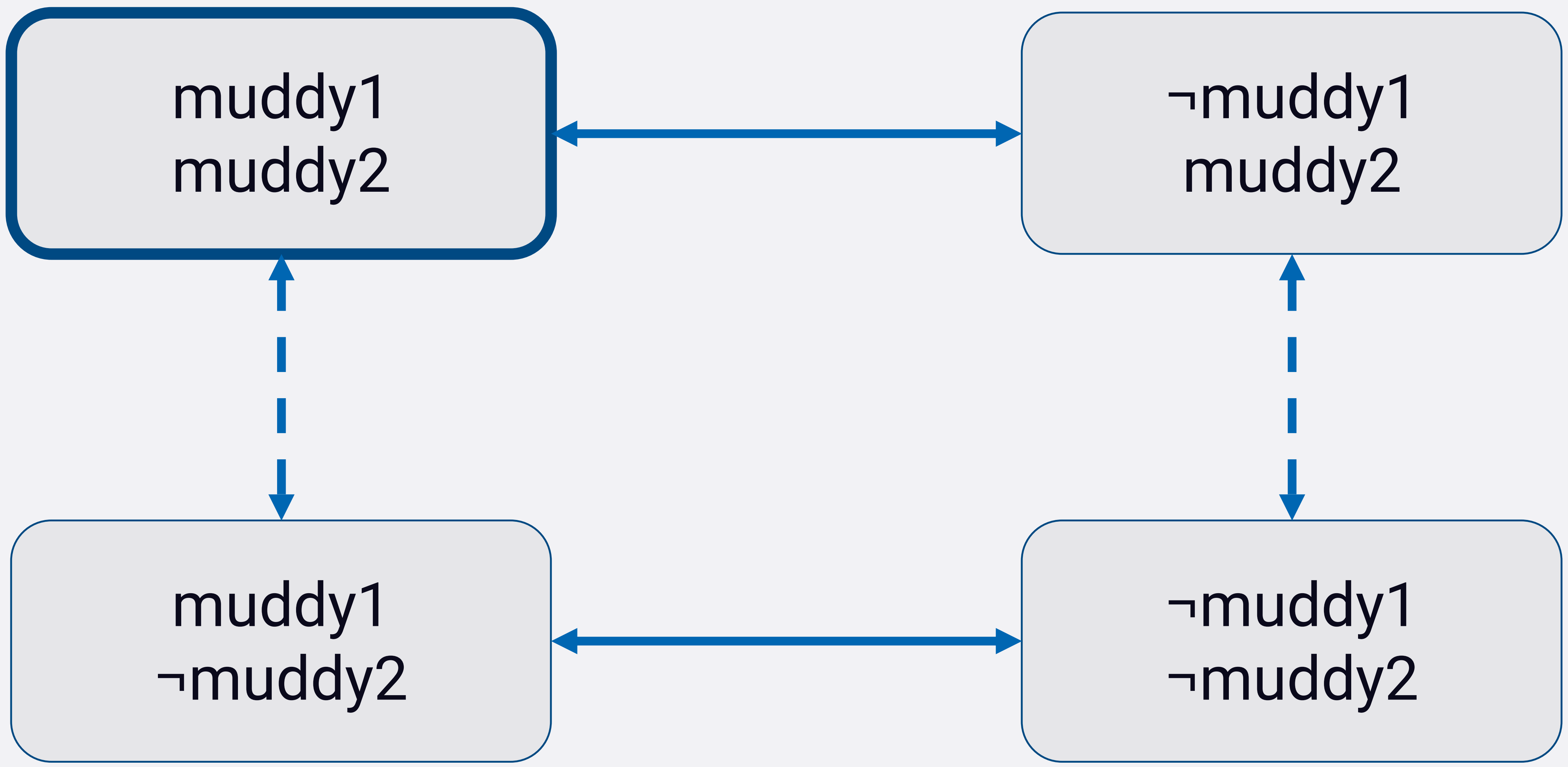
Exercise 12.6



- Bold shape: actual world
- Solid boxes: equivalent classes for agent A_1 (partition for agent 1, i.e., what agent 1 knows)
- Dotted boxes: equivalent classes for agent A_2 (partition for agent 2, i.e., what agent 2 knows)

Exercise 12.6

The Kripke model

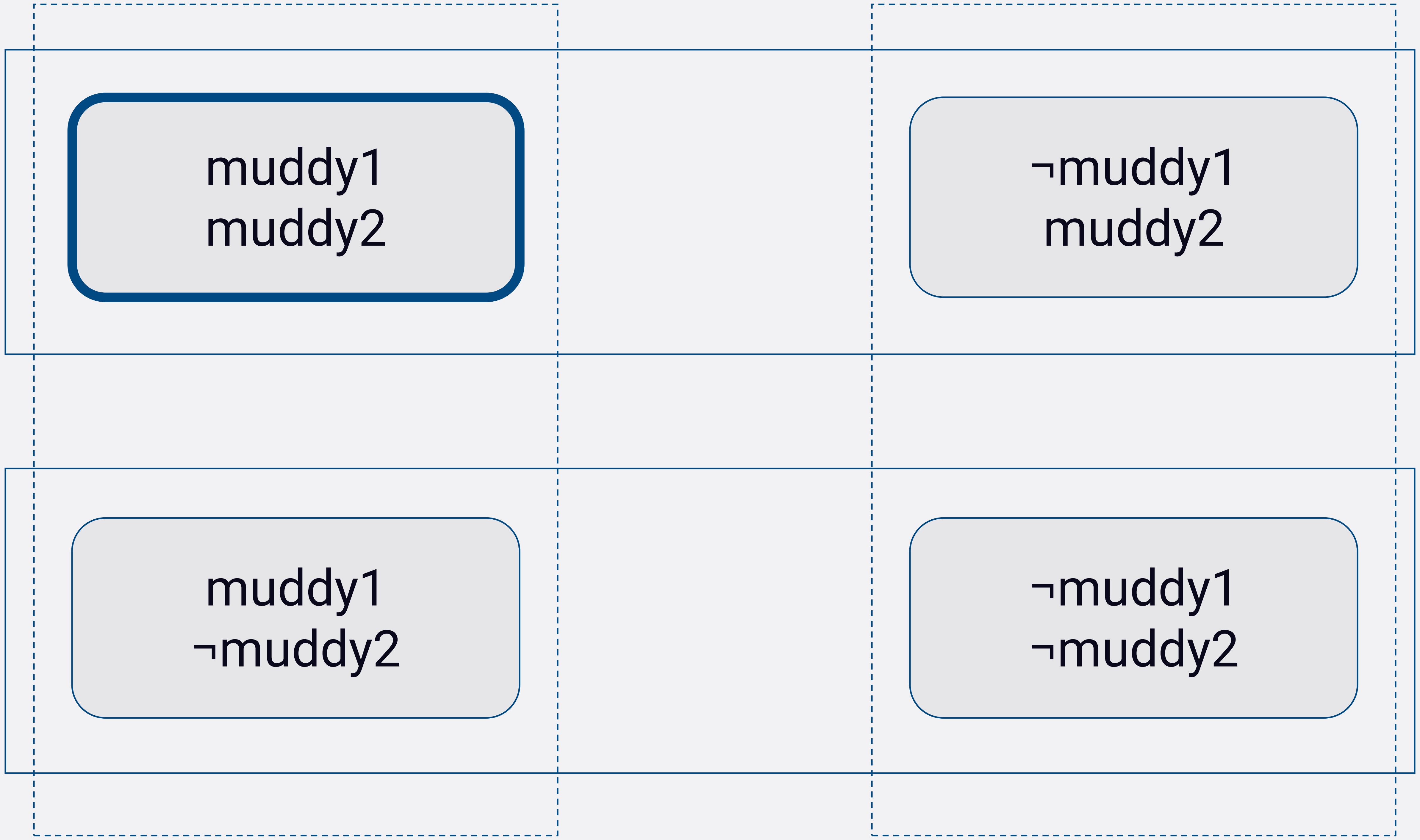


- Bold shape: actual world
- Solid boxes: equivalent classes for agent A_1 (partition for agent 1, i.e., what agent 1 knows)
- Dotted boxes: equivalent classes for agent A_2 (partition for agent 2, i.e., what agent 2 knows)

Exercise 12.6

w1 : muddy1 ∧ muddy2
w2 : muddy1 ∧ ¬muddy2
w3 : ¬muddy1 ∧ muddy2
w4 : ¬muddy1 ∧ ¬muddy2

Note: in w1 we have:
K_1 muddy2
K_2 muddy1
K_1 ¬K_2 muddy2
...
But we don't have:
K_1 muddy1

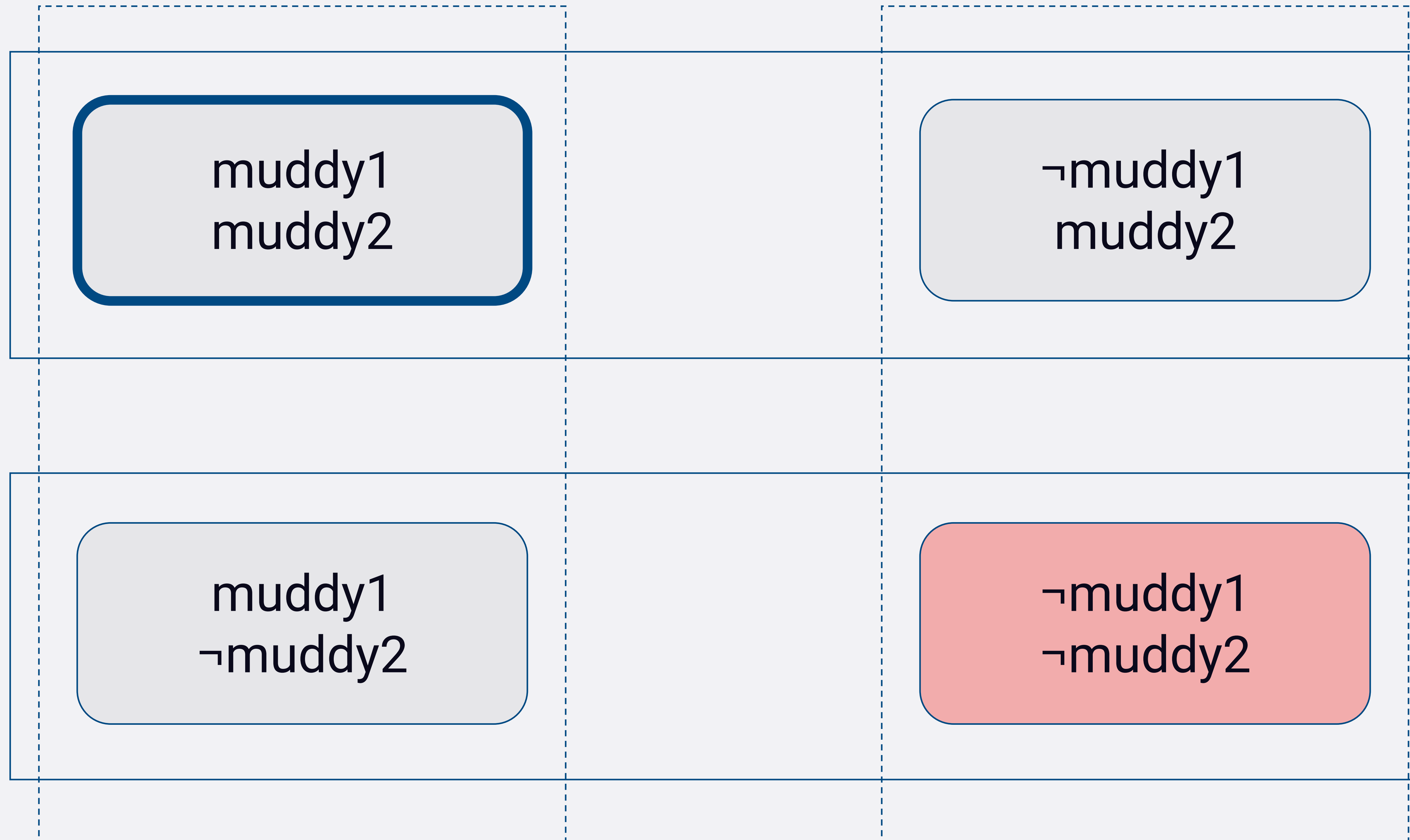


- Bold shape: actual world
- Solid boxes: equivalent classes for agent A_1 (partition for agent 1, i.e., what agent 1 knows)
- Dotted boxes: equivalent classes for agent A_2 (partition for agent 2, i.e., what agent 2 knows)

Exercise 12.6

w1 : muddy1 ∧ muddy2
w2 : muddy1 ∧ ¬muddy2
w3 : ¬muddy1 ∧ muddy2
w4 : ¬muddy1 ∧ ¬muddy2

Note: in w1 we have:
K_1 muddy2
K_2 muddy1
K_1 $\neg K_2$ muddy2
...
But we don't have:
K_1 muddy1

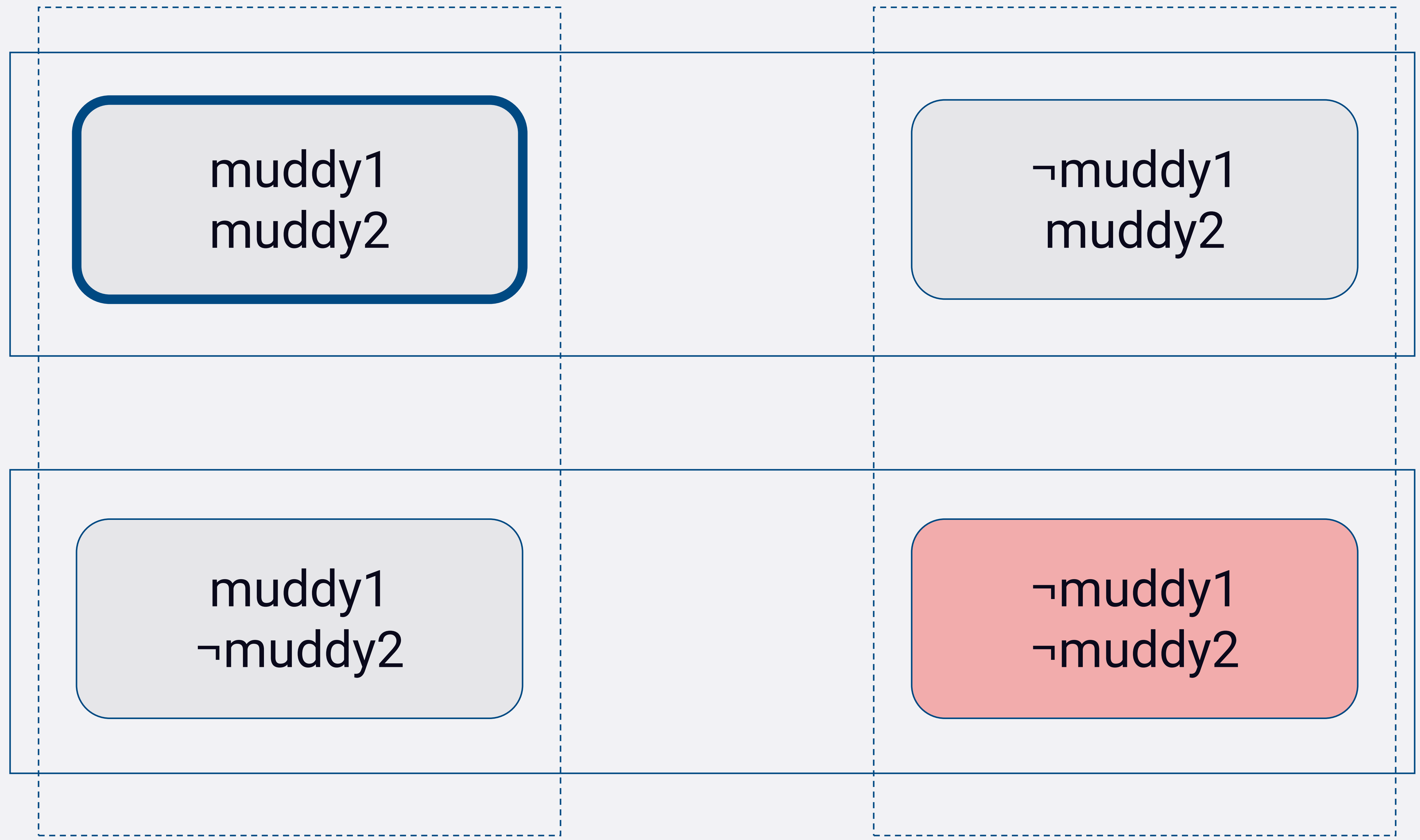


- The father says: "At least one of you has mud on his forehead."
- This eliminates the world: w4 : ¬muddy1 ∧ ¬muddy2

Exercise 12.6

w1 : muddy1 ∧ muddy2
w2 : muddy1 ∧ ¬muddy2
w3 : ¬muddy1 ∧ muddy2
w4 : ¬muddy1 ∧ ¬muddy2

Note: in w1 we have:
K_1 muddy2
K_2 muddy1
$K_1 \neg K_2$ muddy2
...
But we don't have:
K_1 muddy1

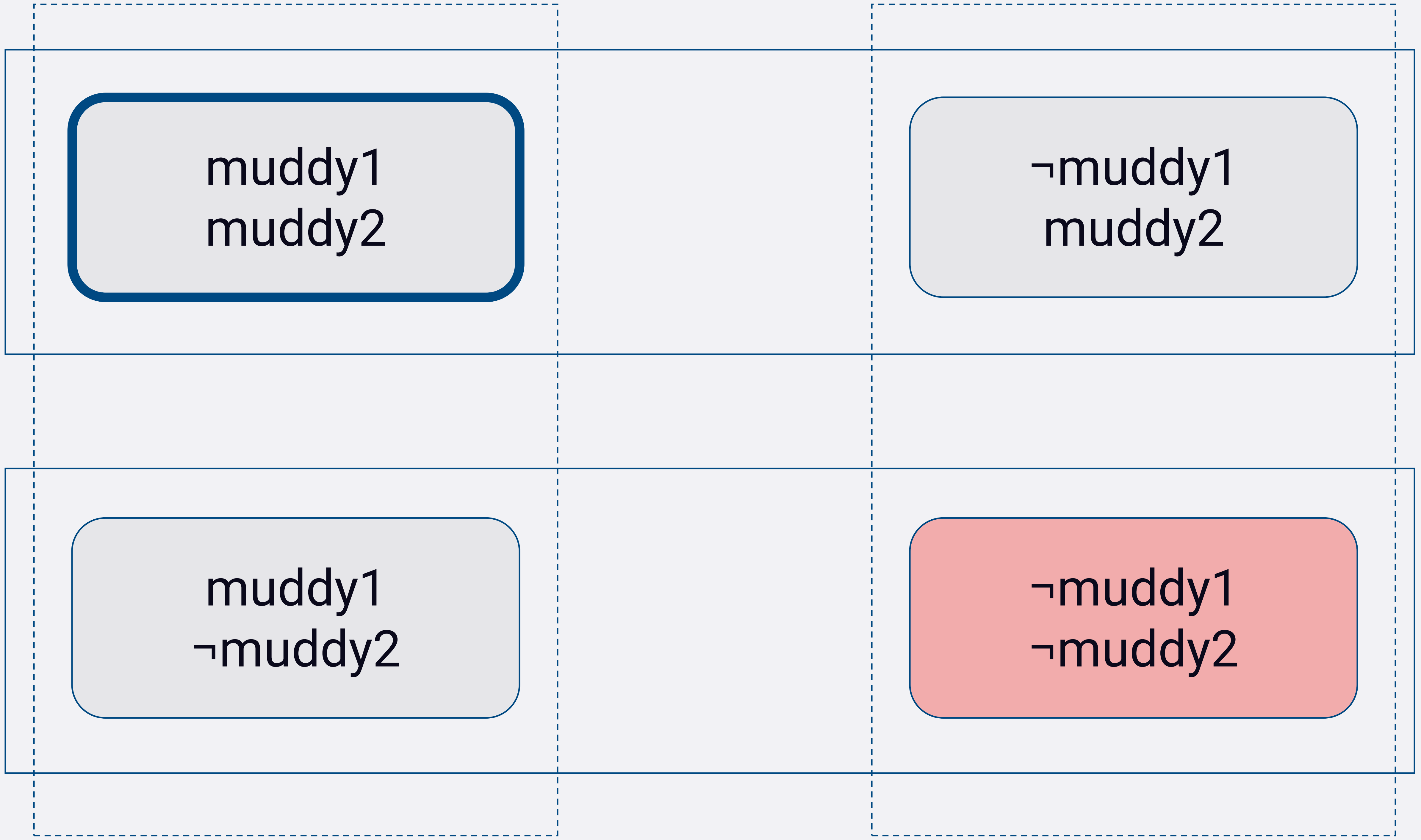


- Now, after father's announcement, the children have only three options:
  1. Other child is muddy
  2. I am muddy
  3. We are both muddy
- For instance in A_2 we see that child 2 thinks as follows:
  1. Either we are both muddy
  2. Or he (child1) is muddy and I (child 2) am not muddy

Exercise 12.6

w1 : muddy1 ∧ muddy2
w2 : muddy1 ∧ ¬muddy2
w3 : ¬muddy1 ∧ muddy2
w4 : ¬muddy1 ∧ ¬muddy2

Note: in w1 we have:
K_1 muddy2
K_2 muddy1
K_1 K_2 muddy2

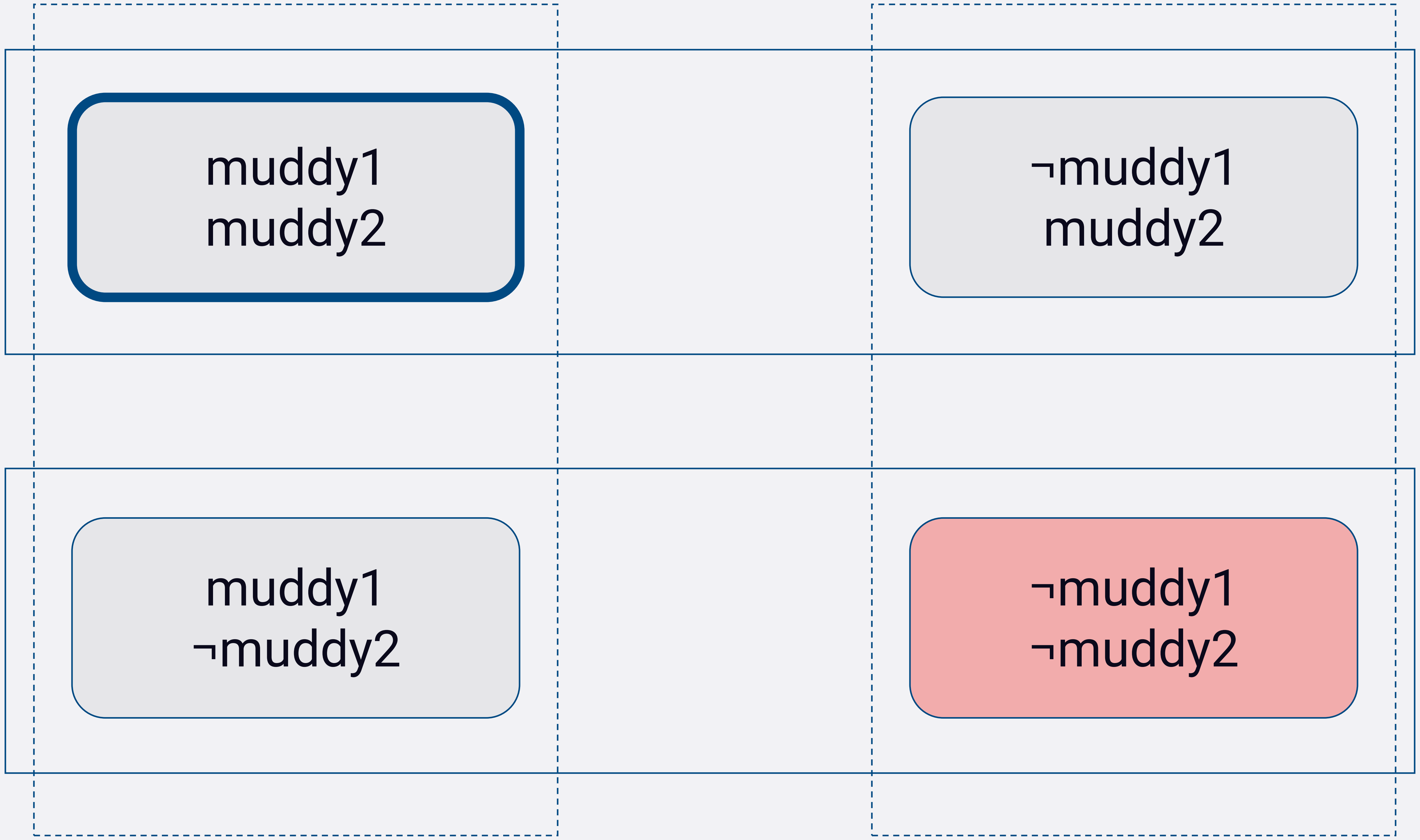


- After the second father's announcement
  1. Child 1 knows he is muddy
  2. Child 2 knows he is muddy
  3. Both children know they are muddy

Exercise 12.6

w1 : muddy1 ∧ muddy2
w2 : muddy1 ∧ ¬muddy2
w3 : ¬muddy1 ∧ muddy2
w4 : ¬muddy1 ∧ ¬muddy2

Note: in w1 we have:
K_1 muddy2
K_2 muddy1
K_1 K_2 muddy2



- After the second father's announcement
  1. Child 1 knows he is muddy
  2. Child 2 knows he is muddy
  3. Both children know they are muddy

As homework, try to do that with n=3 and k=2.

Exercise 12.7

- Define an ontology in first-order logic for tic-tac-toe. The ontology should contain situations, actions, squares, players, marks (X, O, or blank), and the notion of winning, losing, or drawing a game. Also define the notion of a forced win (or draw): a position from which a player can force a win (or draw) with the right sequence of actions. Write axioms for the domain. (Note: The axioms that enumerate the different squares and that characterize the winning positions are rather long. You need not write these out in full, but indicate clearly what they look like.)

Exercise 12.7

- Sortal predicates:
  - Player(p)
  - Mark(m)
  - Square(q)
- Constants:
  - Xp, Op: Players.
  - X, O, Blank: Marks.
  - Q11, Q12 . . . Q33: Squares.
  - S0: Situation.
- Atemporal:
  - MarkOf (p): Function mapping player p to his/her mark.
  - Winning(q1, q2, q3): Predicate. Squares q1, q2, q3 constitute a winning position.
  - Opponent(p): Function mapping player p to his opponent.
- Situation Calculus:
  - Result(a, s).
  - Poss(a, s).
- State:
  - TurnAt(s): Function mapping situation s to the player whose turn it is.
  - Marked(q, s): Function mapping square q and situation s to the mark in q at s.
  - Wins(p, s). Player p has won in situation s.
- Action:
  - Play(p, q): Function mapping player p and square q to the action of p marking q.

Exercise 12.7

- Atemporal axioms:

  A1. $MarkOf(Xp) = X$.
  A2. $MarkOf(Op) = O$.
  A3. $Opponent(Xp) = Op$.
  A4. $Opponent(Op) = Xp$.
  A5. $\forall p\ Player(p) \Leftrightarrow p = Xp \lor p = Op$.
  A6. $\forall m\ Mark(m) \Leftrightarrow m = X \lor m = O \lor m = Blank$.
  A7. $\forall q\ Square(q) \Leftrightarrow q = Q11 \lor q = Q12 \lor \ldots \lor q = Q33$.
  A8. $\forall q1,q2,q3\ WinningPosition(q1, q2, q3) \Leftrightarrow [q1 = Q11 \land q2 = Q12 \land q3 = Q13] \lor$
  $[q1 = Q21 \land q2 = Q22 \land q3 = Q23] \lor$
  . . . (Similarly for the other six winning positions $\lor$
  $[q1 = Q31 \land q2 = Q22 \land q3 = Q13]$.

- Definition of winning:

  A9. $\forall p,s\ Wins(p, s) \Leftrightarrow \exists q1,q2,q3\ WinningPosition(q1, q2, q3) \land MarkAt(q1, s) =$
  $MarkAt(q2, s) = MarkAt(q3, s) = MarkOf\ (p)$

Exercise 12.7

- Causal Axioms:
  A10. $\forall p,q$ Player(p) $\wedge$ Square(q) $\Rightarrow$ MarkAt(q, Result(Play(p, q), s)) = MarkOf (p).
  A11. $\forall p,a,s$ TurnAt(p, s) $\Rightarrow$ TurnAt(Opponent(p), Result(a, s)).

- Precondition Axiom:
  A12. Poss(Play(p, q), s) $\Rightarrow$ TurnAt(s) = p $\wedge$ MarkAt(q, s) = Blank.

- Frame Axiom:
  A13. q1 $\neq$ q2 $\Rightarrow$ MarkAt(q1, Result(Play(p, q2), s)) = MarkAt(q1, s).

- Unique names:
  A14. X $\neq$ O $\neq$ Blank.
  (Note: the unique property on players Xp $\neq$ Op follows from A14, A1, and A2.)

  A15-A50. For each i, j, k, m between 1 and 3 such that either i $\neq$ k or j $\neq$ m assert the axiom Qij $\neq$ Qkm.
  Note: In many theories it is useful to posit unique names axioms between entities of different sorts e.g. $\forall p,q$ Player(p) $\wedge$ Square(q) $\Rightarrow$ p $\neq$ q. In this theory these are not actually necessary; if you want to imagine a circumstance in which player Xp is actually the same entity as square Q23 or the same as the action Play(Xp, Q23) there is no harm in it.