Fundamentals of Artificial Intelligence
Chapter 10: Classical Planning

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Outline

1. The Problem
2. Search Strategies and Heuristics
   - Forward and Backward Search
   - Heuristics
3. Planning Graphs, Heuristics and Graphplan
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   - Heuristics Driven by Planning Graphs
   - The Graphplan Algorithm
4. Other Approaches (hints)
   - Planning as SAT Solving
   - Planning as FOL Inference

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Automated Planning (aka "Planning")

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Synthesize a sequence of actions (plan) to be performed by an agent leading from an initial state of the world to a set of target states (goal)

- Planning is both:
  - an application per se
  - a common activity in many applications (e.g. design & manufacturing, scheduling, robotics,...)
- Similar to problem-solving agents (Ch.03), but with factored/structured representation of states
- "Classical" Planning (this chapter): fully observable, deterministic, static environments with single agents

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Automated Planning [cont.]

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- Given:
  - an initial state
  - a set of actions you can perform
  - a (set of) state(s) to achieve (goal)
- Find:
  - a plan: a partially- or totally-ordered set of actions needed to achieve the goal from the initial state

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A Language for Planning: PDDL

Planning Domain Definition Language (PDDL)

- A state is a conjunction of fluents: ground, function-less atoms
  - ex: $Poor \wedge Unknown$, $At(Truck_1, Melbourne) \wedge At(Truck_2, Sydney)$
  - ex of non-fluents: $At(x,y)$ (non ground), $\neg Poor$ (negated), $At(Father(Fred), Sydney)$ (not function-less)
  - closed-world assumption: all non-mentioned fluents are false
  - unique names assumption: distinct names refer to distinct objects
- Actions are described by a set of action schemata
  - concise description: describe which fluent change

  $\Longrightarrow$ the other fluents implicitly maintain their values
- Action Schema: consists in action name, a list of variables in the schema, the precondition, the effect (aka postcondition)
  - precondition and effect are conjunctions of literals (positive or negated atomic sentences)
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PDDL: Example

- Action schema:
  $Action(Fly(p, from, to),$
  $\quad PRECOND : At(p, from) \land Plane(p) \land Airport(from) \land Airport(to)$
  $\quad EFFECT \quad : \neg At(p, from) \land At(p, to))$
- Action instantiation:
  $Action(Fly(P_1, SFO, JFK),$
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A Language for Planning: PDDL [cont.]

- Precondition: must hold to ensure the action can be executed
  - defines the states in which the action can be executed
  - action is applicable in state s if the preconditions are satisfied by s
- Effect: represent the effects of the action on the world
  - defines the result of executing the action
- Add list (ADD(a)): the positive literals in the action's effects
  - ex: $\{At(p, to)\}$
- Delete list (DEL(a)): (the fluents in) the negative literals in the action's effects
  - ex: $\{At(p, from)\}$
- Result of action a in state s: RESULT(s,a)$\stackrel{\text{def}}{=}$(s$\setminus$DEL(a) $\cup$ ADD(a))
  - start from s
  - remove the fluents that appear as negative literals in effect
  - add the fluents that appear as positive literals in effect
  - ex: $Fly(P_1, SFO, JFK) \Longrightarrow$ remove $At(P_1, SFO)$, add $At(P_1, JFK)$

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- $s : At(P_1, SFO) \wedge Plane(P_1) \wedge Airport(SFO) \wedge Airport(JFK) \wedge ...$

$\Longrightarrow s' : At(P_1, JFK) \wedge Plane(P_1) \wedge Airport(SFO) \wedge Airport(JFK) \wedge ...$

Sometimes we want to propositionalize a PDDL problem: replace each action schema with a set of ground actions.

- Ex: $...At\_P_1\_SFO \wedge Plane\_P_1 \wedge Airport\_SFO \wedge Airport\_JFK)...$

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A Language for Planning: PDDL [cont.]

Time in PDDL

- Fluents do not explicitly refer to time
- Times and states are implicit in the action schemata:
    - the precondition always refers to time t
    - the effect to time t+1.

PDDL Problem

- A set of action schemata defines a planning domain
- PDDL problem: a planning domain, an initial state and a goal
    - the initial state is a conjunction of ground atoms (positive literals)
        - closed-world assumption: any not-mentioned atoms are false
    - the goal is a conjunction of literals (positive or negative)
        - may contain variables, which are implicitly existentially quantified
        - a goal g may represent a set of states (the set of states entailing g)
- Ex: goal: $At(p, SFO) \wedge Plane(p)$:
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        - may contain variables, which are implicitly existentially quantified
        - a goal g may represent a set of states (the set of states entailing g)
- Ex: goal: $At(p, SFO) \land Plane(p)$:
    - “p” implicitly means “for some plane p”
    - the state $Plane(Plane_1) \land At(Plane_1, SFO) \land ...$ entails g

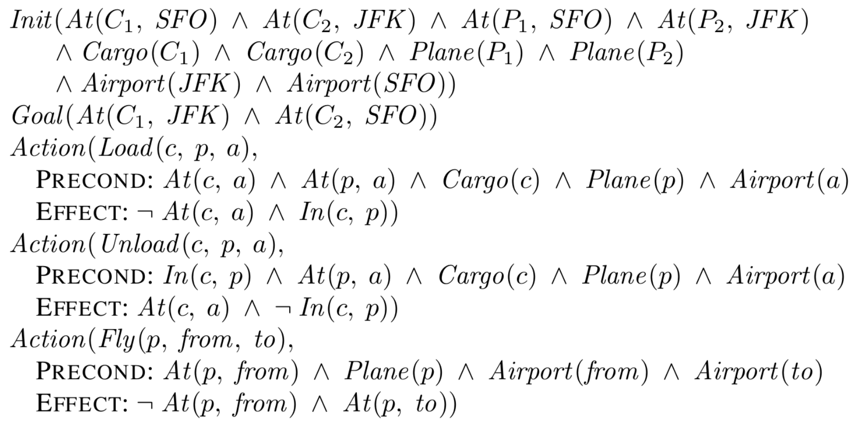
A Language for Planning: PDDL [cont.]

Planning as a search problem

All components of a search problem

- an initial state
- an ACTIONS function
- a RESULT function
- and a goal test

Example: Air Cargo Transport



$Init(At(C_1, SFO) \land At(C_2, JFK) \land At(P_1, SFO) \land At(P_2, JFK)$
$\quad \land Cargo(C_1) \land Cargo(C_2) \land Plane(P_1) \land Plane(P_2)$
$\quad \land Airport(JFK) \land Airport(SFO))$
$Goal(At(C_1, JFK) \land At(C_2, SFO))$
$Action(Load(c, p, a),$
PRECOND: $At(c, a) \land At(p, a) \land Cargo(c) \land Plane(p) \land Airport(a)$
EFFECT: $\neg At(c, a) \land In(c, p))$
$Action(Unload(c, p, a),$
PRECOND: $In(c, p) \land At(p, a) \land Cargo(c) \land Plane(p) \land Airport(a)$
EFFECT: $At(c, a) \land \neg In(c, p))$
$Action(Fly(p, from, to),$
PRECOND: $At(p, from) \land Plane(p) \land Airport(from) \land Airport(to)$
EFFECT: $\neg At(p, from) \land At(p, to))$

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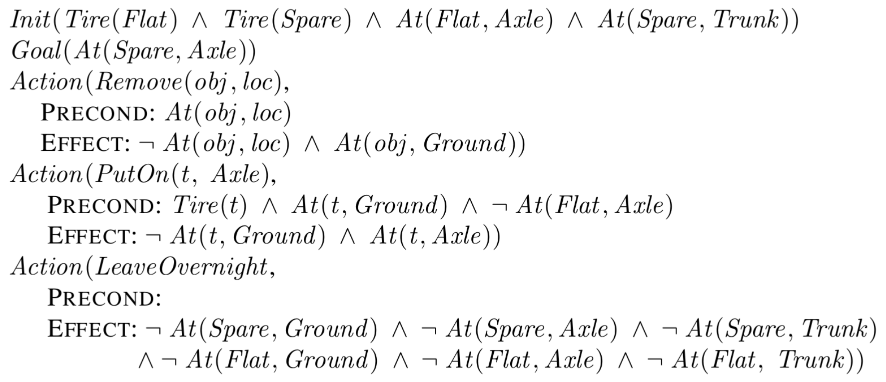
One solution:
$[Load(C_1, P_1, SFO), Fly(P_1, SFO, JFK), Unload(C_1, P_1, JFK),$
$Load(C2, P2, JFK), Fly(P2, JFK, SFO), Unload(C2, P2, SFO)]$

Example: Spare Tire Problem



$Init(Tire(Flat) \wedge Tire(Spare) \wedge At(Flat, Axle) \wedge At(Spare, Trunk))$
$Goal(At(Spare, Axle))$
$Action(Remove(obj, loc),$
  $\textsc{Precond}: At(obj, loc)$
  $\textsc{Effect}: \neg At(obj, loc) \wedge At(obj, Ground))$
$Action(PutOn(t, Axle),$
  $\textsc{Precond}: Tire(t) \wedge At(t, Ground) \wedge \neg At(Flat, Axle)$
  $\textsc{Effect}: \neg At(t, Ground) \wedge At(t, Axle))$
$Action(LeaveOvernight,$
  $\textsc{Precond}:$
  $\textsc{Effect}: \neg At(Spare, Ground) \wedge \neg At(Spare, Axle) \wedge \neg At(Spare, Trunk)$
    $\wedge \neg At(Flat, Ground) \wedge \neg At(Flat, Axle) \wedge \neg At(Flat, Trunk))$

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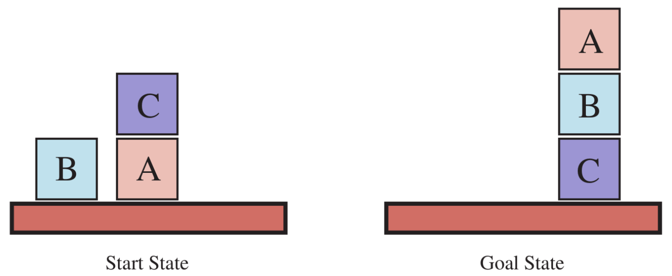
One solution:
[*Remove*(*Flat*, *Axle*), *Remove*(*Spare*, *Trunk*), *PutOn*(*Spare*, *Axle*)]

Example: Blocks World



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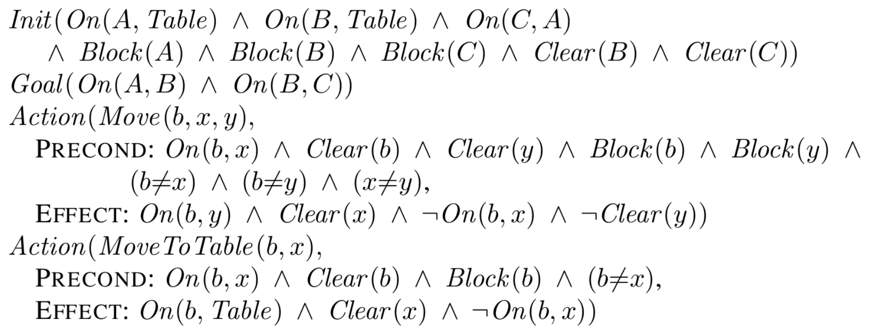
Example: Blocks World [cont.]



$Init(On(A, Table) \land On(B, Table) \land On(C, A)$
$\quad \land Block(A) \land Block(B) \land Block(C) \land Clear(B) \land Clear(C))$
$Goal(On(A, B) \land On(B, C))$
$Action(Move(b, x, y),$
$\quad$ PRECOND: $On(b, x) \land Clear(b) \land Clear(y) \land Block(b) \land Block(y) \land$
$\qquad\qquad (b \neq x) \land (b \neq y) \land (x \neq y),$
$\quad$ EFFECT: $On(b, y) \land Clear(x) \land \neg On(b, x) \land \neg Clear(y))$
$Action(MoveToTable(b, x),$
$\quad$ PRECOND: $On(b, x) \land Clear(b) \land Block(b) \land (b \neq x),$
$\quad$ EFFECT: $On(b, Table) \land Clear(x) \land \neg On(b, x))$

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One solution: $[MoveToTable(C, A), Move(B, Table, C), Move(A, Table, B)]$

Decidability and Complexity

- PlanSAT: the question of whether there exists any plan that solves a planning problem
  - decidable for classical planning
  - with function symbols, the number of states becomes infinite $\Longrightarrow$ undecidable
  - in PSPACE
- Bounded PlanSAT: the question of whether there exists any plan that of a given length k or less
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    - Forward and Backward Search
    - Heuristics

3. Planning Graphs, Heuristics and Graphplan
    - Planning Graphs
    - Heuristics Driven by Planning Graphs
    - The Graphplan Algorithm

4. Other Approaches (hints)
    - Planning as SAT Solving
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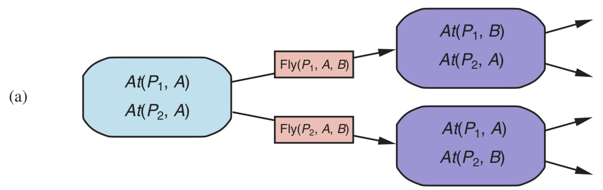
Two Main Approaches

(a) Forward search (aka progression search)

- start in the initial state
- use actions to search forward for a goal state

(b) Backward search (aka regression search)

- start from goal states
- use reverse actions to search forward for the initial state



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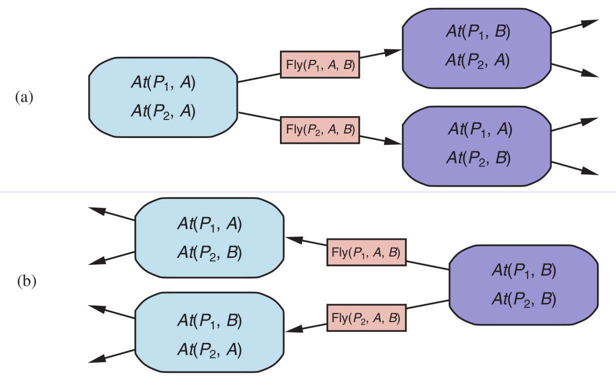

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Forward Search

- Forward search (aka progression search)
    - choose actions whose preconditions are satisfied
    - add positive effects, delete negative
- Goal test: does the state satisfy the goal?
- Step cost: each action costs 1

⟹ We can use any of the search algorithms from Ch. 03, 04
    - need keeping track of the actions used to reach the goal
- Breadth-first and best-first
    - Sound: if they return a plan, then the plan is a solution
    - Complete: if a problem has a solution, then they will return one
    - Require exponential memory wrt. solution length! ⟹ unpractical
- Depth-first search and greedy search
    - Sound
    - Not complete
        - may enter in infinite loops
        - (classical planning only): made complete by loop-checking
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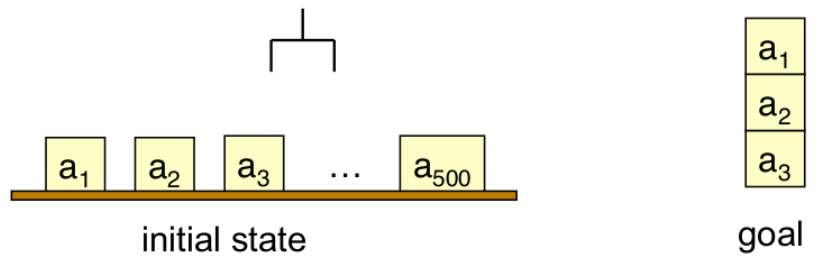
Branching Factor of Forward Search

- Planning problems can have huge state spaces
- Forward search can have a very large branching factor
  - ex: *pickup*(a_1), *pickup*(a_2), ..., *pickup*(a_{500})

⟹ Forward-search can waste time trying lots of irrelevant actions

⟹ Need a good heuristic to guide the search



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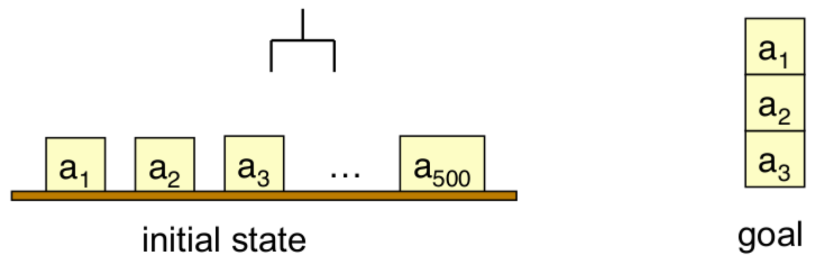

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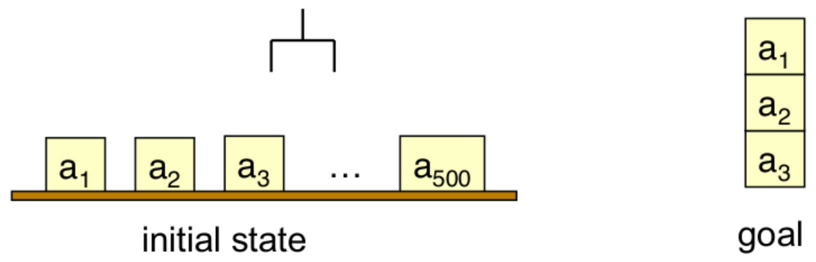

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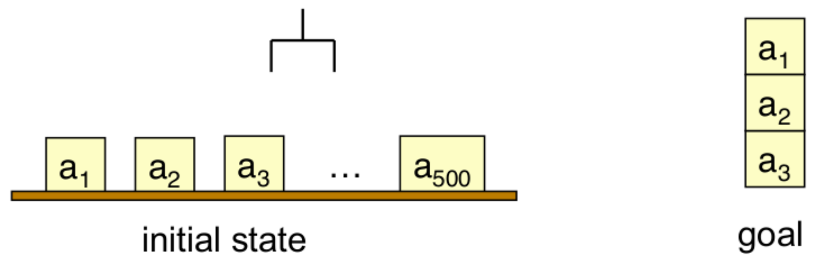

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Backward Search (aka Regression or Relevant-States)

- Predecessor state g' of ground goal g via ground action a:
  $Pos(g') \overset{\text{def}}{=} (Pos(g) \setminus Add(a)) \cup Pos(Precond(a))$
  $Neg(g') \overset{\text{def}}{=} (Neg(g) \setminus Del(a)) \cup Neg(Precond(a))$
- Note: Both g and g' represent many states
  - irrelevant ground atoms unassigned

- Consider the goal $At(C_1, SFO) \wedge At(C_2, JFK)$
- Consider the ground action:
  $Action(Unload(C_1, P_1, SFO),$
  $PRECOND:$
  $In(C_1, P_1) \wedge At(P_1, SFO) \wedge Cargo(C_1) \wedge Plane(P_1) \wedge Airport(SFO)$
  $EFFECT: At(C_1, SFO) \wedge \neg In(C_1, P_1))$
- This produces the sub-goal g':
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Backward Search [cont.]

- Idea: deal with partially un-instantiated actions and states
  - avoid unnecessary instantiations

  $\Longrightarrow$ no need to produce a goal for every possible instantiation
- use the most general unifier
- standardize action schemata first (rename vars into fresh ones)

- Consider the goal $At(C_1, SFO) \wedge At(C_2, JFK)$
- Consider the partially-instantiated action:
  $Action(Unload(C_1, p', SFO),$
  $PRECOND:$
  $In(C_1, p') \wedge At(p', SFO) \wedge Cargo(C_1) \wedge Plane(p') \wedge Airport(SFO)$
  $EFFECT: At(C_1, SFO) \wedge \neg In(C_1, p'))$
- This produces the sub-goal g':
  $In(C_1, p') \wedge At(p', SFO) \wedge Cargo(C_1) \wedge Plane(p') \wedge$
  $Airport(SFO) \wedge At(C_2, JFK)$
- Represents states with all possible planes

  $\Longrightarrow$ no need to produce a subgoal for every plane $P_1, P_2, P_3, ...$

Backward Search [cont.]

- Idea: deal with partially un-instantiated actions and states
  - avoid unnecessary instantiations

  $\Longrightarrow$ no need to produce a goal for every possible instantiation
- use the most general unifier
- standardize action schemata first (rename vars into fresh ones)

- Consider the goal $At(C_1, SFO) \land At(C_2, JFK)$
- Consider the partially-instantiated action:

  $Action(Unload(C_1, p', SFO),$

  $PRECOND:$

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(see AIMA book for formal definition)

- Ex: consider the goal $At(C_1, SFO) \wedge At(C_2, JFK)$

is relevant (previous example)

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Outline

1. The Problem

2. Search Strategies and Heuristics
   - Forward and Backward Search
   - Heuristics

3. Planning Graphs, Heuristics and Graphplan
   - Planning Graphs
   - Heuristics Driven by Planning Graphs
   - The Graphplan Algorithm

4. Other Approaches (hints)
   - Planning as SAT Solving
   - Planning as FOL Inference

Heuristics for (Forward-Search) Planning

- Recall: A^* is a best-first algorithm which
  - uses an evaluation function f(s) = g(s) + h(s),
  - g(s): (exact) cost to reach s
  - h(s): admissible (optimistic) heuristics
    (never overestimates the distance to the goal)
- A technique for admissible heuristics: problem relaxation

  $\Longrightarrow$ h(s): the exact cost of a solution to the relaxed problem
- Forms of problem relaxation exploiting problem structure
  - Add arcs to the search graph $\Longrightarrow$ make it easier to search
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Ignore-Preconditions Heuristics

- Ignore all preconditions drops all preconditions from actions
  - every action is applicable in any state
  - any single goal literal can be satisfied in one step (or there is no solution)
  - fast, but over-optimistic
- Remove all preconditions & effects, except literals in the goal
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Ignore-Preconditions Heuristics: Example

Sliding tiles

$Action(Slide(t, s_1, s_2),$
$\quad PRECOND: On(t, s_1) \land Tile(t) \land Blank(s_2) \land Adjacent(s_1, s_2)$
$\quad EFFECT: On(t, s_2) \land Blank(s_1) \land \neg On(t, s_1) \land \neg Blank(s_2))$

- Remove the preconditions $Blank(s_2) \land Adjacent(s_1, s_2)$

  $\Longrightarrow$ we get the number-of-misplaced-tiles heuristics

- Remove the precondition $Blank(s_2)$

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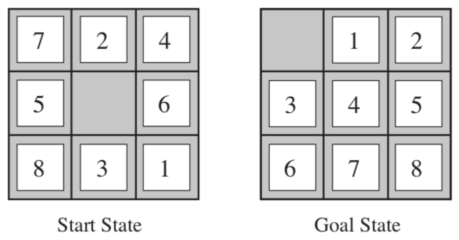

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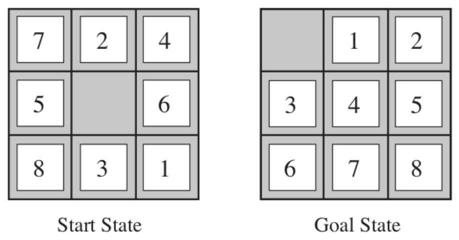

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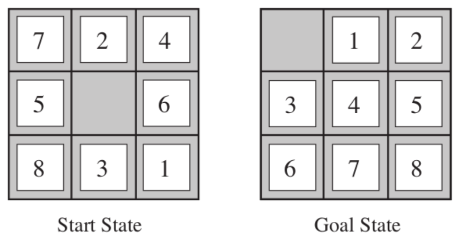

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Ignore Delete-list Heuristics

- Assumption: goals & preconditions contain only positive literals
  - reasonable in many domains
- Idea: Remove the delete lists from all actions
  - No action will ever undo the effect of actions,

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- Still NP-hard to find the optimal solution of the relaxed problem
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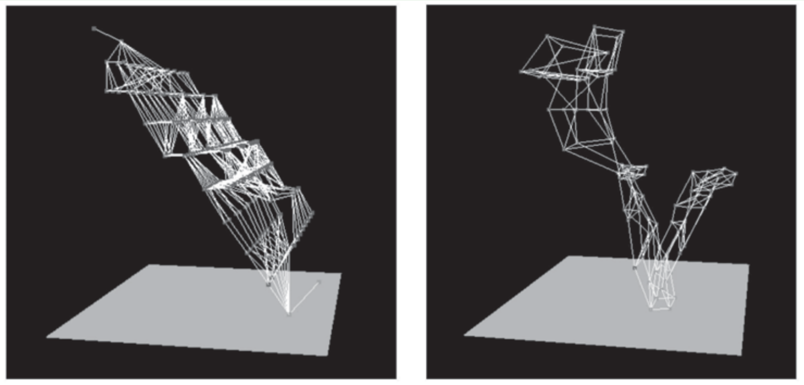
- Still NP-hard to find the optimal solution of the relaxed problem
  - can be approximated in polynomial time, with hill-climbing
- Can be very effective for some problems

Ignore Delete-list Heuristics: Example (Hoffmann’05)

- Planning state spaces with ignore-delete-lists heuristic
  - height above the bottom plane is the heuristic score of a state
  - states on the bottom plane are goals

$\Longrightarrow$ No local minima, non dead-ends, non backtracking

$\Longrightarrow$ Search for the goal is straightforward



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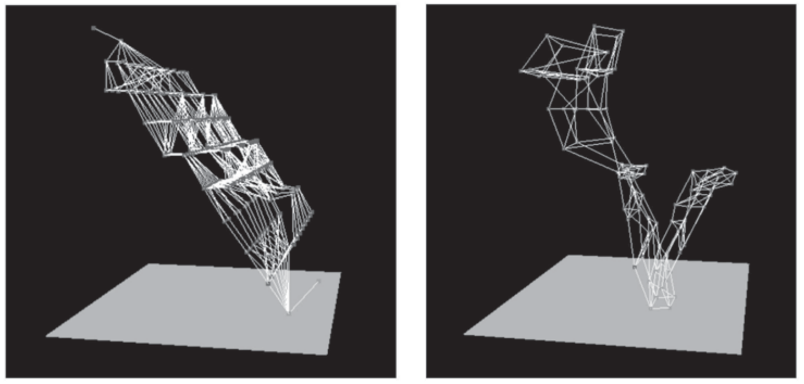

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State Abstractions

- Many-to-one mapping from states in the ground/original representation of the problem to a more abstract representation
  - drastically reduces the number of states
- Common strategy: ignore some (less-relevant) fluents
  - drop k fluents $\Longrightarrow$ reduce search space by 2^k factors
  - relevance based on (heuristic) evaluation or domain knowledge

- Air cargo problem: 10 airports, 50 planes, 200 pieces of cargo
  $\Longrightarrow 50^{10} \cdot 200^{50+10} \approx 10^{155}$ states
- Consider particular problem in that domain
  - all packages are at 5 airports
  - all packages at a given airport have the same destination
- Abstraction: drop all "At" fluents except for these involving one plane and one package at each of the the 5 airports
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Other strategies to define heuristics

- Problem decomposition
  - “divide & conquer” problem into subproblem
  - solve subproblems independently
- Using a data structure called “planning graphs” (next section)

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Outline

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1. The Problem

2. Search Strategies and Heuristics
   - Forward and Backward Search
   - Heuristics

3. Planning Graphs, Heuristics and Graphplan
   - Planning Graphs
   - Heuristics Driven by Planning Graphs
   - The Graphplan Algorithm

4. Other Approaches (hints)
   - Planning as SAT Solving
   - Planning as FOL Inference

Planning Graph

Generalities

- A data structure which is a rich source of information:
  - can be used to give better heuristic estimates h(s)
  - can drive an algorithm called Graphplan
- A polynomial size approximation to the (exponential) search tree
  - can be constructed very quickly

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Planning Graph: Definition

- A directed graph, built forward and organized into levels
  - level S_0: contain each ground fluent that holds in the initial state
  - level A_0: contains each ground action applicable in S_0
  - ...
  - level A_i: contains all ground actions with preconditions in S_{i-1}
  - level S_{i+1}: all the effects of all the actions in A_i
    - each S_i may contain both P_j and $\neg P_j$
- Persistence actions (aka maintenance actions, no-ops)
  - say that a literal l persists if no action negates it
- Mutual exclusion links (mutex) connect
  - incompatible pairs of actions
  - incompatible pairs of literals

Deals with ground states and actions only

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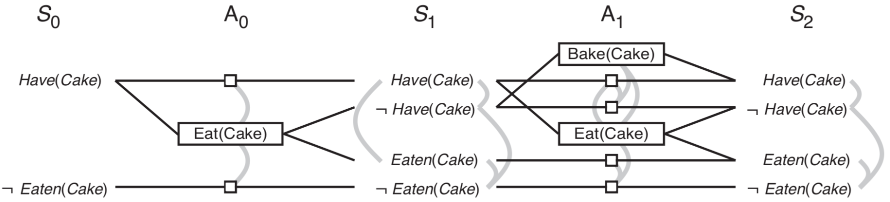
Planning Graph: Example

Init(*Have*(*Cake*))
Goal(*Have*(*Cake*) ∧ *Eaten*(*Cake*))
Action(*Eat*(*Cake*)
  PRECOND: *Have*(*Cake*)
  EFFECT: ¬ *Have*(*Cake*) ∧ *Eaten*(*Cake*))
Action(*Bake*(*Cake*)
  PRECOND: ¬ *Have*(*Cake*)
  EFFECT: *Have*(*Cake*))

You would like to eat your cake and still have a cake. Fortunately, you can bake a new one.

Rectangles indicate actions
Small squares persistence actions (**no-ops**)
Straight lines indicate preconditions and effects
Mutex links are shown as curved gray lines



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Mutex Computation

- Two actions at the same action-level have a mutex relation if
  - Inconsistent effects: an effect of one negates an effect of the other
  - Interference: one deletes a precondition of the other
  - Competing needs: they have mutually exclusive preconditions
- Otherwise they don't interfere with each other

  ⟹ both may appear in a solution plan
- Two literals at the same state-level have a mutex relation if
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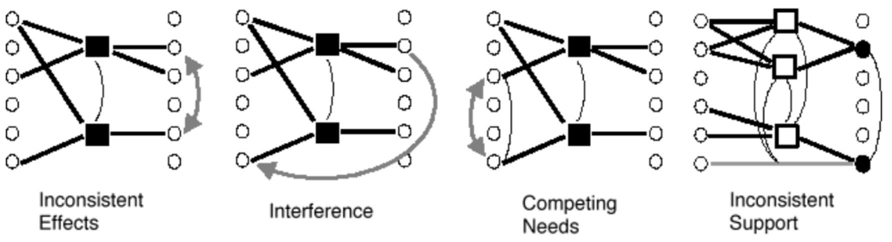

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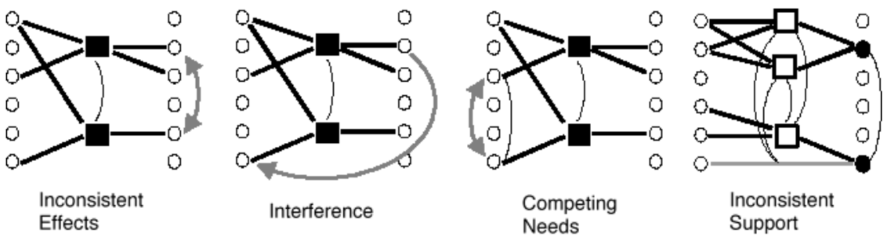

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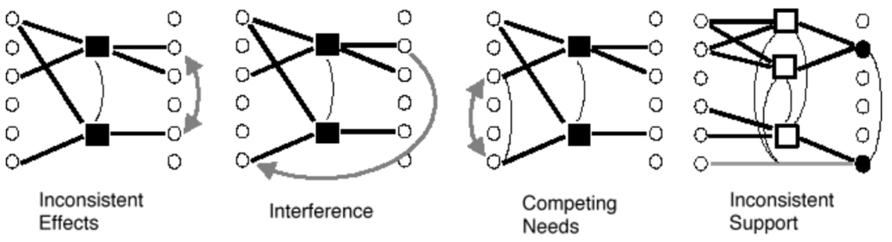

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    ex: persistence of *Have*(*Cake*), *Eat*(*Cake*) have competing effects
    ex: *Bake*(*Cake*), *Eat*(*Cake*) have competing effects
  - Interference: one deletes a precondition of the other
    ex: *Eat*(*Cake*) interferes with the persistence of *Have*(*Cake*)
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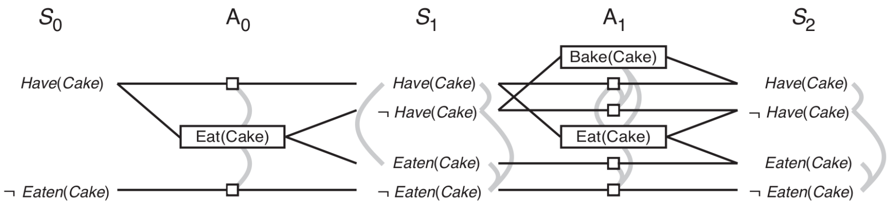
    ex: *Bake*(*Cake*) and *Eat*(*Cake*)



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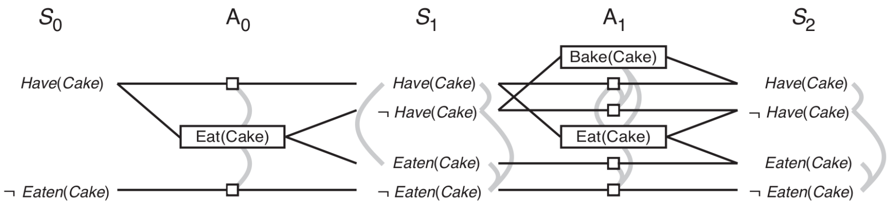

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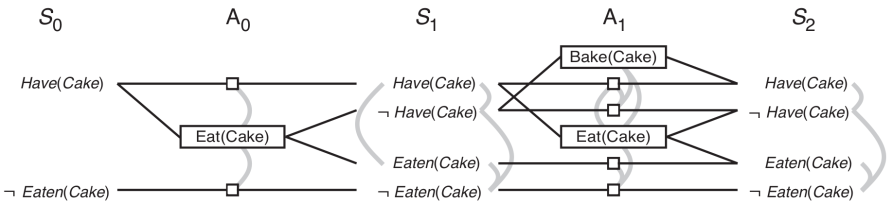

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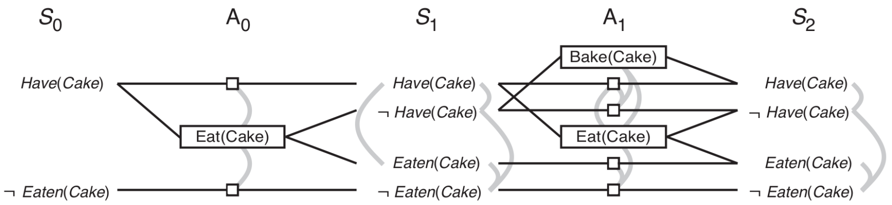

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    ex.: *Have*(*Cake*), ¬*Have*(*Cake*)
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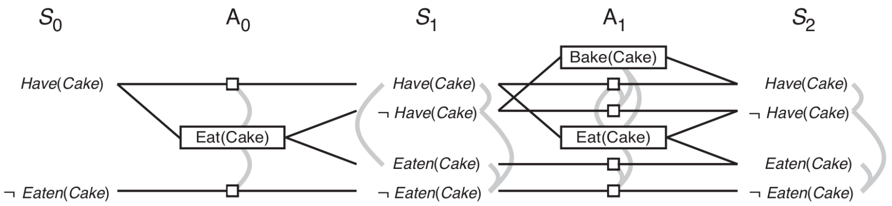

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Building of the Planning Graph

Create initial layer S_0:

1. insert into S_0 all literals in the initial state

Repeat for increasing values of $i = 0, 1, 2, ...$:

Create action layer A_i:

1. for each action schema, for each way to unify its preconditions to non-mutually exclusive literals in S_i, enter an action node into A_i
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Planning Graphs: Complexity

- A planning graph is polynomial in the size of the problem:
  - a graph with n levels, a actions, l literals, has size $O(n(a+l)^2)$
  - time complexity is also $O(n(a+l)^2)$

$\Longrightarrow$ The process of constructing the planning graph is very fast

  - does not require choosing among actions

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Planning Graphs for Heuristic Estimation

Information provided by Planning Graphs

- Each level S_i represents a set of possible belief states
  - two literals connected by a mutex belong to different belief state
- A literal not appearing in the final level of the graph cannot be achieved by any plan

  ⟹ if a goal literal is not in the final level, the problem is unsolvable
- The level S_j a literal l appears first is never greater than the level it can be achieved in a plan
  - j is called the level cost of literal l
- the level cost of a literal g_i in the graph constructed starting from state s, is an estimate of the cost to achieve it from s (i.e. h(g))
  - this estimate is admissible
  - ex: from s_0 Have(cake) has cost 0 and Eaten(cake) has cost 1
- Planning graph admits several actions per level

  ⟹ inaccurate estimate
- Serialization: enforcing only one action per level (adding mutex)

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Planning Graphs for Heuristic Estimation [cont.]

Estimating the heuristic cost of a conjunction of goal literals

- Max-level heuristic: the maximum level cost of the sub-goals
  - admissible
- Level-sum heuristic: the sum of the level costs of the goals
  - can be inadmissible when goals are not independent,
  - it may work well in practice
- Set-level heuristic: the level at which all goal literals appear together, without pairwise mutexes
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function GRAPHPLAN(problem) returns solution or failure
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      if solution $\neq$ failure then return solution
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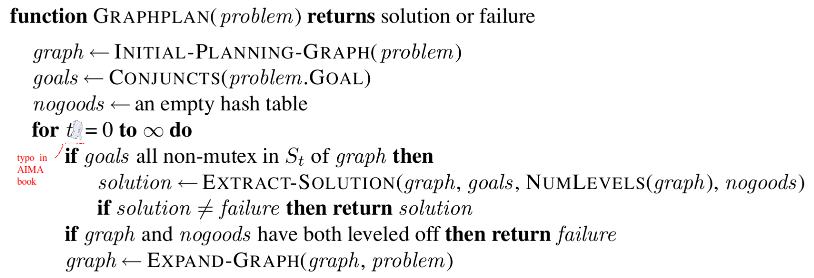

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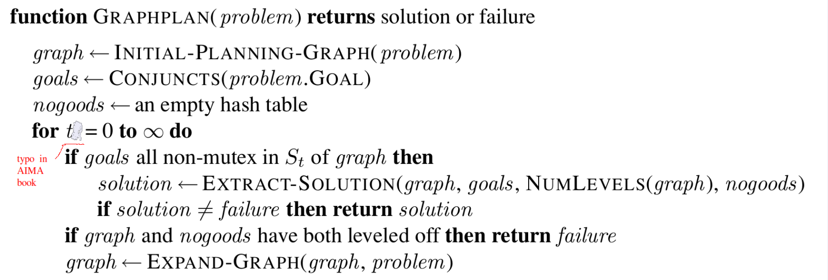

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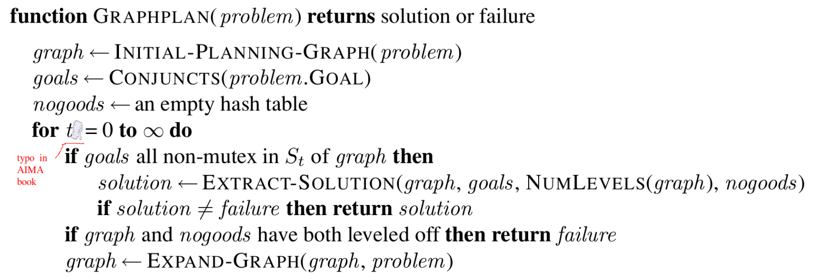

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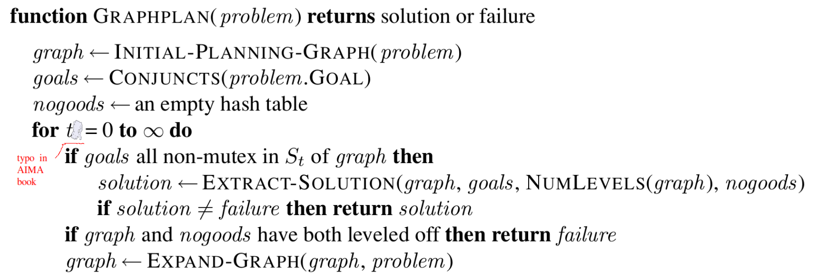

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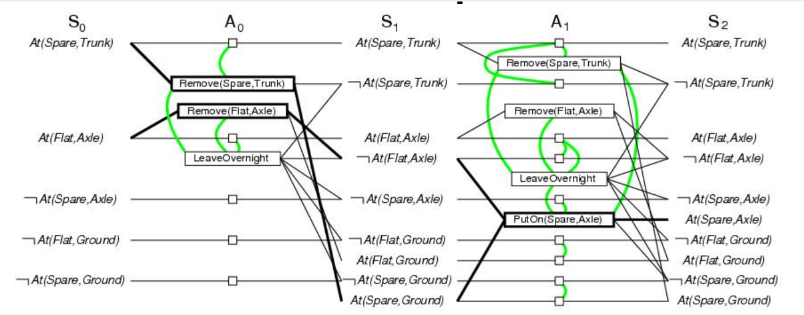
Graphplan: Example

Spare Tire problem

- Initial plan 5 literals from initial state and the CWA literals (S_0).
  - fixed literals (e.g. *Tire*(*Flat*)) ignored here
  - irrelevant literals ignored here
- Goal *At*(*Spare*, *Axle*) not present in S_0

  $\Longrightarrow$ no need to call EXTRACT-SOLUTION
- Graph and nogoods not leveled off $\Longrightarrow$ invoke EXPAND-GRAPH



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Graphplan: Example [cont.]

Spare Tire problem

- Invoke EXPAND-GRAPH
  - add actions A_0, persistence actions and mutexes
  - add fluents S_1 and mutexes
- Goal *At(Spare, Axle)* not present in S_1

  $\Longrightarrow$ no need to call EXTRACT-SOLUTION
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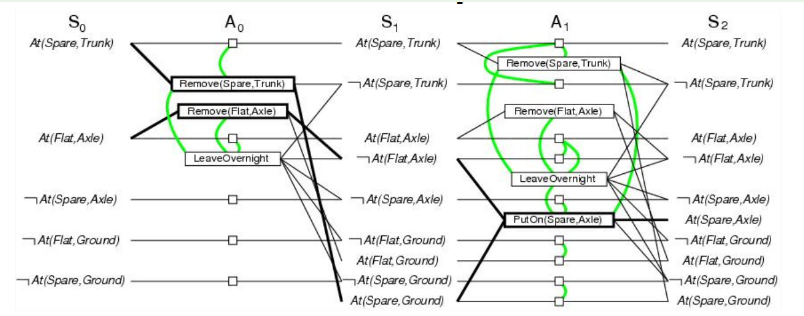

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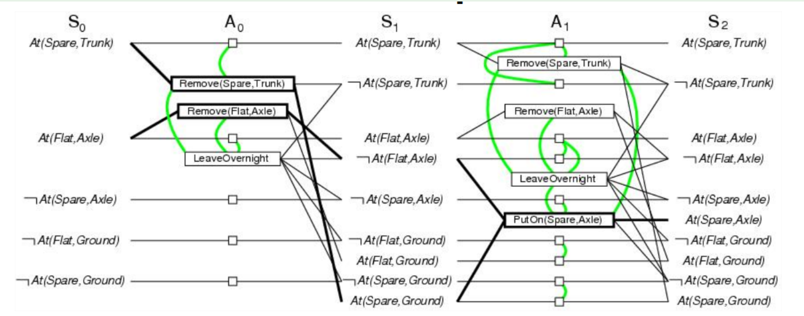
Graphplan: Example [cont.]

Spare Tire problem

- Invoke Expand-Graph
  - add actions A_1, persistence actions and mutexes
  - add fluents S_2 and mutexes
- Goal $At(Spare, Axle)$ present in S_2
  - call Extract-Solution
- Solution found!



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Exercise

- Consider the following variant of the Spare Tire problem: add $At(Flat, Trunk)$ to the goal
- Write the (non-serialized) planning graph
- Extract a plan from the graph
- Do the same with the serialized planning graph

The Graphplan Algorithm [cont.]

Graphplan "family" of algorithms, depending on approach used in EXTRACT-SOLUTION(...)

About EXTRACT-SOLUTION(...)

- Can be formulated as an (incremental) SAT problem
  - one proposition for each ground action and fluent
  - clauses represent preconditions, effects, no-ops and mutexes
- Can be formulated as a backward search problem
- Planning problem restricted to planning graph
  - mutexes found by EXPAND-GRAPH prune paths in the search tree

  $\Longrightarrow$ much faster than unrestricted planning
- (if P.G. not serialized) may produce partial order plans

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Partial-Order Plans

Partial-Order vs. Total-Order Plans

- Total-order plans: strictly linear sequences of actions
  - disregards the fact that some action are mutually independent
- Partial-order plans: set of precedence constraints between action pairs
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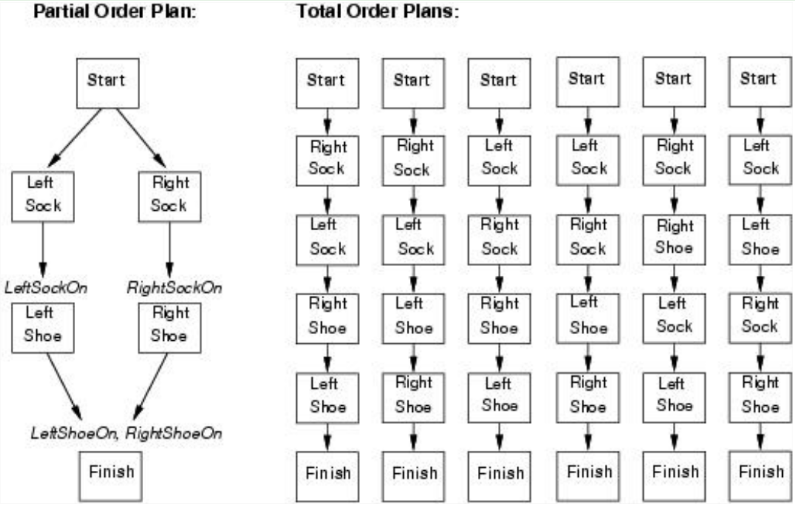
Partial-Order Plans: Example

Socks & Shoes Examples



(Courtesy of Michela Milano, UniBO)

Termination of Graphplan

- Theorem: If the graph and the no-goods have both leveled off, and no solution is found we can safely terminate with failure
- Intuition (proof sketch):
  - Literals and actions increase monotonically and are finite $\Longrightarrow$ we eventually reach a level where they stabilize
  - Mutex and no-goods decrease monotonically (and cannot become less than zero) $\Longrightarrow$ so they too eventually must level off

  $\Longrightarrow$ When we reach this stable state, if one of the goals is missing or is mutex with another goal, then it will remain so

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Exercise

- Socks & Shoes example:
  1. Formalize the Socks & Shoes example in PDDL
  2. Write the non-serialized planning graph
  3. Compute the level cost for every fluent
  4. Choose some states, compute h(s) using the three heuristics
  5. Extract a plan from the graph in (2)
  6. Compare h(s) with the level they occur in the plan
  7. Write the serialized planning graph
  8. Repeat steps (3)-(6) with the serialized graph
- Do same steps (1)-(8) for the Air Cargo Transport example

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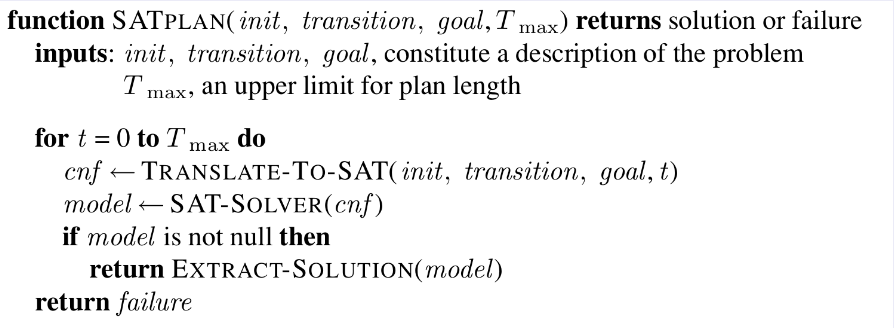
Planning as SAT Solving

- Encode bounded planning problem into a propositional formula

⟹ Solve it by (incremental) calls to a SAT solver

- A model for the formula (if any) is a plan of length t
- Many variants in the encoding
- Extremely efficient with many problems of interest



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    $cnf \leftarrow$ TRANSLATE-TO-SAT($init$, $transition$, $goal$, $t$)
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    if $model$ is not null then
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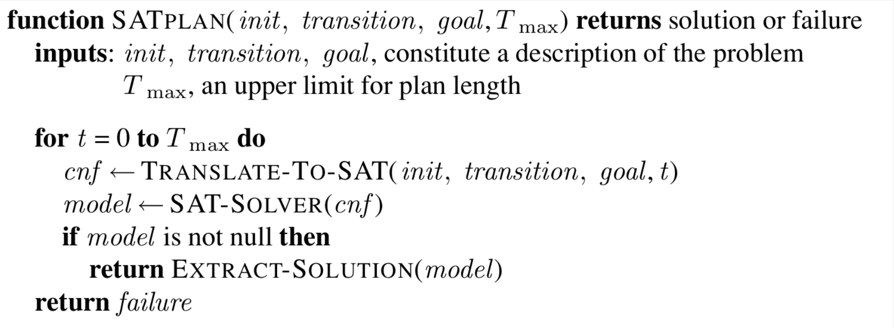

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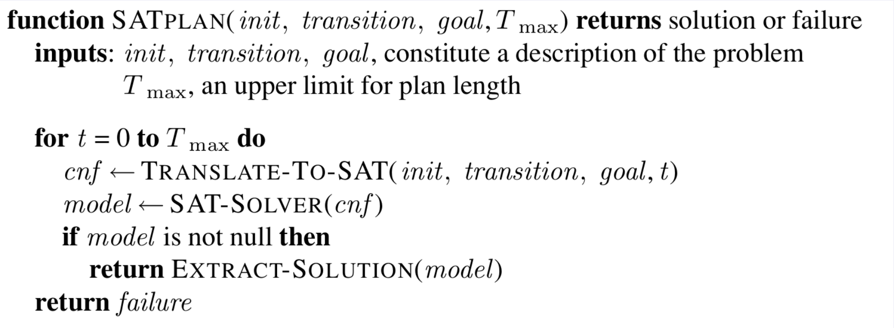

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function SATPLAN($init$, $transition$, $goal$, $T_{\max}$) returns solution or failure
  inputs: $init$, $transition$, $goal$, constitute a description of the problem
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  for $t = 0$ to $T_{\max}$ do
     $cnf$ ← TRANSLATE-TO-SAT($init$, $transition$, $goal$, $t$)
     $model$ ← SAT-SOLVER($cnf$)
     if $model$ is not null then
        return EXTRACT-SOLUTION($model$)
  return $failure$
```

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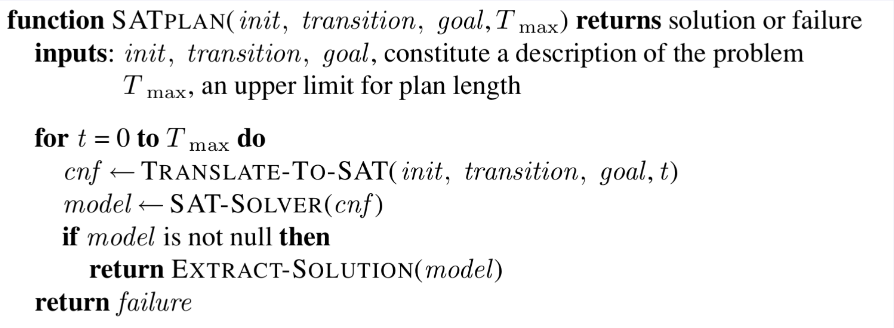
Planning as SAT Solving

- Encode bounded planning problem into a propositional formula

⟹ Solve it by (incremental) calls to a SAT solver

- A model for the formula (if any) is a plan of length t
- Many variants in the encoding
- Extremely efficient with many problems of interest



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Planning as SAT Solving [cont.]

- TRANSLATE-TO-SAT(INIT,TRANSITION, GOAL, T):
  - ground fluents & actions at each step are propositionalized
    - ex: $\langle At(P_1, SFO), 3\rangle \Longrightarrow At\_P_1\_SFO\_3$
    - ex: $\langle Fly(P_1, SFO, JFK), 3\rangle \Longrightarrow Fly\_P_1\_SFO\_JFK\_3$
  - returns propositional formula: $Init^0 \land (\bigwedge_{i=1}^{t-1} Transition^{i,i+1}) \land Goal^t$
- $Init^0$ and $Goal^t$: conjunctions of literals at step 0 and t resp.
  - ex: $Init^0$: $At\_P_1\_SFO\_0 \land At\_P_2\_JFK\_0$
  - ex: $Goal^3$: $At\_P_1\_JFK\_3 \land At\_P_2\_SFO\_3$
- $Transition^{i,i+1}$: encodes transition from steps i to $i+1$
  - Actions: $Action^i \to (Precond^i \land Effects^{i+1})$
    ex: $Fly\_P_1\_SFO\_JFK\_2 \to (At\_P_1\_SFO\_2 \land At\_P_1\_JFK\_3$
  - No-Ops: for each fluent F and step i:

$$F^{i+1} \leftrightarrow \bigvee_k ActionCausingF_k^i \lor (F^i \land \bigwedge_j \neg ActionCausingNotF_j^i)$$

  - Mutex constraints: $\neg Action_1^i \lor \neg Action_2^i$
    ex: $\neg Fly\_P_1\_SFO\_JFK\_2 \lor \neg Fly\_P_1\_SFO\_Newark\_2$
  - If serialized: add mutex between each pair of actions at each step

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Consider the socks & shoes example

- Trenslate it into SAT for t=0,1,2
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Outline

1. The Problem
2. Search Strategies and Heuristics
   - Forward and Backward Search
   - Heuristics
3. Planning Graphs, Heuristics and Graphplan
   - Planning Graphs
   - Heuristics Driven by Planning Graphs
   - The Graphplan Algorithm
4. Other Approaches (hints)
   - Planning as SAT Solving
   - Planning as FOL Inference

Planning via FOL Inference: Situation Calculus

Situation Calculus in a nutshell

- Idea: formalize planning into FOL

$\Longrightarrow$ use resolution-based inference for planning

\+ Admit quantifications $\Longrightarrow$ very expressive

  - allows formalizing sentences like "move all the cargos from A to B regardless of how many pieces of cargo there are"

\- Frame problem (no-ops) complicate to handle

\- Not very efficient! (cannot compete against s.o.a. planners)
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Planning via FOL Inference: Situation Calculus [cont.]

Basic concepts

- Situation:
  - the initial state is a situation
  - if s is a situation and a is an action, then $Result(s, a)$ is a situation
  - $Result()$ injective: $Result(s, a) = Result(s', a') \leftrightarrow (s = s' \land a = a')$
  - a solution is a situation that satisfies the goal
- Action preconditions: $\Phi(s) \rightarrow Poss(a, s)$
  - $\Phi(s)$ describes preconditions
  - ex: $(Alive(Agent, s) \land Have(Agent, Arrow, s)) \rightarrow Poss(Shoot, s)$
- Successor-state axioms (similar to propositional case):

$$\text{[Action is possible]} \rightarrow \left[\begin{array}{l}\text{[Fluent is true in result state]} \leftrightarrow \\ \text{([Action's effect made it true]}\lor \\ \ \text{([It was true before]} \land \text{[action left it alone]))}\end{array}\right]$$

  - ex: $Poss(a, s) \rightarrow$
    $$\left[\begin{array}{l} Holding(Agent, g, Result(a, s)) \leftrightarrow \\ a = Grab(g) \lor (Holding(Agent, g, s) \land a \neq Release(g)) \end{array}\right]$$
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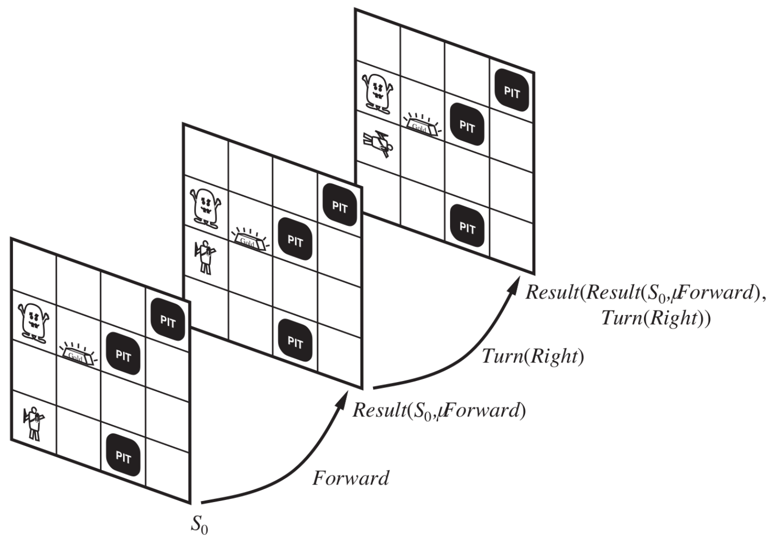
Situation Calculus: Example

Situations as the results of actions in the Wumpus world



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