

Fundamentals of Artificial Intelligence

Chapter 06: Constraint Satisfaction Problems

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Outline

- 1 Defining Constraint Satisfaction Problems (CSPs)
- 2 Inference in CSPs: Constraint Propagation
- 3 Backtracking Search with CSPs
- 4 Local Search with CSPs
- 5 Exploiting Structure of CSPs

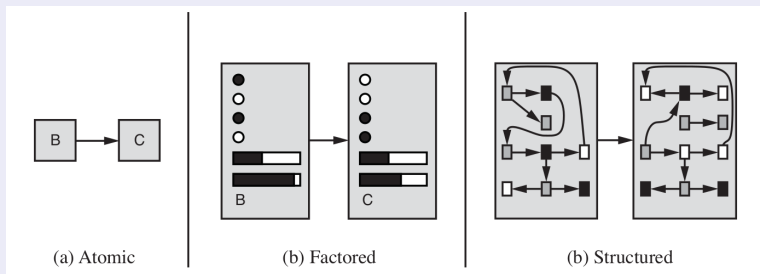
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Recall: State Representations [Ch. 02]

Representations of states and transitions

- Three ways to represent states and transitions between them:
 - **atomic**: a state is a black box with no internal structure
 - **factored**: a state consists of a vector of attribute values
 - **structured**: a state includes objects, each of which may have attributes of its own as well as relationships to other objects
- increasing expressive power and computational complexity
- reality represented at different levels of abstraction



Constraint Satisfaction Problems (CSPs): Generalities

Constraint Satisfaction Problems (CSPs)

(aka **Constraint Satisfiability Problems**)

- Search problem so far: **Atomic representation of states**
 - black box with no internal structure
 - goal test as set inclusion
- Henceforth: use a **Factored representation of states**
 - **state** is defined by a **set of variables values** from some domains
 - **goal test** is a **set of constraints** specifying allowable combinations of values for subsets of variables
 - a set of variable values is a goal iff the values verify all constraints
- **CSP Search Algorithms**
 - take advantage of the **structure of states**
 - use **general-purpose heuristics** rather than **problem-specific** ones
 - main idea: **eliminate large portions of the search space all at once**
 - identify variable/value combinations that violate the constraints

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CSPs: Definitions

CSPs

- A Constraint Satisfaction Problem is a tuple $\langle X, D, C \rangle$:
 - a set of variables $X \stackrel{\text{def}}{=} \{X_1, \dots, X_n\}$
 - a set of (non-empty) domains $D \stackrel{\text{def}}{=} \{D_1, \dots, D_n\}$, one for each X_i
 - a set of constraints $C \stackrel{\text{def}}{=} \{C_1, \dots, C_m\}$
 - specify allowable combinations of values for the variables in X
- Each D_i is a set of allowable values $\{v_i, \dots, v_k\}$ for variable X_i
- Each C_i is a pair $\langle \text{scope}, \text{rel} \rangle$
 - **scope** is a tuple of variables that participate in the constraint
 - **rel** is a relation defining the values that such variables can take
- A relation is
 - an explicit list of all tuples of values that satisfy the constraint (most often inconvenient), or
 - an abstract relation supporting two operations:
 - test if a tuple is a member of the relation
 - enumerate the members of the relation
- We need a language to express constraint relations!

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CSPs: Definitions [cont.]

States, Assignments and Solutions

- A **state** in a CSP is an **assignment of values to some or all of the variables** $\{X_i = v_{x_i}\}_i$ s.t $X_i \in X$ and $v_{x_i} \in D_i$
- An assignment is
 - **complete** or **total**, if every variable is assigned a value
 - **incomplete** or **partial**, if some variable is assigned a value
- An assignment that does not violate any constraints in the CSP is called a **consistent** or **legal assignment**
- A **solution** to a CSP is a **consistent and complete assignment**
- A CSP consists in finding one solution (or state there is none)
- **Constraint Optimization Problems (COPs)**: CSPs requiring solutions that maximize/minimize an objective function

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Example: Sudoku

- 81 Variables: (each square) X_{ij} ,
 $i = A, \dots, I$; $j = 1 \dots 9$
- Domain: $\{1, 2, \dots, 8, 9\}$
- Constraints:
 - $AllDiff(X_{i1}, \dots, X_{i9})$ for each row i
 - $AllDiff(X_{A_j}, \dots, X_{I_j})$ for each column j
 - $AllDiff(X_{A1}, \dots, X_{A3}, X_{B1}, \dots, X_{C3})$ for each 3×3 square region(alternatively, a long list of pairwise inequality constraints:
 $X_{A1} \neq X_{A2}, X_{A1} \neq X_{A3}, \dots$)
- Solution: total value assignment satisfying all the constraints:
 $X_{A1} = 4, X_{A2} = 8, X_{A3} = 3, \dots$

	1	2	3	4	5	6	7	8	9
A			3		2		6		
B	9			3		5			1
C			1	8		6	4		
D			8	1		2	9		
E	7								8
F			6	7		8	2		
G			2	6		9	5		
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Example: Map-Coloring

- Variables WA, NT, Q, NSW, V, SA, T
- Domain $D_i = \{\text{red}, \text{green}, \text{blue}\}, \forall i$
- Constraints: adjacent regions must have different colours
 - e.g. (explicit enumeration): $\langle \text{WA}, \text{NT} \rangle \in \{\langle \text{red}, \text{green} \rangle, \langle \text{red}, \text{blue} \rangle, \}$
or (implicit, if language allows it): $\text{WA} \neq \text{NT}$
- Solution:
{WA=red, NT=green, Q=red, NSW=green, V=red, SA=blue, T=green}



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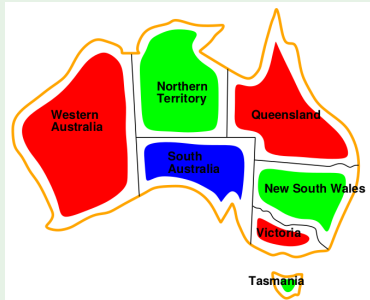
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Constraint Graphs

- Useful to visualize a CSP as a **constraint graph** (aka **network**)
 - the **nodes of the graph** correspond to variables of the problem
 - an **edge** connects any two variables that participate in a constraint
- CSP algorithms use the graph structure to speed up search
 - Ex: Tasmania is an independent subproblem!

Constraint Graphs

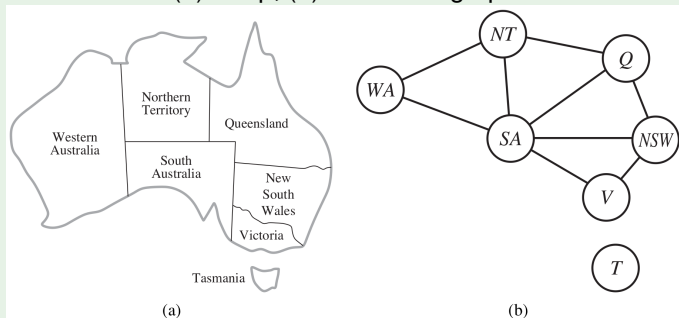
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Example: Map Coloring

(a): map; (b) constraint graph



Varieties of CSPs

- Discrete variables
 - Finite domains (ex: Booleans, bounded integers, lists of values)
 - domain size $d \implies d^n$ complete assignments (candidate solutions)
 - e.g. Boolean CSPs, incl. Boolean satisfiability (NP-complete)
 - possible to define constraints by enumerating all combinations (although unpractical)
 - Infinite domains (ex: unbounded integers)
 - infinite domain size \implies infinite # of complete assignments
 - e.g. job scheduling: variables are start/end days for each job
 - need a constraint language (ex: $StartJob_1 + 5 \leq StartJob_3$)
 - linear constraints \implies solvable (but NP-Hard)
 - non-linear constraints \implies undecidable (ex: $x^n + y^n = z^n, n > 2$)
- Continuous variables (ex: reals, rationals)
 - linear constraints solvable in poly time by LP methods
 - non-linear constraints solvable (e.g. by Cylindrical Algebraic Decomposition) but dramatically hard

The same problem may have distinct formulations as CSP!

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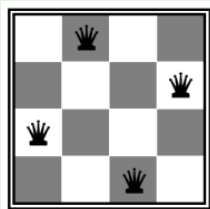
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Example: N-Queens

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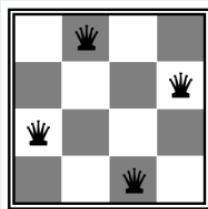
- variables X_{ij} , $i, j = 1..N$
- domains: $\{0, 1\}$
- constraints (explicit):
 - $\forall i, j, k \langle X_{ij}, X_{ik} \rangle \in \{\langle 0, 0 \rangle, \langle 1, 0 \rangle, \langle 0, 1 \rangle\}$ (row)
 - $\forall i, j, k \langle X_{ij}, X_{kj} \rangle \in \{\langle 0, 0 \rangle, \langle 1, 0 \rangle, \langle 0, 1 \rangle\}$ (column)
 - $\forall i, j, k \langle X_{ij}, X_{i+k, j+k} \rangle \in \{\langle 0, 0 \rangle, \langle 1, 0 \rangle, \langle 0, 1 \rangle\}$
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- explicit representation
- very inefficient



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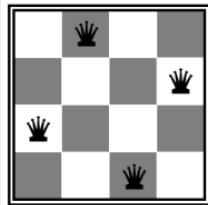
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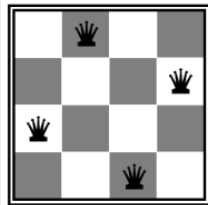
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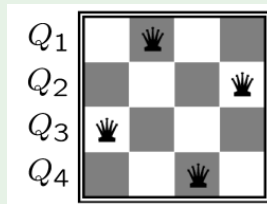
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Formulation 2

- variables Q_k , $k = 1..N$ (row)
- domains: $\{1..N\}$ (column position)
- constraints (implicit): *Nonthreatening*($Q_k, Q_{k'}$):
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 - $Q_i \neq Q_j$ (column)
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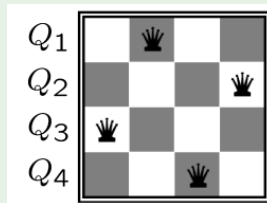


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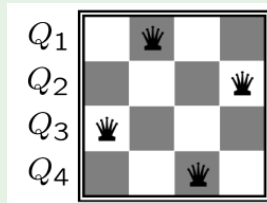


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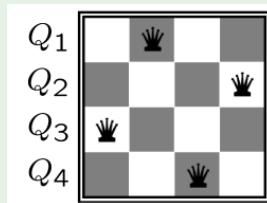


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Varieties of Constraints

- **Unary constraints**: involve one single variable
 - ex: ($SA \neq \text{green}$)
 - **Binary constraints**: involve pairs of variables
 - ex: ($SA \neq WA$)
 - **Higher-order constraints**: involve ≥ 3 variables
 - ex: cryptarithmic column constraints
 - can be represented by **constraint hypergraphs** (hypernodes represent n-ary constraints, squares in cryptarithmic example)
 - **Global constraints**: involve an **arbitrary number of variables**
 - ex: $AllDiff(X_1, \dots, X_k)$
 - note: maximum domain size $\geq k$, otherwise $AllDiff()$ unsatisfiable
 - compact, specialized routines for handling them
 - **Preference constraints** (aka **soft constraints**): describe preferences between/among solutions
 - ex: "I'd rather WA in red than in blue or green"
 - can often be encoded as **costs/rewards** for variables/constraints:
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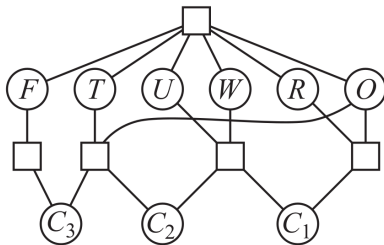
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Example: Cryptarithmic

- Variables: F, T, U, W, R, O , plus C_1, C_2, C_3 (carry)
- Domains: $F, T, U, W, R, O \in \{0, 1, \dots, 9\}$; $C_1, C_2, C_3 \in \{0, 1\}$
- Constraints:
$$\left\{ \begin{array}{l} \text{AllDiff}(F, T, U, W, R, O), \\ O + O = R + 10 \cdot C_1 \\ W + W + C_1 = U + 10 \cdot C_2 \\ T + T + C_2 = 10 \cdot C_3 + O \\ F = C_3, F \neq 0, T \neq 0 \end{array} \right\}$$
- (one) solution: $\{F=1, T=7, U=2, W=1, R=8, O=4\}$ ($714+714=1428$)

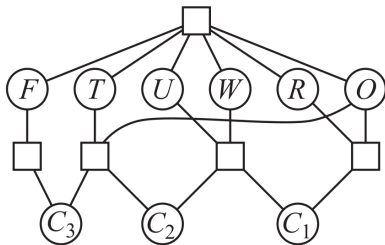
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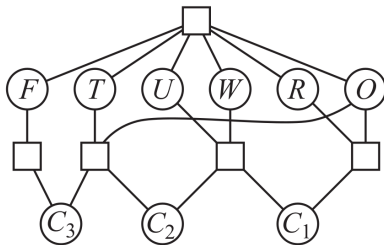
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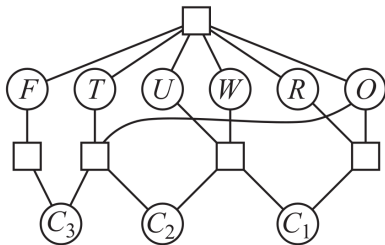
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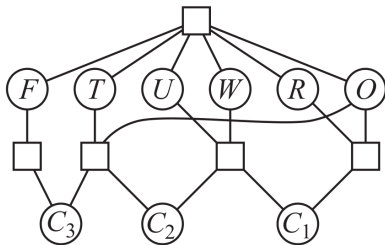
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Example: Job-Shop Scheduling

- Scheduling the assembling of a car requires several tasks
 - ex: installing axles, installing wheels, tightening nuts, put on hubcap, inspect
- Variables X_t (for each task t): **starting times of the tasks**
- Domain: **(bounded) integers** (time units)
- Constraints:
 - Precedence: $(X_T + \text{duration}_T \leq X_{T'})$ (task T precedes task T')
 - duration_T constant value (ex: $(X_{\text{axleA}} + 10 \leq X_{\text{axleB}})$)
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Real-World CSPs

- Task-Assignment problems
 - Ex: who teaches which class?
- Timetabling problems
 - Ex: which class is offered when and where?
- Hardware configuration
 - Ex: which component is placed where? with which connections?
- Transportation scheduling
 - Ex: which van goes where?
- Factory scheduling
 - Ex: which machine/worker takes which task? in which order?
- ...

Remarks

- many real-world problems involve **real/rational-valued variables**
- many real-world problems involve **combinatorics and logic**
- many real-world problems require **optimization**

Real-World CSPs

- Task-Assignment problems
 - Ex: who teaches which class?
- Timetabling problems
 - Ex: which class is offered when and where?
- Hardware configuration
 - Ex: which component is placed where? with which connections?
- Transportation scheduling
 - Ex: which van goes where?
- Factory scheduling
 - Ex: which machine/worker takes which task? in which order?
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Outline

- 1 Defining Constraint Satisfaction Problems (CSPs)
- 2 Inference in CSPs: Constraint Propagation**
- 3 Backtracking Search with CSPs
- 4 Local Search with CSPs
- 5 Exploiting Structure of CSPs

k-ary constraints can be transformed into sets of binary constraints

⇒ often CSP solvers work with binary constraints only

- In this chapter (unless specified otherwise) we assume we have only binary constraints in the CSP
- we call **neighbours** two variables sharing a binary constraint

Constraint Propagation

- In state-space search, an algorithm can only search
- With CSPs, an algorithm can
 - search: pick a new variable assignment
 - infer (apply constraint propagation):
use the constraints to reduce the set of legal values for a variable
- Constraint propagation can either:
 - be interleaved with search
 - be performed as a preprocessing step
- Intuition: preserve and propagate local consistency
 - enforcing local consistency in each part of the constraint graph
⇒ inconsistent values eliminated throughout the graph
- Different types of local consistency:
 - node consistency (aka 1-consistency)
 - arc consistency (aka 2-consistency)
 - path consistency (aka 3-consistency)
 - k-consistency and strong k-consistency, $k \geq 1$

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Node Consistency (aka 1-Consistency)

- X_i is node-consistent if all the values in the variable's domain satisfy the variable's unary constraints
- A CSP is node-consistent if every variable is node-consistent
- Node-consistency propagation: remove all values from the domain D_i of X_i which violate unary constraints on X_i
 - ex: if the constraint $WA \neq green$ is added to map-coloring problem then WA domain $\{red, green, blue\}$ is reduced to $\{red, blue\}$
- Unary constraints can be removed a priori by node consistency propagation

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- A CSP is arc-consistent if every variable is arc consistent with every other variable
- Forward Checking: remove values from unassigned variables which are not arc consistent with assigned variable
 - ensure arcs from assigned to unassigned variables are consistent
- Arc-consistency propagation: remove all values from the domains of every variable which are not arc-consistent with these of some other variables
 - ensure all arcs are consistent!
- A well-known algorithm: AC-3
 - ⇒ every arc is arc-consistent, or some variable domain is empty
 - complexity: $O(|C| \cdot |D|^3)$ worst-case
 - AC-4 is $O(|C| \cdot |D|^2)$ worst-case, but worse than AC-3 on average
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Forward Checking

- Simplest form of propagation
- Idea: **propagate information from assigned to unassigned vars**
 - pick variable assignment
 - update remaining legal values for unassigned variables
- Does not provide early detection for all failures
- If X loses a value, neighbors of X need to be rechecked!
 - ex: SA single value is incompatible with NT single value
- Can we conclude anything?
 - NT and SA cannot both be blue!
- Why didn't we detect this inconsistency yet?



WA	NT	Q	NSW	V	SA
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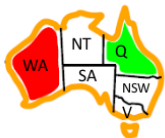


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The Arc-Consistency Propagation Algorithm AC-3

function AC-3(*csp*) **returns** false if an inconsistency is found and true otherwise

inputs: *csp*, a binary CSP with components (X , D , C)

local variables: *queue*, a queue of arcs, initially all the arcs in *csp*

while *queue* is not empty **do**

 (X_i , X_j) \leftarrow REMOVE-FIRST(*queue*)

if REVISE(*csp*, X_i , X_j) **then**

if size of $D_i = 0$ **then return** false

for each X_k **in** X_i .NEIGHBORS - $\{X_j\}$ **do**

 add (X_k , X_i) to *queue*

return true

function REVISE(*csp*, X_i , X_j) **returns** true iff we revise the domain of X_i

revised \leftarrow false

for each x **in** D_i **do**

if no value y in D_j allows (x,y) to satisfy the constraint between X_i and X_j **then**

 delete x from D_i

revised \leftarrow true

return *revised*

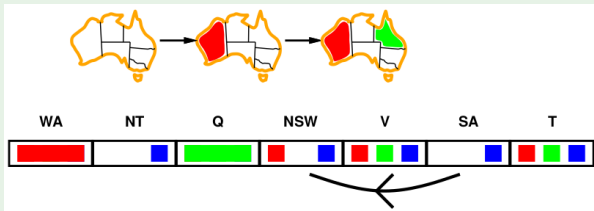
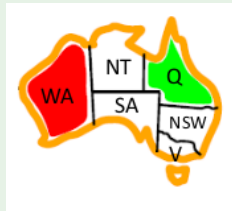
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note: “queue” is LIFO \implies revises first the neighbours of revised vars

Arc Consistency Propagation AC-3: Example

- If X loses a value, neighbors of X need to be rechecked
 - ex: SA single value
- Empty domain!

⇒ Arc consistency detects failure earlier than forward checking

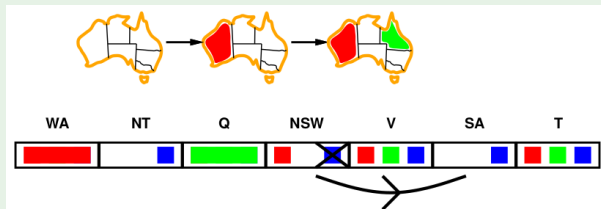
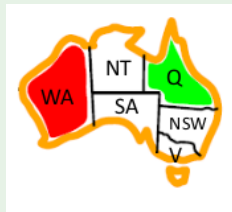


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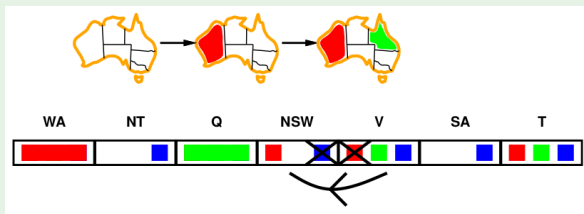


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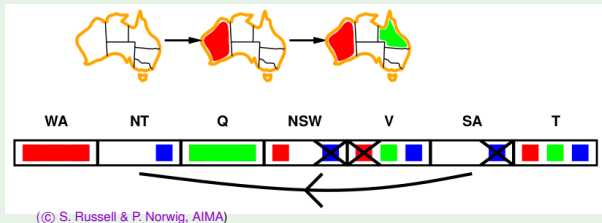


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(consider *AllDiff()* as a set of binary constraints)

Apply arc consistency:

- What about E6?

- arc-consistency on 6:
drop 2,3,5,6,8,9
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drop 1,7 \implies E6=4

- What about I6?

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- ...

- AC-3 solves the whole puzzle

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K-Consistency

- A CSP is k -consistent iff for any set of $k - 1$ variables and for any consistent assignment to those variables, a consistent value can always be assigned to any other k -th variable
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Arc vs. Path Consistency

- Can we say anything about X1?

We can drop red & blue from D1

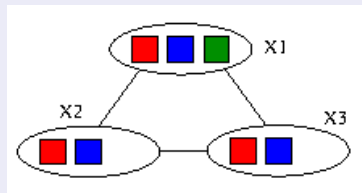
⇒ Infers the assignment $C1 = \text{green}$

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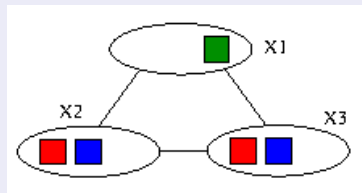
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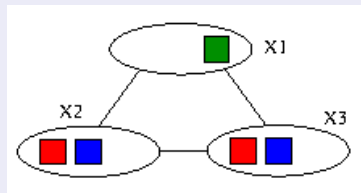
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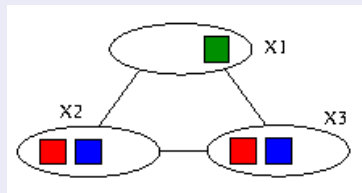
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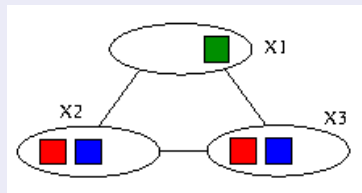
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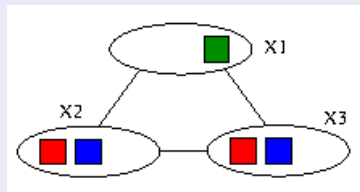
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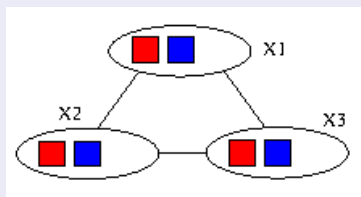


Arc vs. Path Consistency [cont.]

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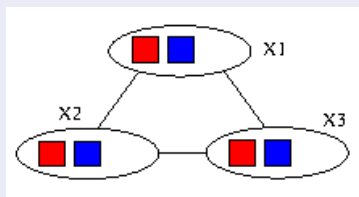


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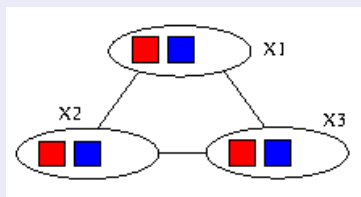
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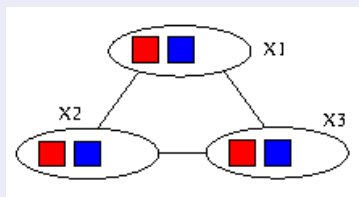
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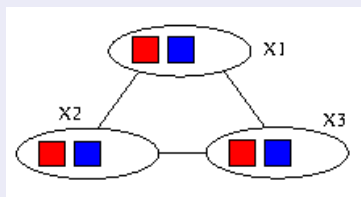
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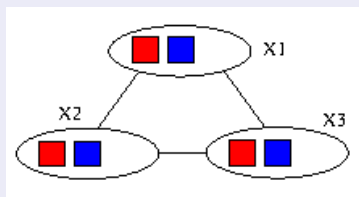
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Outline

- 1 Defining Constraint Satisfaction Problems (CSPs)
- 2 Inference in CSPs: Constraint Propagation
- 3 Backtracking Search with CSPs**
- 4 Local Search with CSPs
- 5 Exploiting Structure of CSPs

Backtracking Search: Generalities

Backtracking Search

- Basic uninformed algorithm for solving CSPs
- Idea 1: **Pick one variable at a time**
 - variable assignments are commutative \implies fix an ordering
 - ex: $\{WA = \text{red}, NT = \text{green}\}$ same as $\{NT = \text{green}, WA = \text{red}\}$ \implies **can consider assignments to a single variable at each step**
 - reasons on **partial assignments**
- Idea 2: **Check constraints as long as you proceed**
 - pick only **values which do not conflict with previous assignments**
 - requires some computation to check the constraints \implies “incremental goal test”
 - can detect if a partial assignments violate a goal
 - \implies early detection of inconsistencies
- **Backtracking search**: DFS with the two above improvements

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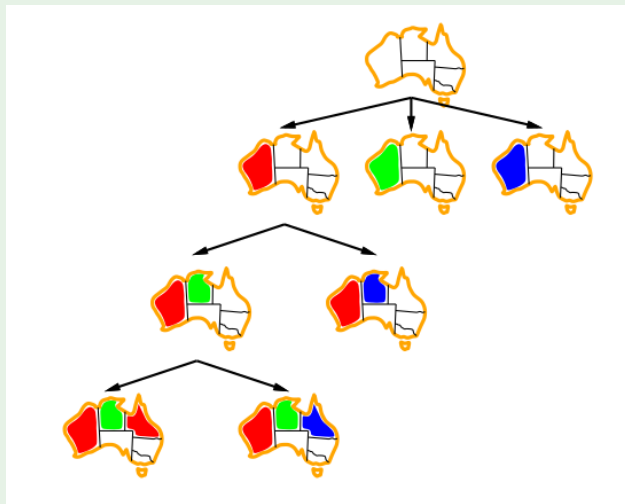
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Backtracking Search: Example

(Part of) Search Tree for Map-Coloring



(© S. Russell & P. Norwig, AIMA)

Backtracking Search Algorithm

function BACKTRACKING-SEARCH(*csp*) **returns** a solution, or failure
 return BACKTRACK({ }, *csp*)

function BACKTRACK(*assignment*, *csp*) **returns** a solution, or failure

if *assignment* is complete **then return** *assignment*

var \leftarrow SELECT-UNASSIGNED-VARIABLE(*csp*)

for each *value* **in** ORDER-DOMAIN-VALUES(*var*, *assignment*, *csp*) **do**

if *value* is consistent with *assignment* **then**

 add {*var* = *value*} to *assignment*

inferences \leftarrow INFERENCE(*csp*, *var*, *value*)

if *inferences* \neq failure **then**


 add *inferences* to *assignment*

result \leftarrow BACKTRACK(*assignment*, *csp*)

if *result* \neq failure **then**

return *result*

inside first "if"

 **remove** {*var* = *value*} and *inferences* from *assignment*

return failure

Backtracking Search Algorithm [cont.]

- General-purpose algorithm for generic CSPs
- The representation of CSPs is standardized
 - ⇒ no need to provide a domain-specific initial state, action function, transition model, or goal test
- *BacktrackingSearch()* keeps a single representation of a state
 - alters such representation rather than creating new ones
- We can add some sophistication to the unspecified functions:
 - *SelectUnassignedVariable()*: which variable should be assigned next?
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Variable Selection Heuristics

Minimum Remaining Values (MRV) heuristic

- Aka **most constrained variable** or **fail-first** heuristic
- MRV: **Choose the variable with the fewest legal values**
 - ⇒ pick a variable that is most likely to cause a failure soon
- If X has no legal values left, MRV heuristic selects X
 - ⇒ **failure detected immediately**
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- Used as tie-breaker in combination with MRV
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Least Constraining Value (LCS) heuristic

- pick the value that rules out the fewest choices for the neighboring variables
⇒ tries maximum flexibility for subsequent variable assignments
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- Ex: MRV+DH+LCS allow for solving 1000-queens

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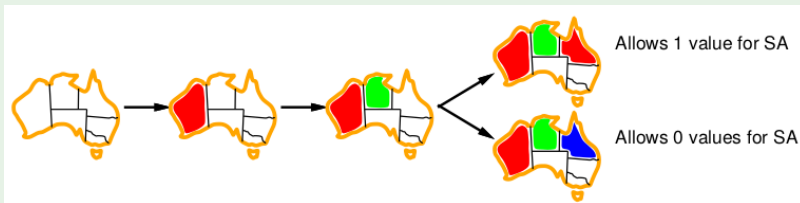
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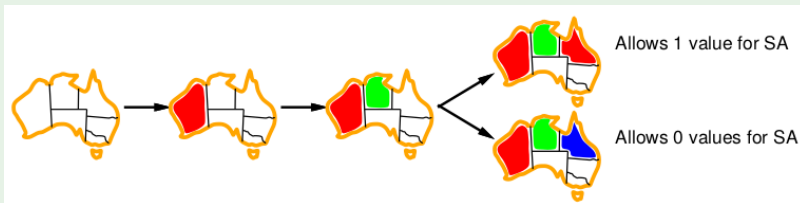


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Inference

Interleaving search and inference

- After a choice, **infer new domain reductions on other variables**
 - detect inconsistencies earlier
 - reduce search spaces
 - may produce unary domains (deterministic steps)
⇒ returned as assignments (“inferences”)
- Tradeoff between effectiveness and efficiency
- **Forward checking**
 - cheap
 - ensures arc consistency of $\langle \text{assigned}, \text{unassigned} \rangle$ variable pairs
- **AC-3**
 - more expensive
 - ensure arc consistency of all variable pairs
 - strategy (MAC):
 - after X_i is assigned, start AC-3 with only the arcs $\langle X_j, X_i \rangle$ s.t. X_j unassigned neighbour variables of X_i
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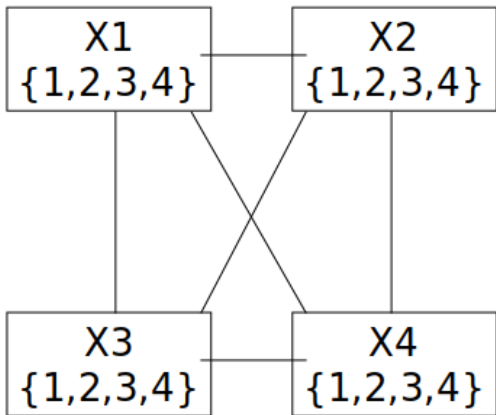
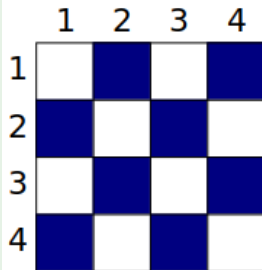
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Backtracking w. Forward Checking: Example

4-Queens

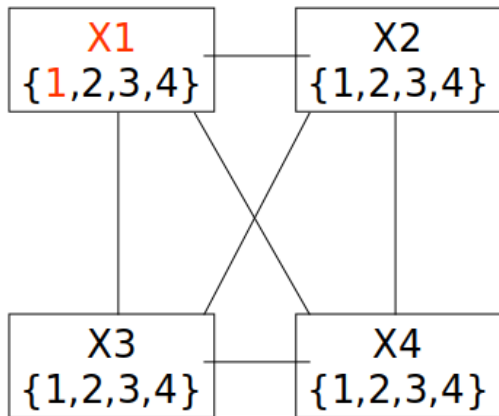
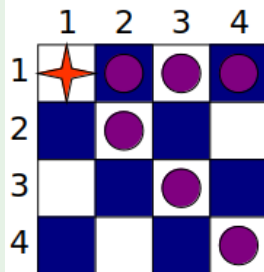


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...(after trying $X_2 = 4$, failing and backtracking)...

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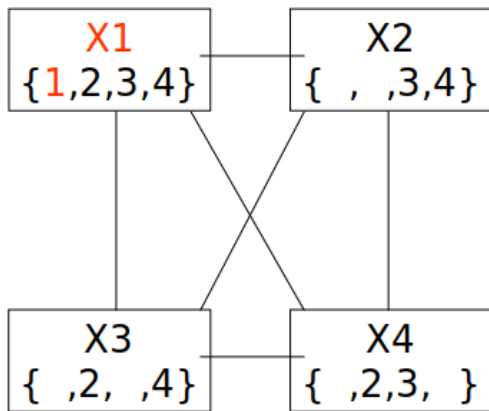
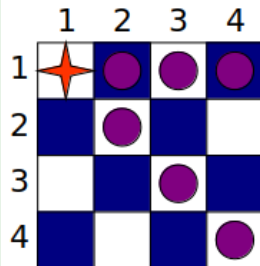


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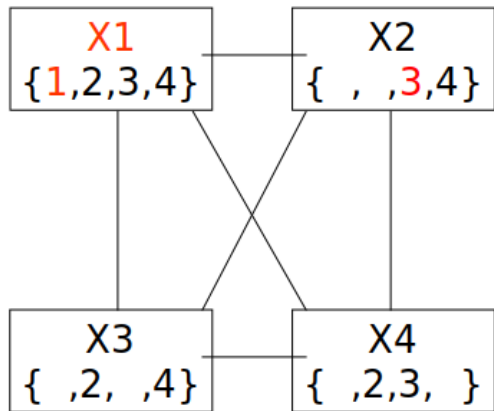
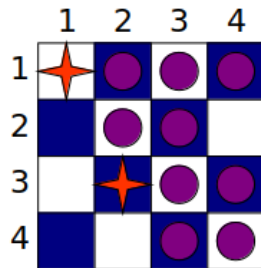


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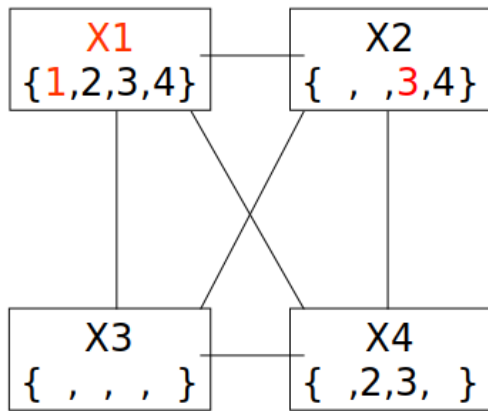
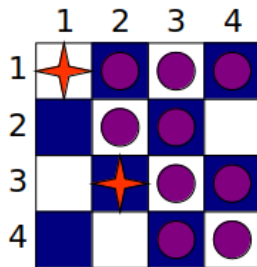


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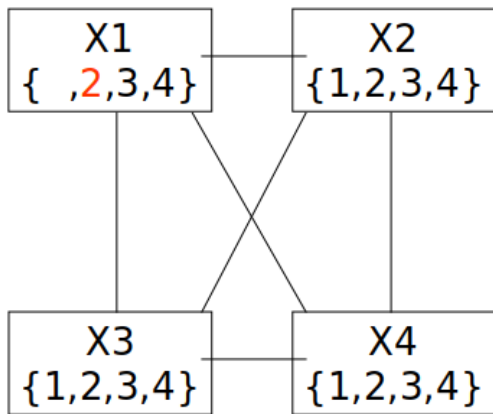
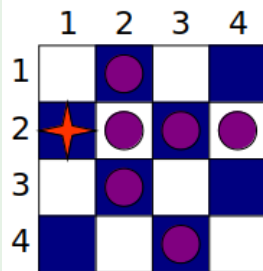


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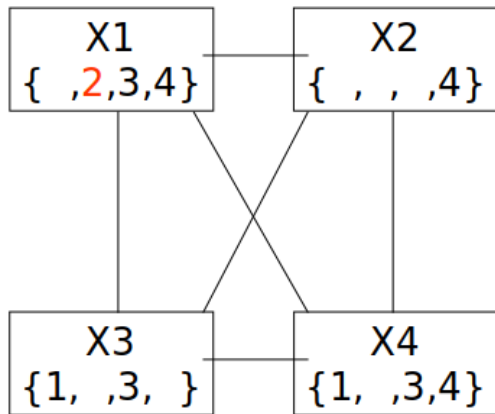
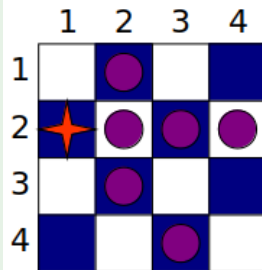


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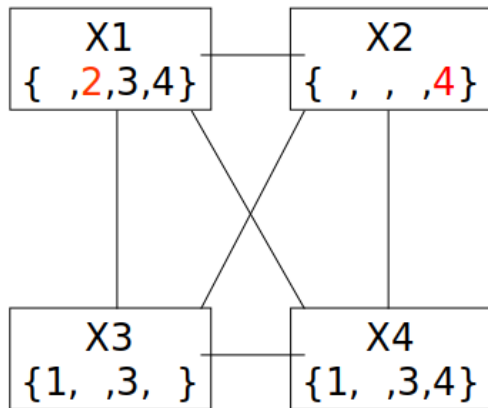
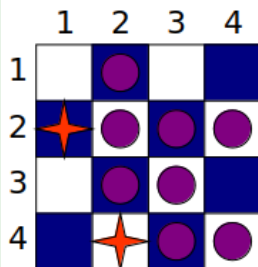


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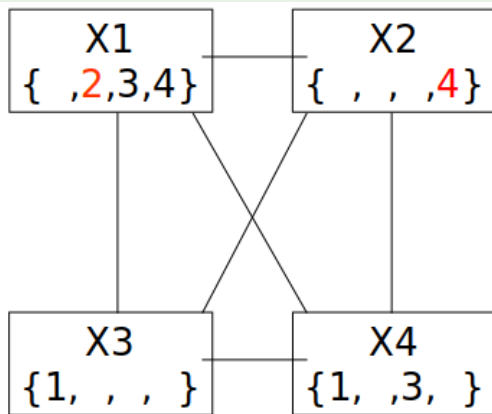
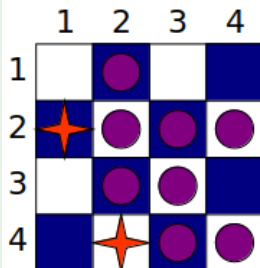


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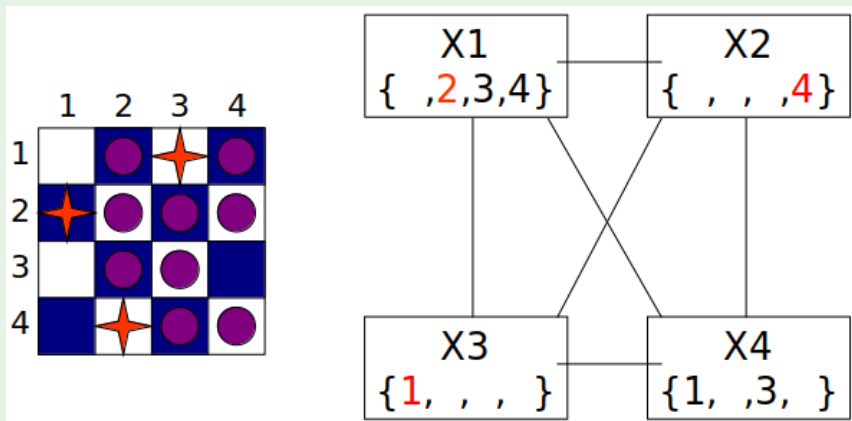


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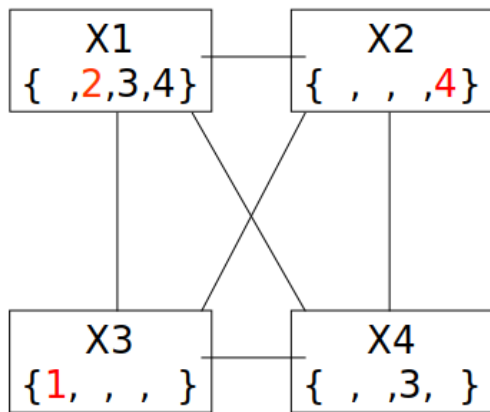
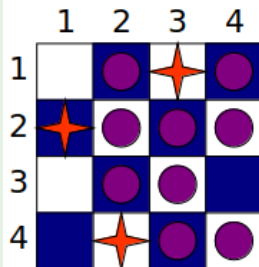


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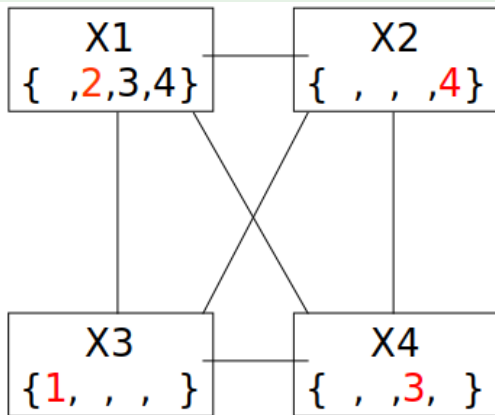
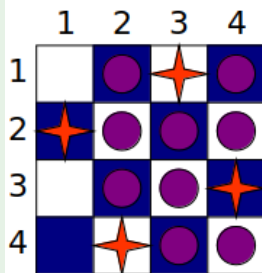


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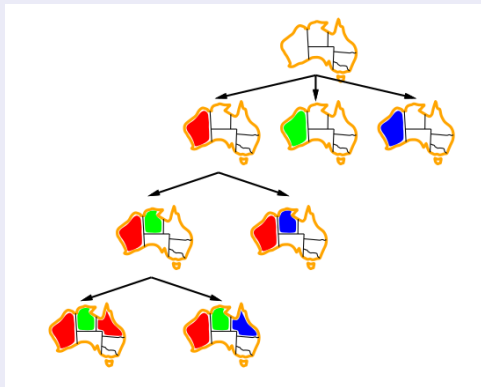
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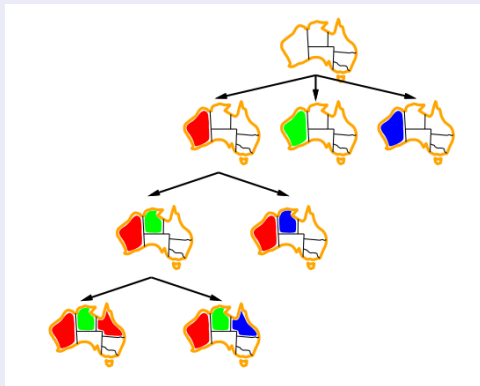
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Standard Chronological Backtracking



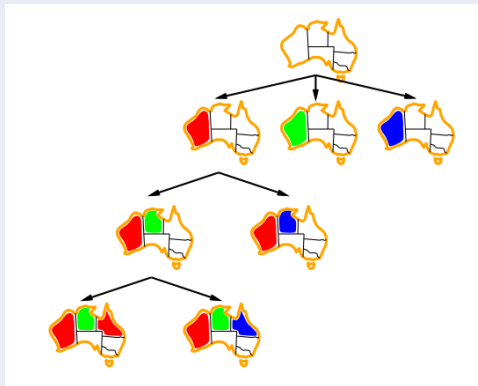
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- When a branch fails (empty domain for variable X_i):
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- Problem: lots of search wasted!



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Standard Chronological Backtracking: Example

Assume variable selection order: WA, NSW, T, NT, Q, V, SA

- failed branch:

	<i>step</i>	<i>assignment</i> [<i>domain</i>]
(1)	<i>pick</i>	WA = <i>r</i> [<i>rbg</i>]
(2)	<i>pick</i>	NSW = <i>r</i> [<i>rbg</i>]
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- backtrack to (5), pick $V = g \Rightarrow (7)$ again
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\Rightarrow backtrack to (1), then assign NSW another value

\Rightarrow lots of useless search on T and V values

- source of inconsistency not identified: $\{WA = r, NSW = r\}$

Standard Chronological Backtracking: Example

Assume variable selection order: WA, NSW, T, NT, Q, V, SA

- failed branch:

	<i>step</i>	<i>assignment</i> [<i>domain</i>]
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(7)	\xRightarrow{fc}	$SA = \{\}$ []



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Nogoods & Conflict Sets

- **Nogood**: subassignment which cannot be part of any solution
 - ex: $\{WA = r, NSW = r\}$ (see previous example)
- **Conflict set for X_j** (aka **explanations**): (minimal) set of value assignments in direct conflict with some values of X_j
 - cause reduction of D_j via forward checking
 - ex: $NSW=r, NT=g$ in conflict with r and g values for Q resp.
 \implies domain of Q reduced to $\{b\}$ via f.c.
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Conflict-Driven Backjumping

- Idea: When a branch fails (empty domain for variable X_i):
 - 1 identify nogood which caused the failure deterministically, via forward checking
 - 2 backtrack to the most-recently assigned element in nogood,
 - 3 change its value

⇒ May jump much higher, lots of search saved

- Identify nogood:
 - 1 take the conflict set C_i of empty-domain X_i (initial nogood)
 - 2 backward-substitute (deterministic) unit assignments with their respective conflict set
- Many different strategies & variants available

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- failed branch:

step	assign.[domain]	$\leftarrow \{\text{conflict set}\}$
(1) pick	WA = r [rbg]	$\leftarrow \{\}$
(2) pick	NSW = r [rbg]	$\leftarrow \{\}$
(3) pick	T = r [rbg]	$\leftarrow \{\}$
(4) pick	NT = g [bg]	$\leftarrow \{WA = r\}$
(5) \xRightarrow{fc}	Q = b [b]	$\leftarrow \{NSW = r, NT = g\}$
(6) pick	V = b [b, g]	$\leftarrow \{WA = r\}$
(7) \xRightarrow{fc}	SA = \emptyset []	$\leftarrow \{WA = r, NT = g, Q = b\}$

- backward-substitute assignments

$$\begin{array}{c} \emptyset \quad (7) \\ \hline \{WA = r, NT = g, Q = b\} \quad (5) \\ \hline \{WA = r, NT = g, NSW = r\} \end{array}$$

⇒ backtrack till (3) $T = r$, then assign $NT = b$

⇒ saves useless search on V values



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(4) pick	NT = g [bg]	$\leftarrow \{ \text{WA} = r \}$
(5) \xRightarrow{fc}	Q = b [b]	$\leftarrow \{ \text{NSW} = r, \text{NT} = g \}$
(6) pick	V = b [b, g]	$\leftarrow \{ \text{WA} = r \}$
(7) \xRightarrow{fc}	SA = \emptyset []	$\leftarrow \{ \text{WA} = r, \text{NT} = g, \text{Q} = b \}$

- backward-substitute assignments

$$\frac{\emptyset \quad (7)}{\frac{\{ \text{WA} = r, \text{NT} = g, \text{Q} = b \} \quad (5)}{\{ \text{WA} = r, \text{NT} = g, \text{NSW} = r \}}}$$

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(7) \xRightarrow{fc}	SA = \emptyset []	$\leftarrow \{ \text{WA} = r, \text{NT} = g, \text{Q} = b \}$

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$$\frac{\emptyset \quad (7)}{\frac{\{ \text{WA} = r, \text{NT} = g, \text{Q} = b \} \quad (5)}{\{ \text{WA} = r, \text{NT} = g, \text{NSW} = r \}}}$$

⇒ backtrack till (3) $T = r$, then assign
 $\text{NT} = b$

⇒ saves useless search on V values



Conflict-Driven Backjumping: Example [cont.]

- new failed branch:

step	assign.[domain]	$\leftarrow \{ \text{conflict set} \}$
(1) pick	WA = r [rbg]	$\leftarrow \{ \}$
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(3) pick	T = r [rbg]	$\leftarrow \{ \}$
(4) pick	NT = b [b]	$\leftarrow \{ \text{WA} = r \}$
(5) \xRightarrow{fc}	Q = g [g]	$\leftarrow \{ \text{NSW} = r, \text{NT} = b \}$
(6) pick	V = b [b, g]	$\leftarrow \{ \text{WA} = r \}$
(7) \xRightarrow{fc}	SA = \emptyset []	$\leftarrow \{ \text{WA} = r, \text{NT} = b, \text{Q} = g \}$

- backward-substitute assignments

$$\begin{array}{c}
 \emptyset \quad (7) \\
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 \end{array}$$

\Rightarrow backtrack till (1) $\text{WA} = r$, then assign NSW another value

\Rightarrow saves useless search on T values

\Rightarrow overall, saves lots of search wrt. chronological backtracking



Conflict-Driven Backjumping: Example [cont.]

- new failed branch:

step	assign.[domain]	$\leftarrow \{ \text{conflict set} \}$
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\Rightarrow backtrack till (1) $\text{WA} = r$, then assign NSW another value

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 \{ \text{WA} = r, \text{NSW} = r \}
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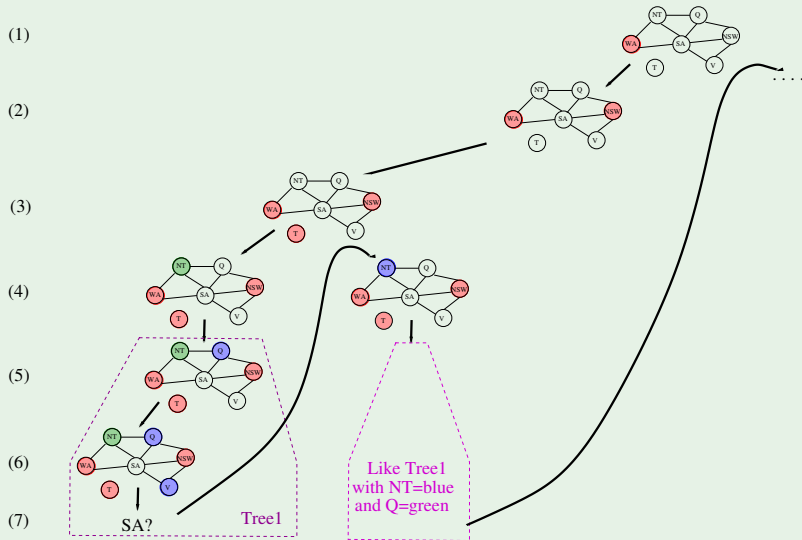
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Conflict-Driven Backjumping: Example [cont.]

Search Tree



Learning Nogoods

- Nogood can be *learned* (stored) for future search pruning:
 - added to constraints (e.g. “($WA \neq r$) or ($NSW \neq r$)”)
 - added to explicit nogood list
- As soon as assignment contains all but one element of a nogood, drop the value of the remaining element from variable's domain
- Example:
 - given nogood: $\{WA=r, NSW=r\}$
 - as soon as $\{NSW=r\}$ is added to assignment r is dropped from WA domain
- Allows for
 - early-reveal inconsistencies
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- Nogoods can be learned either temporarily or permanently
 - pruning effectiveness vs. memory consumption & overhead
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 - early-reveal inconsistencies
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- 1 Defining Constraint Satisfaction Problems (CSPs)
- 2 Inference in CSPs: Constraint Propagation
- 3 Backtracking Search with CSPs
- 4 Local Search with CSPs**
- 5 Exploiting Structure of CSPs

Local Search with CSPs

- Extension of Local Search to CSPs straightforward
- Use complete-state representation (**complete assignments**)
 - allow states with unsatisfied constraints
 - “**neighbour states**” **differ for one variable value**
 - steps: reassign variable values
- **Min-conflicts heuristic** in hill-climbing:
 - Variable selection: **randomly select any conflicted variable**
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 - Improvement: adaptive strategies giving different weights to constraints according to their criticality
- SLC variants [see Ch. 4] apply to CSPs as well
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The Min-Conflicts Heuristic

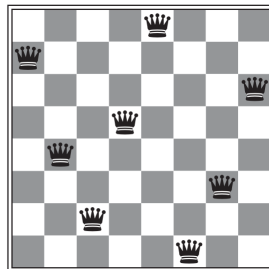
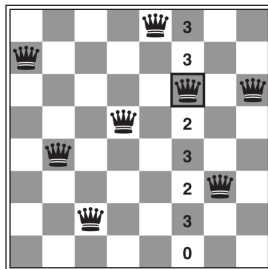
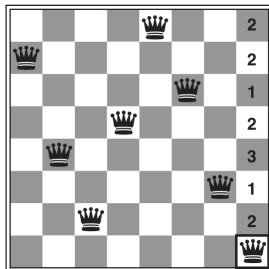
```
function MIN-CONFLICTS(csp, max_steps) returns a solution or failure
inputs: csp, a constraint satisfaction problem
         max_steps, the number of steps allowed before giving up

current  $\leftarrow$  an initial complete assignment for csp
for i = 1 to max_steps do
    if current is a solution for csp then return current
    var  $\leftarrow$  a randomly chosen conflicted variable from csp.VARIABLES
    value  $\leftarrow$  the value v for var that minimizes CONFLICTS(var, v, current, csp)
    set var = value in current
return failure
```

(© S. Russell & P. Norwig, AIMA)

The Min-Conflicts Heuristic: Example

Two steps solution of 8-Queens problem



(© S. Russell & P. Norwig, AIMA)

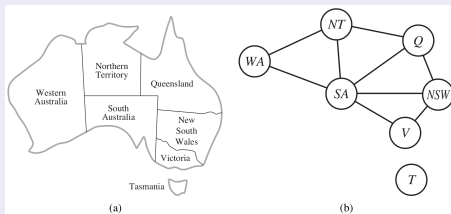
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Partitioning CFPs

“Divide & Conquer” CSPs

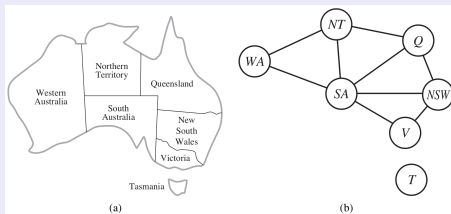
- Idea (when applicable): Partition a CSP into independent CSPs
 - identify strongly-connected components in constraint graph
 - e.g. by Tarjan's algorithms (linear!)
- Ex: Tasmania and mainland are independent subproblems
- E.g. partition n -variable CSP into n/c CSPs w. c variables each:
 - from d^n to $n/c \cdot d^c$ steps in worst-case
 - if $n = 80, d = 2, c = 20$, then from $2^{80} \approx 10^{24}$ to $4 \cdot 2^{20} \approx 4 \cdot 10^6$
 \implies from 4 billion years to 0.4 secs at 10million steps/sec



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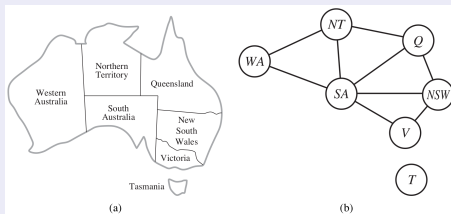
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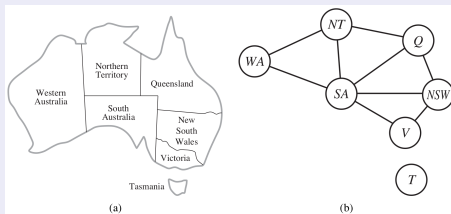
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Solving Tree-structured CSPs

Theorem:

- If the constraint graph has no loops, the CSP can be solved in $O(nd^2)$ time in worst case
 - general CSPs can be solved $O(d^n)$ time worst-case

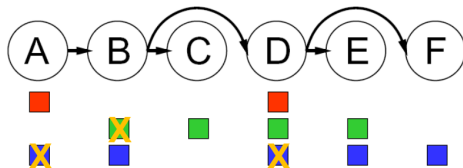
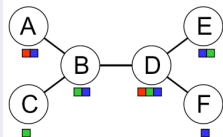
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Algorithm

- 1 Choose a variable as root, order variables from root to leaves
- 2 For $j \in n..2$ apply MAKEARCONSISTENT(PARENT(X_j), X_j)
- 3 For $j \in 2..n$, assign X_j consistently with PARENT(X_j)



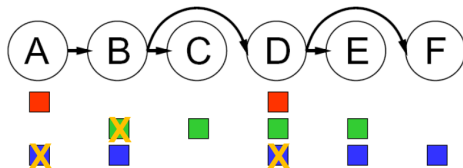
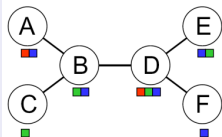
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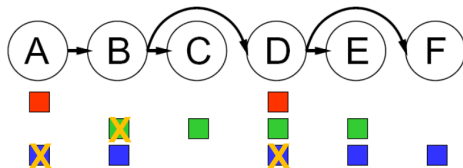
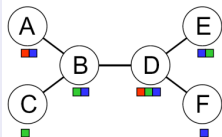
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Solving Tree-structured CSPs [cont.]

function TREE-CSP-SOLVER(*csp*) **returns** a solution, or failure

inputs: *csp*, a CSP with components X , D , C

$n \leftarrow$ number of variables in X

assignment \leftarrow an empty assignment

root \leftarrow any variable in X

$X \leftarrow$ TOPOLOGICALSORT(X , *root*)

for $j = n$ **down to** 2 **do**

 MAKE-ARC-CONSISTENT(PARENT(X_j), X_j)

if it cannot be made consistent **then return** *failure*

for $i = 1$ **to** n **do**

assignment[X_i] \leftarrow any consistent value from D_i

if there is no consistent value **then return** *failure*

return *assignment*

Solving Nearly Tree-Structured CSPs

Cutset Conditioning

- 1 Identify a (small) **cycle cutset** S : a set of variables s.t. the remaining constraint graph is a tree
 - finding smallest cycle cutset is NP-hard
 - fast approximated techniques known
- 2 For each possible consistent assignment to the variables in S
 - a) remove from the domains of the remaining variables any values that are inconsistent with the assignment for S
 - b) apply the tree-structured CSP algorithm
- 3 If $c \stackrel{\text{def}}{=} |S|$, then runtime is $O(d^c \cdot (n - c)d^2)$
 \Rightarrow much smaller than d^n if c small

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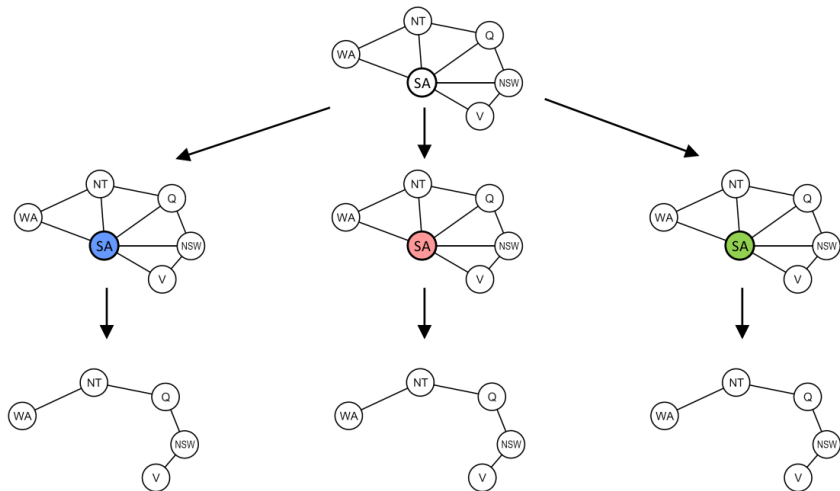
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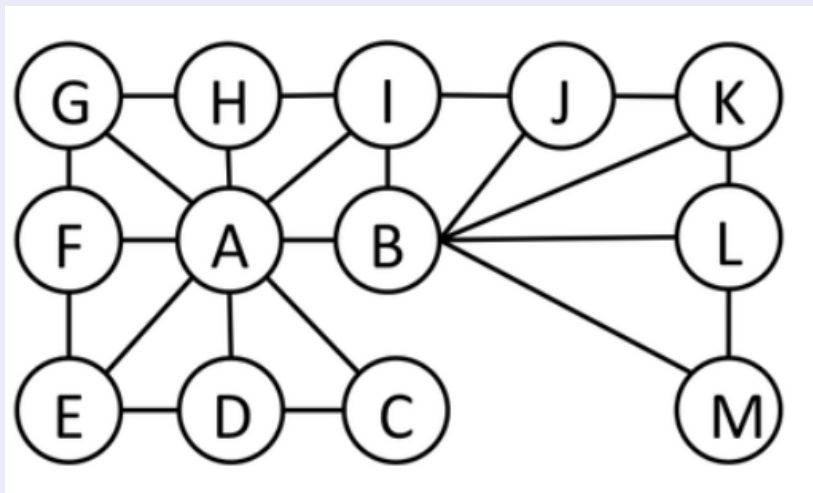
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Cutset Conditioning: Example



Exercise

- Solve the following 3-coloring problem by Cutset Conditioning



Breaking Value Symmetry

- Value symmetry: if domain size is n and no unary constraints
 - every solution has $n!$ solutions obtained by permuting color names
 - ex: 3-coloring, $3! = 6$ permutations for every solutions
- Symmetry Breaking: add symmetry-breaking constraints s.t. only one of the $n!$ solution is possible
 - ⇒ reduce search space by $n!$ factor
- Add value-ordering constraints on n variables:
 - give an ordering of values (ex: $r < b < g$)
 - impose an ordering on the values of n variables s.t. $x_i \neq x_j$ (ex: $WA < NT < SA$)
 - ⇒ only one solution out of $n!$

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