Course "**Fundamentals of Artificial Intelligence**"
EXAM TEXT

Prof. Roberto Sebastiani
DISI, Università di Trento, Italy

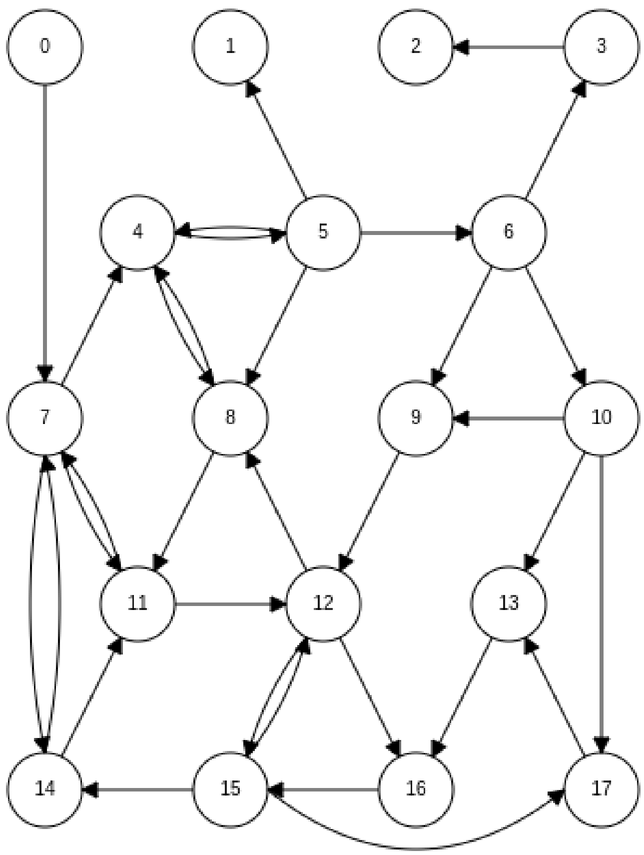
January 12^{th}, 2021

27534516

[COPY WITH SOLUTIONS]

1

Consider the graph shown below.



For each of the following facts, say if it is true or false. Graph nodes are explored by following the ascending order of their labels.

(*a*) By supposing to have the start node in 0 and the goal state in 15, the BFS algorithm reaches the goal before the DFS one.
[Solution: true]

(*b*) By supposing to have the start node in 0 and the goal state in 15 and that the BFS performs the goal test before the generation of new node, the DFS algorithm generates a number of nodes lower than the BFS one.
[Solution: false]

(*c*) The BFS algorithm reaches the node 6 before the node 10.
[Solution: true]

(*d*) The DFS algorithm reaches the node 10 before the node 15.
[Solution: false]

2

Let φ be a generic Boolean formula, and let $\varphi_1 \stackrel{\text{def}}{=} CNF_{label}(\varphi)$, s.c. $CNF_{label}()$ is the "labeling" CNF conversion. Let $|\varphi|$ and $|\varphi_1|$ denote the size of φ and φ_1 respectively.

For each of the following sentences, say if it is true or false.

(a) $|\varphi_1|$ is in worst-case polynomial in size wrt. $|\varphi|$. [Solution: True.]

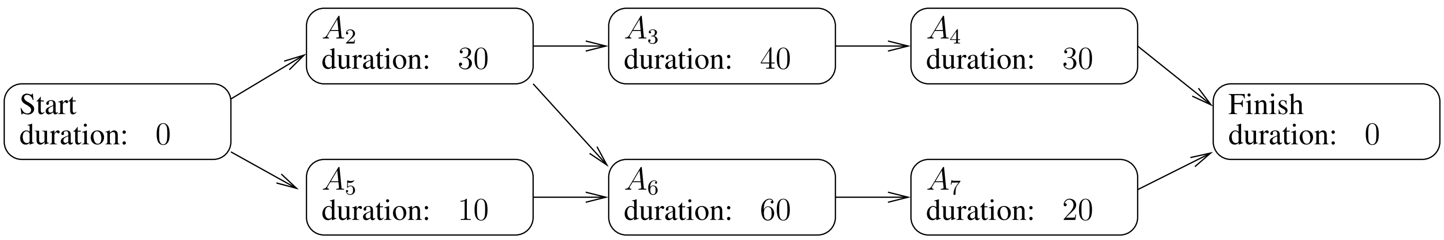
(b) φ_1 has the same number of distinct Boolean variables as φ has. [Solution: False.]

(c) A model for φ_1 (if any) is also a model for φ, and vice versa. [Solution: False.]

(d) φ_1 is valid if and only if φ is valid. [Solution: False.]

3

Consider the following partial-order plan, with respective predicted duration of each action.



For each of the following facts, say if it is true or false

(a) Action A_4 in on the critical path
[Solution: false]

(b) If Action A_7 takes longer than its predicted duration, then the whole process takes longer than the predicted optimal total duration.
[Solution: true]

(c) In an optimum schedule, action A_4 must necessarily start earlier than action A_7.
[Solution: true]

(d) In an optimum schedule, action A_6 cannot start later than action A_3.
[Solution: true]

4

Consider (normal) modal logics. Let $\mathsf{IsRed}(\mathsf{Pen}), \mathsf{IsOnTable}(\mathsf{Pen})$ be possible facts, let $Mary, John$ be agents and let $\mathbf{K}_{Mary}, \mathbf{K}_{John}$ denote the modal operators "Mary knows that..." and "John knows that..." respectively.
For each of the following facts, say if it is true or false.

(*a*) If $\mathbf{K}_{Mary}\neg\mathsf{IsRed}(\mathsf{Pen})$ holds, then $\neg\mathbf{K}_{Mary}\mathsf{IsRed}(\mathsf{Pen})$ holds
[Solution: true]

(*b*) If $\neg\mathbf{K}_{Mary}\mathsf{IsRed}(\mathsf{Pen})$ holds, then $\mathbf{K}_{Mary}\neg\mathsf{IsRed}(\mathsf{Pen})$ holds
[Solution: false]

(*c*) If $\mathbf{K}_{John}\mathsf{IsRed}(\mathsf{Pen})$ and $\mathsf{IsRed}(\mathsf{Pen}) \leftrightarrow \mathsf{IsOnTable}(\mathsf{Pen})$ hold, then $\mathbf{K}_{John}\mathsf{IsOnTable}(\mathsf{Pen})$ holds
[Solution: false]

(*d*) If $\mathbf{K}_{Mary}\mathsf{IsRed}(\mathsf{Pen})$ and $\mathbf{K}_{Mary}(\mathsf{IsRed}(\mathsf{Pen}) \rightarrow \mathbf{K}_{\mathsf{John}}\mathsf{IsRed}(\mathsf{Pen}))$ hold, then $\mathbf{K}_{Mary}\mathbf{K}_{John}\mathsf{IsRed}(\mathsf{Pen}))$ holds
[Solution: true]

5

Given the following Sudoku scenario.

	1	2	3	4	5	6	7	8	9
A					8	7			1
B									2
C									3
D	2	3	5	6				1	4
E								3	5
F									6
G							1	2	7
H							3	4	
I					9		5		

For each of the following facts, say if it is true or false

(*a*) I8=6 can be derived by one arc-consistency propagation step.
[Solution: false]

(*b*) D5=7 can be derived by one arc-consistency propagation step
[Solution: true]

(*c*) I8=6 can be derived by one path-consistency propagation step.
[Solution: true]

(*d*) D6=8 can be derived by one path-consistency propagation step.
[Solution: false]

6

Given the following set of propositional clauses Γ:

$$
\begin{array}{l}
(\ \ F \vee \ \ E \vee \neg A) \\
(\neg D \vee \ \ B) \\
(\ \ F) \\
(\ \ C \vee \ \ B \vee \neg G) \\
(\neg G) \\
(\neg B \vee \ \ A) \\
(\neg F \vee \neg C \vee \ \ D) \\
(\ \ C) \\
(\neg C \vee \neg D \vee \ \ E) \\
(\neg C \vee \neg A \vee \ \ G)
\end{array}
$$

Produce a PL-resolution proof that Γ is unsatisfiable.
Such proof must be written as a sequence of resolution steps in the form:

$[Clause_{11},\ Clause_{12}] \Longrightarrow Resolvent\_Clause_1;$
$[Clause_{21},\ Clause_{22}] \Longrightarrow Resolvent\_Clause_2;$
...;
$[Clause_{k1},\ Clause_{k2}] \Longrightarrow Resolvent\_Clause_k;$

s.t. $Resolvent\_Clause_k$ is the empty clause, and each $Clause_{ij}$ is either in Γ or is a resolvent clause $Resolvent\_Clause_m$ resulting from previous steps, i.e. s.t. $m < i$.

[Solution:

$[(\ \ F),\ (\neg F \vee \neg C \vee \ \ D)] \ \Longrightarrow (\neg C \vee \ \ D);$
$[(\ \ C),\ (\neg C \vee \ \ D)] \ \Longrightarrow (\ \ D);$
$[(\ \ D),\ (\neg D \vee \ \ B)] \ \Longrightarrow (\ \ B);$
$[(\ \ B),\ (\neg B \vee \ \ A)] \ \Longrightarrow (\ \ A);$
$[(\ \ C),\ (\neg C \vee \neg A \vee \ \ G)] \ \Longrightarrow (\neg A \vee \ \ G);$
$[(\ \ A),\ (\neg A \vee \ \ G)] \ \Longrightarrow (\ \ G);$
$[(\neg G),\ (\ \ G)] \ \Longrightarrow ();$

]

7

Translate the following English sentences into FOL formulas

(a) Charles owns some expensive pens.
[Solution: $\exists x.(Pen(x) \wedge Expensive(x) \wedge Owns(Charles, x))$
or any formula which is logically equivalent to it]

(b) Every handsome prince marries a pretty princess.
[Solution:
$\forall x.((Prince(x) \wedge Handsome(x)) \rightarrow \exists y.(Princess(y) \wedge Pretty(y) \wedge Marries(x, y)))$
or any formula which is logically equivalent to it]

(c) Nobody loves a man who beats a woman.
[Solution:
$\forall x, y.((Man(y) \wedge \exists z.(Woman(z) \wedge Beats(y, z))) \rightarrow \neg Loves(x, y))$
or any formula which is logically equivalent to it, like, eg,
$\forall y.((Man(y) \wedge \exists z.(Woman(z) \wedge Beats(y, z))) \rightarrow \forall x.\neg Loves(x, y))$
or, equivalently,
$\forall x, y, z.((Man(y) \wedge Woman(z) \wedge Beats(y, z)) \rightarrow \neg Loves(x, y))$
]

(d) Everyone who respects all persons is appreciated by someone.
[Solution:
$\forall x.([\forall y.(Person(y) \rightarrow Respects(x, y))] \rightarrow [\exists z.Appreciate(z, x)])$
or any formula which is logically equivalent to it

]

8

Consider the following FOL KB:

$\forall x, y, z. ((Passerby(x) \wedge Jacket(y) \wedge Homeless(z) \wedge Gives(x, y, z)) \rightarrow GoodHearted(x))$
$\exists x.(Owns(Paula, x) \wedge Jacket(x))$
$\forall x.((Jacket(x) \wedge Owns(Paula, x)) \rightarrow Gives(Anne, x, Paula))$
$Passerby(Anne)$
$Homeless(Paula)$

(a) Compute the CNF-ization of the KB & standardize variables
(b) Write a FOL-resolution inference of the query $GoodHearted(Anne)$ from the CNF-ized KB

[Solution:

(a) Compute the CNF-ization of the KB & standardize variables:
$(\neg Passerby(x) \vee \neg Jacket(y) \vee \neg Homeless(z) \vee \neg Gives(x, y, z) \vee GoodHearted(x))$
$(Owns(Paula, F_1))$
$(Jacket(F_1))$
$(\neg Jacket(x_1) \vee \neg Owns(Paula, x_1) \vee Gives(Anne, x_1, Paula))$
$(Passerby(Anne))$
$(Homeless(Paula))$
where F_1 is a Skolem constant.

(b) Infer the query $GoodHearted(Anne)$ from the CNF-ized KB via FOL resolution:

$[(Passerby(Anne)),\ (\neg Passerby(x) \vee \neg Jacket(y) \vee \neg Homeless(z) \vee \neg Gives(x, y, z) \vee GoodHearted(x))]$
$\Longrightarrow (\neg Jacket(y) \vee \neg Homeless(z) \vee \neg Gives(Anne, y, z) \vee GoodHearted(Anne));$

$[(Jacket(F_1)),\ (\neg Jacket(y) \vee \neg Homeless(z) \vee \neg Gives(Anne, y, z) \vee GoodHearted(Anne))]$
$\Longrightarrow (\neg Homeless(z) \vee \neg Gives(Anne, F_1, z) \vee GoodHearted(Anne));$

$[(Homeless(Paula)),\ (\neg Homeless(z) \vee \neg Gives(Anne, F_1, z) \vee GoodHearted(Anne))]$
$\Longrightarrow (\neg Gives(Anne, F_1, Paula) \vee GoodHearted(Anne));$

$[(Jacket(F_1)),\ (\neg Jacket(x_1) \vee \neg Owns(Paula, x_1) \vee Gives(Anne, x_1, Paula))]$
$\Longrightarrow (\neg Owns(Paula, F_1) \vee Gives(Anne, F_1, Paula));$

$[(Owns(Paula, F_1)),\ (\neg Owns(Paula, F_1) \vee Gives(Anne, F_1, Paula))]$
$\Longrightarrow (Gives(Anne, F_1, Paula));$

$[(Gives(Anne, F_1, Paula)),\ (\neg Gives(Anne, F_1, Paula) \vee GoodHearted(Anne))]$
$\Longrightarrow (GoodHearted(Anne));$
]

9

Given the random propositional variables A, B, C and their joint probability distribution $\mathbf{P(A, B, C)}$ described as follows:

A	B	C	$\mathbf{P(A,B,C)}$
T	T	T	0.072
T	T	F	0.036
T	F	T	0.256
T	F	F	0.192
F	T	T	0.008
F	T	F	0.084
F	F	T	0.064
F	F	F	0.288

(a) Using marginalization, compute the probability $P(B)$

(b) Using normalization, compute the probability $P(A|B, C)$

[Solution:

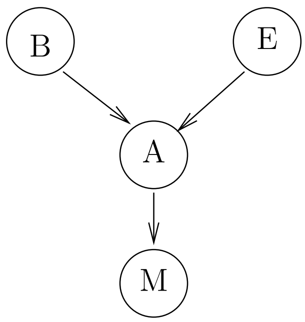
(a) $P(B) = P(A, B, C) + P(A, B, \neg C) + P(A, B, C) + P(\neg A, B, \neg C) =$
$= 0.072 + 0.036 + 0.008 + 0.084 = 0.2$

(b) $P(A|B, C) = \alpha \cdot P(A, B, C) = \overbrace{\frac{1}{P(A, B, C) + P(\neg A, B, C)}}^{\alpha} \cdot P(A, B, C) =$
$= \frac{1}{0.072 + 0.008} \cdot 0.072 = 0.9$

]

10

Consider the following simple Bayesian network, where $\mathbf{P}(\mathbf{B})$, $\mathbf{P}(\mathbf{E})$, $\mathbf{P}(\mathbf{A}|\mathbf{B},\mathbf{E})$, $\mathbf{P}(\mathbf{M}|\mathbf{A})$ are defined in the following annex file (columns M-N, reported also in columns E-H):
2021.01.12-27534516-Bayes.xlsx



(a) Compute the full distribution $\mathbf{P}(\mathbf{A},\mathbf{B},\mathbf{E},\mathbf{M})$ and write it in column I of the above-mentioned file.

(b) Can you say something about $\mathbf{P}(\mathbf{M}|\mathbf{A},\mathbf{B},\mathbf{E})$ without any further computation?

(c) Compute by normalization the partial distribution $\mathbf{P}(\mathbf{M}|\mathbf{A},\mathbf{B},\mathbf{E})$ and write it in column J and the normalization values α in column K of the above-mentioned file. Verify that the result complies with your answer of point (b)

You can use the arithmetic excel operators $(=,+,-,*,/)$
[Solution:
Consider the annex file 2021.01.12-27534516-Bayes_sol.xlsx

(a) The full distribution $\mathbf{P}(\mathbf{A},\mathbf{B},\mathbf{E},\mathbf{M}) \stackrel{\text{def}}{=} \mathbf{P}(\mathbf{B})\cdot\mathbf{P}(\mathbf{E})\cdot\mathbf{P}(\mathbf{A}|\mathbf{B},\mathbf{E})\cdot\mathbf{P}(\mathbf{M}|\mathbf{A})$ is reported in column I of the above-mentioned file.

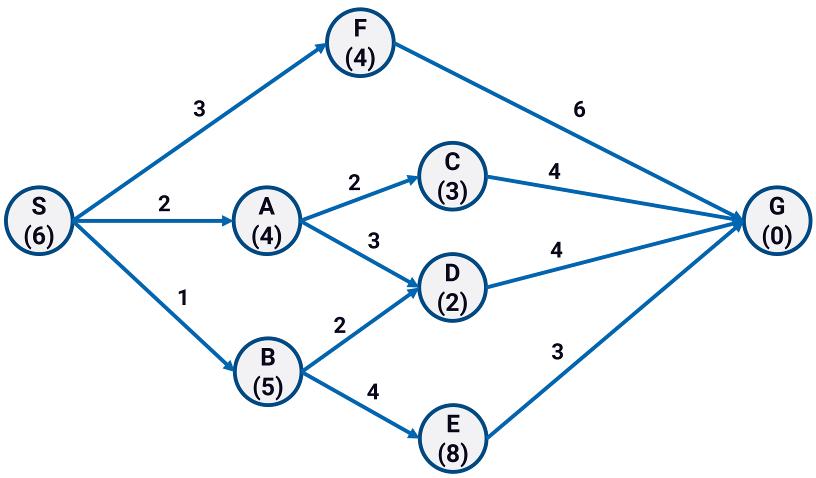
(b) By construction of Bayesian networks, M is independent on B and E given A, so that we have that $\mathbf{P}(\mathbf{M}|\mathbf{A},\mathbf{B},\mathbf{E}) = \mathbf{P}(\mathbf{M}|\mathbf{A})$.

(c) By normalization, the partial distribution $\mathbf{P}(\mathbf{M}|\mathbf{A},\mathbf{B},\mathbf{E}) = \alpha\cdot\mathbf{P}(\mathbf{A},\mathbf{B},\mathbf{E},\mathbf{M})$, where, for each truth value combination of $\langle A,B,E\rangle$, $\alpha \stackrel{\text{def}}{=} 1/(P(M,A,B,E)+P(\neg M,A,B,E))$. The result is reported in column J of the above-mentioned file, and the values of α in column K.
Notice that $\mathbf{P}(\mathbf{M}|\mathbf{A},\mathbf{B},\mathbf{E}) = \mathbf{P}(\mathbf{M}|\mathbf{A})$, –possibly modulo numerical roundings– by comparing column E with column J.

]

11

The graph in the figure below shows the state space of a hypothetical search problem. States are denoted by letters; Arcs are labeled with the cost of traversing them; the estimated cost to a goal (i.e., the h function) is reported inside nodes (so that lower scores are better). Considering S as the initial state, solve the above search problem using A* search. Report each step of the resolution process. Then, explain if the heuristic adopted is admissible or not.



Notice and notations:

- The solution format has to be provided as shown in the example contained in the file 2021-01-12-a-star-example.pdf

[Solution:

]

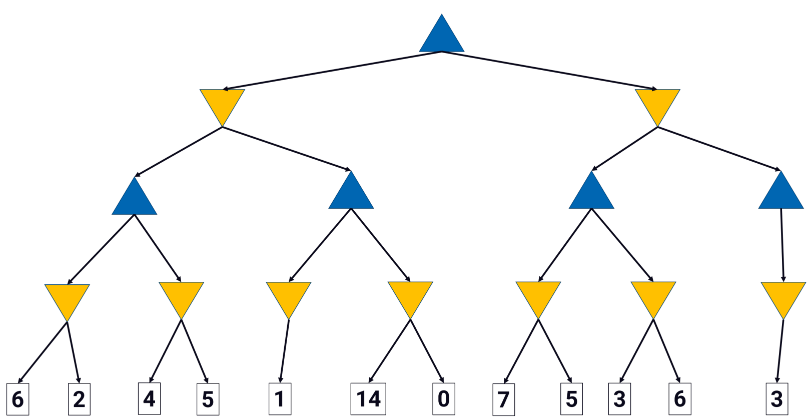
Step	Queue	Processed Nodes	Children
1	S(6)	S(6)	A(2+4) B(1+5) F(3+4)
2	A(6) B(6) F(7)	A(6)	C(4+3) D(5+2)
3	B(6) C(7) D(7) F(7)	B(6)	D(3+2) E(5+8)
4	D(5) C(7) F(7) E(13)	D(5)	G(7+0)
5	C(7) F(7) G(7) E(13)	C(7)	G(8+0)
6	F(7) G(7) E(13)	F(7)	G(9+0)
7	G(7) E(13)	G(7)	

Node	h	h*
S	6	7
A	4	6
B	5	6
C	3	4
D	2	4
E	**8**	**3**
F	4	6

The **h** function is **not admissible** because for the **node E** the actual cost for reaching the goal is higher than the estimated one.

12

Given the tree below, apply the alpha-beta pruning strategy for finding the optimal choice that the MAX agent has to perform. Report the values of alpha and beta for each MIN and MAX nodes. Mark the pruned branches.

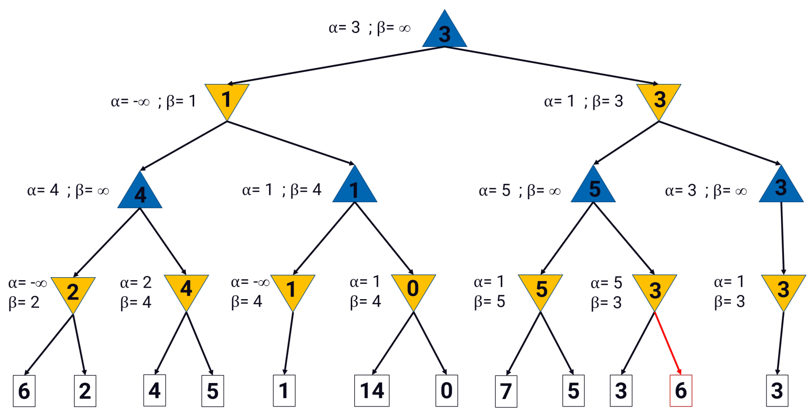


Notice and notations:

- Blue triangles are MAX nodes.
- Yellow triangles are MIN nodes.
- Each triangles has to be labeled with the format L + top down level number + I + left-right index. For example, the top MAX node is labeled as $L0I0$, the third from the left MIN node in the last level as $L3I2$.
- Pruned branches have to be provided with the notation: $Pruned(Node, Side)$, where $Node$ is the start node of the pruned branch and $Side$ can assume the values $Left$ or $Right$. For

example, with the notation $Pruned(L1I0, Right)$ means that you pruned everything under the right side of the first MIN node from the left on the second level from the top.

[Solution:

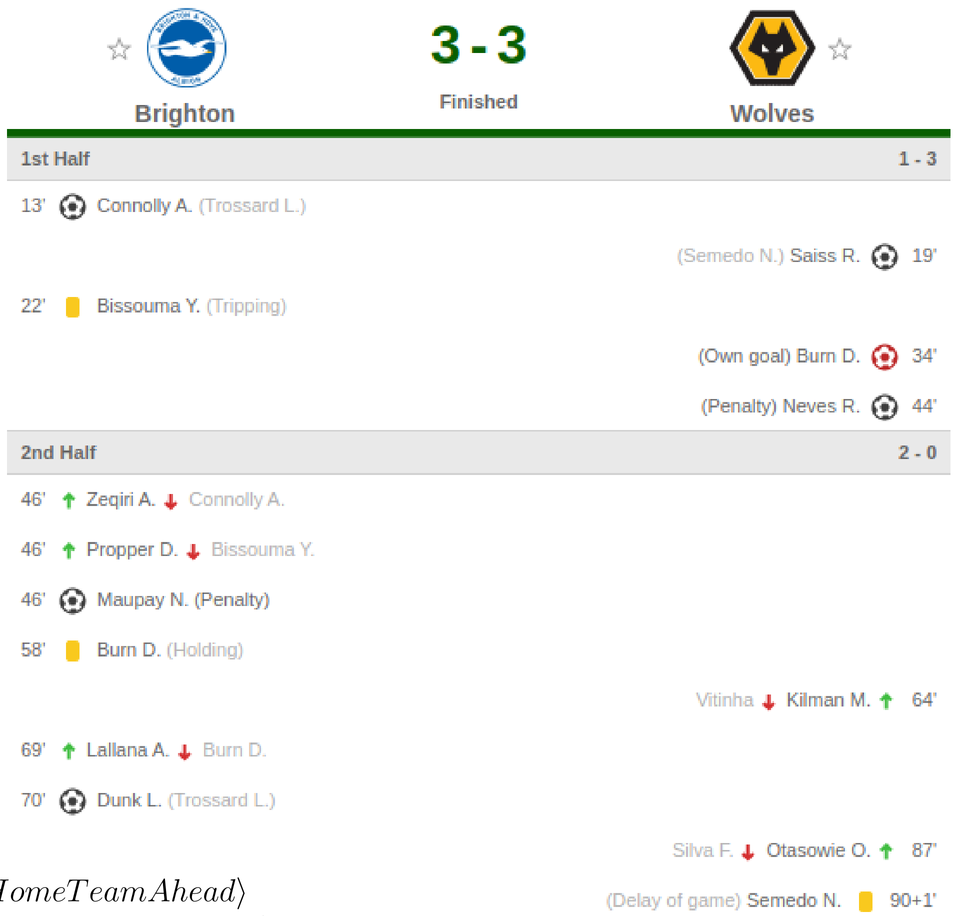


No pruned branches.

]

13

Given the image below, please state the interval-algebra relations that hold between the provided pairs related to the described real-world event:



Pairs:
$\langle FirstHalf, HomeTeamAhead\rangle$
$\langle BissoumaWarned, SaissScored\rangle$
$\langle BissoumaPlayed, VitinhaPlayed\rangle$
$\langle BurnWarned, AwayTeamAhead\rangle$
$\langle DunkScored, SecondHalf\rangle$
$\langle DunkScored, KilmanPlayed\rangle$
$\langle FirstHalf, SecondHalf\rangle$
$\langle BurnWarned, HomeTeamAhead\rangle$

Notice and notations:

- events like goals, have not to be intended as instantaneous events, but like events during a certain (small) amount of time;
- a player is intended to be warned from the moment in which he received the yellow card, until the end of the match or until the moment in which he is substituted;
- you have to assume that the halftime break exists.
- the list of relations have to be provided by using the format: $Relation(Event1, Event2)$

[Solution:

$During(FirstHalf, HomeTeamAhead)$
$Before(SaissScored, BissoumaWarned)$
$Starts(BissoumaPlayed, VitinhaPlayed)$
$Overlap(BurnWarned, AwayTeamAhead)$
$During(DunkScored, SecondHalf)$
$During(DunkScored, KilmanPlayed)$
$Before(FirstHalf, SecondHalf)$
$Before(HomeTeamAhead, BurnWarned)$

]

14

Given the plan described below:

Init(Have(Pizza))

Goal(Have(Pizza) ∧ Eaten(Pizza))

Action(Eat(Pizza)) PRECOND: Have(Pizza) EFFECT: ¬Have(Pizza) ∧ Eaten(Pizza))

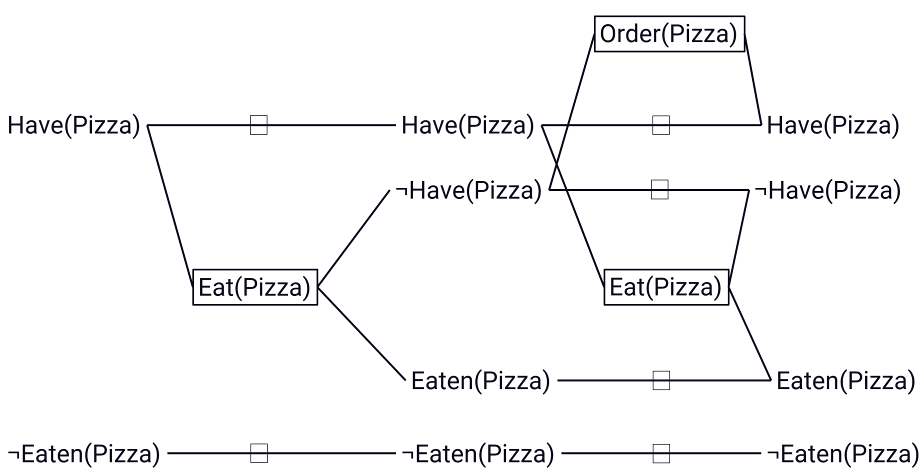
Action(Order(Pizza)) PRECOND: ¬Have(Pizza) EFFECT: Have(Pizza))

Draw the planning graph by using the following notation:

- Rectangles indicate actions.
- Small squares persistent actions (no-ops).
- Straight lines indicate preconditions and effects.
- Arcs indicate mutex links.

Within the planning graph, please enumerate the added mutex links and provide a description of the kind of mutex relation.

[Solution:



Inconsistent effect:
Persistence of Have(Pizza), Eat(Pizza)
¬Eaten(Pizza), Eat(Pizza)
Order(Pizza), Eat(Pizza)

Interference:
Persistence of Have(Pizza), Eat(Pizza)
Persistence of ¬Have(Pizza), Order(Pizza)
Persistence of Have(Pizza), Persistence of ¬Have(Pizza)
Persistence of Eaten(Pizza), Persistence of ¬Eaten(Pizza)

Competing needs:
Order(Pizza), Eat(Pizza)
Persistence of Have(Pizza), Persistence of ¬Have(Pizza)
Persistence of Eaten(Pizza), Persistence of ¬Eaten(Pizza)

Inconsistent support:
Have(Pizza), ¬Have(Pizza)
Eaten(Pizza), ¬Eaten(Pizza)

All ways of achieving them are pairwise mutex:
Have(Pizza), Eaten(Pizza)
¬Have(Pizza), ¬Eaten(Pizza)
]

15

TEXT

Consider the following binary constraint network:
There are 4 variables: X_1, X_2, X_3, X_4
Domains: $D_1 = \{1, 2, 5, 7\}, D_2 = \{1, 2, 6, 8, 9\}, D_3 = \{2, 4, 6, 7, 8, 9\}, D_4 = \{1, 2, 3, 8, 9\}$ The constraints are

$$X_1 = X_2$$

$$X_2 < X_3 \text{ } or \text{ } X_2 - X_3 = 3$$

$$X_3 > X_4$$

Questions:

(a) Is the network arc-consistent? If not, compute the arc-consistent network.

(b) Is the network consistent? If yes, give a solution.

[Solution:
Enforce arc consistency between X_1 and X_2 led to $D_1 = \{1, 2\}$ and $D_2 = \{1, 2\}$
Enforce arc consistency between X_3 and X_4 led to $D_4 = \{1, 2, 3, 8\}$
One possible solution: $X_1 = 1$, $X_2 = 1$, $X_3 = 8$, $X_4 = 1$]