Course "Introduction to SAT & SMT"
TEST

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1

Let φ be a generic Boolean formula, and let $\varphi_{nnf}^{tree} \stackrel{\text{def}}{=} NNF^{tree}(\varphi)$ and $\varphi_{nnf}^{dag} \stackrel{\text{def}}{=} NNF^{dag}(\varphi)$, s.c. $NNF()^{tree}$ and $NNF()^{dag}$ are the conversion into negative normal form using a tree and a DAG representation of the formulas respectively.

Let $|\varphi|$, $|\varphi_{nnf}^{tree}|$ and $|\varphi_{nnf}^{dag}|$ denote the size of φ, φ_{nnf}^{tree} and φ_{nnf}^{dag} respectively.

For each of the following sentences, say if it is true or false.

(a) $|\varphi_{nnf}^{tree}|$ is in worst-case polynomial in size wrt. $|\varphi|$.

(b) $|\varphi_{nnf}^{dag}|$ is in worst-case polynomial in size wrt. $|\varphi|$.

(c) φ_{nnf}^{dag} has the same number of distinct Boolean variables as φ has.

(d) A model for φ_{nnf}^{dag} (if any) is also a model for φ, and vice versa.

2

Using the variable ordering " A_1, A_2, A_3, A_4", draw the OBDD corresponding to the following formulas:

$$A_1 \wedge (\neg A_1 \vee \neg A_2) \wedge (A_2 \vee A_3) \wedge (\neg A_3 \vee A_4)$$

3

Using the semantic tableaux algorithm, decide whether the following formula is satisfiable or not. (Write the search tree.)

$$
\begin{array}{rcrcrl}
 & & & & (\neg A_1) & \wedge \\
 & & (\ A_1 & \vee & \neg A_2) & \wedge \\
(\ A_1 & \vee & A_2 & \vee & A_3) & \wedge \\
(\ A_4 & \vee & \neg A_3 & \vee & A_6) & \wedge \\
(\ A_4 & \vee & \neg A_3 & \vee & \neg A_6) & \wedge \\
(\neg A_3 & \vee & \neg A_4 & \vee & A_7) & \wedge \\
(\neg A_3 & \vee & \neg A_4 & \vee & \neg A_7) &
\end{array}
$$

(Literal-selection criteria to your choice.)

4

Consider the following piece of a much bigger formula, which has been fed to a CDCL SAT solver:

$$\begin{array}{l} c_1 : \neg A_7 \lor \quad A_2 \\ c_2 : \quad A_4 \lor \quad A_1 \lor \quad A_{11} \\ c_3 : \quad A_8 \lor \neg A_6 \lor \neg A_4 \\ c_4 : \neg A_5 \lor \neg A_1 \\ c_5 : \quad A_7 \lor \neg A_8 \\ c_6 : \quad A_7 \lor \quad A_6 \lor \quad A_9 \\ c_7 : \neg A_7 \lor \quad A_3 \lor \neg A_{12} \\ c_8 : \quad A_4 \lor \quad A_5 \lor \quad A_{10} \\ \ldots \end{array}$$

Suppose the solver has decided, in order, the following literals (possibly interleaved by others not occurring in the above clauses):

$\{..., \neg A_9, ... \neg A_{10}, ... \neg A_{11}, ... \quad A_{12}, ... \quad A_{13}, ..., \neg A_7\}$

(a) List the sequence of unit-propagations following after the last decision, each literal tagged (in square brackets) by its antecedent clause

(b) Derive the conflict clause via conflict analysis by means of the 1st-UIP technique

(c) Using the 1st-UIP backjumping strategy, update the list of literals above after the backjumping step and the unit-propagation of the UIP

5

Consider the following CNF formula:

$$
\begin{array}{lcccccl}
(& A_7 & & & & &) \wedge \\
(& A_8 & \vee & \neg A_7 & & &) \wedge \\
(& \neg A_4 & \vee & \neg A_7 & \vee & \neg A_5 &) \wedge \\
(& \neg A_8 & \vee & \neg A_6 & \vee & A_1 &) \wedge \\
(& A_2 & \vee & \neg A_7 & \vee & \neg A_6 &) \wedge \\
(& \neg A_8 & \vee & A_6 & \vee & \neg A_2 &) \wedge \\
(& \neg A_1 & \vee & \neg A_5 & \vee & \neg A_2 &) \wedge \\
(& A_2 & \vee & \neg A_6 & \vee & \neg A_8 &) \wedge \\
(& \neg A_1 & \vee & \neg A_5 & \vee & A_8 &) \wedge \\
(& A_2 & \vee & A_7 & \vee & \neg A_6 &) \wedge \\
(& \neg A_6 & \vee & A_4 & \vee & \neg A_6 &) \wedge \\
(& A_3 & \vee & A_8 & \vee & \neg A_7 &)
\end{array}
$$

Decide <u>*quickly*</u> if it is satisfiable or not, and briefly explain why.

6

Consider the following Boolean formulas:

$$\begin{array}{rll} \varphi_1 \stackrel{\text{def}}{=} & (\neg A_7 \vee \neg A_3) & \wedge \\ & (\ \ A_7 \vee \neg A_3) & \wedge \\ & (\ \ A_2) & \wedge \\ & (\neg A_2 \vee \neg A_4) & \end{array}$$

$$\begin{array}{rll} \varphi_2 \stackrel{\text{def}}{=} & (\ \ A_3 \vee \ \ A_5) & \wedge \\ & (\ \ A_4 \vee \neg A_1) & \wedge \\ & (\neg A_5 \vee \ \ A_1) & \end{array}$$

which are such that $\varphi_1 \wedge \varphi_2 \models \perp$. For each of the following formulas, say if it is a Craig interpolant for (φ_1, φ_2) or not.

(a)

$$\begin{array}{ll} (\neg A_7 \vee \neg A_3) & \wedge \\ (\ \ A_7 \vee \neg A_3) & \wedge \\ (\neg A_4) & \end{array}$$

(b)

$$(\neg A_4)$$

(c)

$$(\neg A_3 \wedge \neg A_4)$$

7

Consider the following formula in the theory $\mathcal{EUF}$ of linear arithmetic on the Rationals.

$$\begin{array}{rl}\varphi = & \{(f(x) = f(f(y))) \ \vee A_2\} \ \ \wedge \\ & \{\neg(h(x, f(y)) = h(g(x), y)) \ \vee \underline{\neg(h(x, g(z) = h(f(x), y)))} \ \vee \neg A_1\} \ \ \wedge \\ & \{A_1 \vee (h(x, y) = h(y, x))\} \ \ \wedge \\ & \{\underline{(x = f(x))} \ \vee A_3 \vee \neg A_1\} \ \ \wedge \\ & \{\underline{\neg(w(x) = g(f(y)))} \ \vee A_1\} \wedge \\ & \{\underline{\neg A_2} \vee (w(g(x)) = w(f(x)))\} \wedge \\ & \{A_1 \vee \underline{(y = g(z))} \ \vee A_2\}\end{array}$$

and consider the partial truth assignment μ given by the underlined literals above:

$$\{\neg(w(x) = g(f(y))), \neg A_2, \neg(h(x, g(z) = h(f(x), y))), (x = f(x)), (y = g(z))\}.$$

1. Does (the Boolean abstraction of) μ propositionally satisfy (the Boolean abstraction of) φ?
2. Is μ satisfiable in $\mathcal{EUF}$?
   (a) If no, find a minimal conflict set for μ and the corresponding conflict clause C.
   (b) If yes, show one unassigned literal which can be deduced from μ, and show the corresponding deduction clause C.

8

Consider the following set of clauses φ in the theory of linear arithmetic on the Integers $\mathcal{EUF}$.

$$\left\{ \begin{array}{l} (\neg(x = y) \vee \quad (f(x) = f(y))), \\ (\neg(x = y) \vee \neg(f(x) = f(y))), \\ (\quad (x = y) \vee \quad (f(x) = f(y))), \\ (\quad (x = y) \vee \neg(f(x) = f(y))) \end{array} \right\}$$

Say which of the following sets is a $\mathcal{EUF}$-unsatisfiable core of φ and which is not. For each one, explain why.

(a)

$$\left\{ \begin{array}{l} (\neg(x = y) \vee \neg(f(x) = f(y))), \\ (\quad (x = y) \vee \quad (f(x) = f(y))), \\ (\quad (x = y) \vee \neg(f(x) = f(y))) \end{array} \right\}$$

(b)

$$\left\{ \begin{array}{l} (\neg(x = y) \vee \quad (f(x) = f(y))), \\ (\quad (x = y) \vee \quad (f(x) = f(y))), \\ (\quad (x = y) \vee \neg(f(x) = f(y))) \end{array} \right\}$$

(c)

$$\left\{ \begin{array}{l} (\neg(x = y) \vee \neg(f(x) = f(y))), \\ (\quad (x = y) \vee \quad (f(x) = f(y))), \\ (\quad (x = y) \vee \neg(f(x) = f(y))), \\ ((x = f(y))) \end{array} \right\}$$

9

Let $\mathcal{LRA}$ be the logic of linear arithmetic over the rationals and $\mathcal{EUF}$ be the logic of equality and uninterpreted functions. Consider the following pure formula φ in the combined logic $\mathcal{LRA} \cup \mathcal{EUF}$:

$$(x = 1.0) \wedge (h = 1.0) \wedge (k = 1.0) \wedge (y = 2h - k) \wedge (z < w) \quad (1)$$
$$(z = f(x)) \wedge (w = f(y)) \quad (2)$$

Say which variables are interface variables, list the interface equalities for this formula (modulo symmetry), and decide whether this formulas is $\mathcal{LRA} \cup \mathcal{EUF}$-satisfiable or not, using either Nelson-Oppen or Delayed Theory Combination.

10

Consider the following formulas in difference logic ($\mathcal{DL}$):

$$
\begin{array}{rll}
\varphi_1 \stackrel{\text{def}}{=} & (x_2 - x_3 \le -4) & \wedge \\
& (x_3 - x_4 \le -6) & \wedge \\
& (x_5 - x_6 \le 4) & \wedge \\
& (x_6 - x_1 \le 2) & \wedge \\
& (x_6 - x_7 \le -2) & \wedge \\
& (x_7 - x_8 \le 1) &
\end{array}
$$

$$
\begin{array}{rll}
\varphi_2 \stackrel{\text{def}}{=} & (x_4 - x_9 \le 2) & \wedge \\
& (x_9 - x_5 \le 0) & \wedge \\
& (x_1 - x_2 \le 1) &
\end{array}
$$

which are such that $\varphi_1 \wedge \varphi_2 \models_{\mathcal{DL}} \perp$. For each of the following formulas, say if it is a Craig interpolant in $\mathcal{DL}$ for (φ_1, φ_2), and explain why.

(a) $\quad (x_2 - x_3 + x_6 - x_1 \le -2)$

(b) $\quad (x_2 - x_4 \le -10)$

(c) $\quad (x_2 - x_4 \le -10) \quad \wedge$
$\quad\quad (x_5 - x_1 \le 6)$