Course "Automated Reasoning"
TEST

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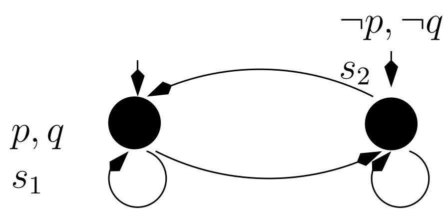
[COPY WITH SOLUTIONS]

1

Consider the following Kripke Model M:



For each of the following facts, say if it is true or false in CTL*.
[Solution: Recall that an LTL formula φ represents the same property as the CTL* formula $\mathbf{A}\varphi$.]

(a) $M \models \mathbf{A}(\mathbf{GF}p \rightarrow \mathbf{GF}q)$
[Solution: true]

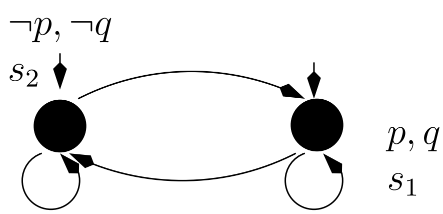
(b) $M \models \mathbf{A}(\mathbf{GF}p)$
[Solution: false]

(c) $M \models \mathbf{A}(\mathbf{FG}\neg p)$
[Solution: false]

(d) $M \models \mathbf{A}(\neg p \mathbf{U} q)$
[Solution: false]

2

Consider the following Kripke Model M:



For each of the following facts, say if it is true or false in CTL.

(a) $M \models \mathbf{EG}p$
[Solution: false]

(b) $M \models \mathbf{AF}\neg p$
[Solution: false]

(c) $M \models \mathbf{AGAF}q$
[Solution: false]

(d) $M \models \mathbf{E}(\neg p \mathbf{U} q)$
[Solution: true]

3

Let p, q be Boolean atoms. For each of the following LTL formulas, say if there exists a CTL formula representing the same property.

(a) $\bot\mathbf{R}(\mathbf{F}q)$
[Solution: Yes It rewrites into $\mathbf{GF}q$, which is equivalent to $\mathbf{AGAF}q$.]

(b) $\top\mathbf{U}(\mathbf{G}q)$
[Solution: No.It rewrites into $\mathbf{FG}q$, which is equivalent to no CTL formula.]

(c) $\mathbf{FG}p \to q$
[Solution: Yes. It rewrites into $\mathbf{GF}\neg p \lor q$, which is equivalent to $\mathbf{AGAF}\neg p \lor q$]

(d) $\mathbf{GF}p \to q$
[Solution: No. It rewrites into $\mathbf{FG}\neg p \lor q$, which is equivalent to no CTL formula.]

4

Consider CDCL SAT solving. For each of the following sentences, say if it is true or false.

(a) Let φ be the CNF input Boolean formula, and C denote a generic clause learned during the process. Then $\varphi \models C$.

[Solution: True]

(b) During the CDCL SAT solving process, the formula may contain an exponential number of learned clauses.

[Solution: False. Clauses are discharged according to their activity to avoid exponential blowups.]

(c) Let C be a conflict clause learned using the original backjumping&learning strategy. Then C contains at least one literal whose negation was unit-propagated in the current branch.

[Solution: False. In the decision criterion used by original CDCL solvers, C contains only decision literals.]

(d) Let C be a conflict clause learned using the state-of-the-art backjumping&learning strategy. Then C contains at most one literal whose negation was unit-propagated in the current branch.

[Solution: False. In the 1st-UIP criterion used by state-of-the-art CDCL solvers, C contains at most one literal whose negation was unit-propagated *at the last decision level* in the current branch.]

5

For each of the following facts regarding theories of interest for SMT, say if it is true or false

(a) The theory of equality and uninterpreted function symbols ($\mathcal{EUF}$) is stably-infinite.
[Solution: true]

(b) The theory of fixed-width bit-vectors ($\mathcal{BV}$) is stably-infinite.
[Solution: false]

(c) The theory of linear arithmetic over the rationals ($\mathcal{LRA}$) is convex.
[Solution: true]

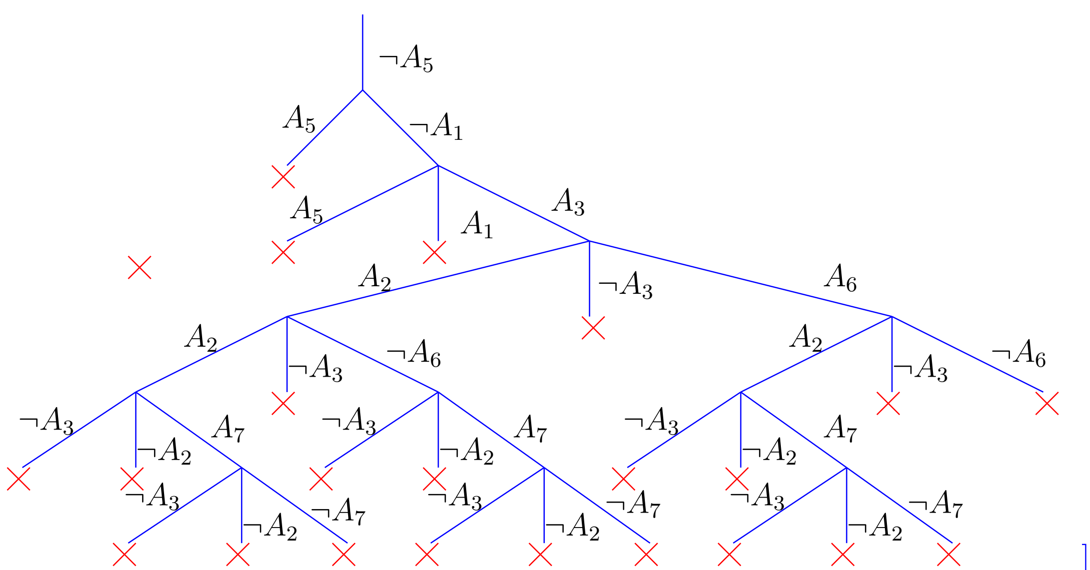
(d) The theory of linear arithmetic over the integers ($\mathcal{LIA}$) is convex.
[Solution: false]

6

Consider the following CNF formula in PL:

$$
\begin{array}{rcl}
(A_5 \vee A_1 \vee A_3) \wedge \\
(A_2 \vee \neg A_3 \vee A_6) \wedge \\
(A_5 \vee \neg A_1) \wedge \\
(A_2 \vee \neg A_3 \vee \neg A_6) \wedge \\
(\neg A_5) \wedge \\
(\neg A_3 \vee \neg A_2 \vee A_7) \wedge \\
(\neg A_3 \vee \neg A_2 \vee \neg A_7)
\end{array}
$$

(a) Draw the search tree obtained by applying to the above formula the tableaux algorithm.
Hint: when no better choice is available, clauses should be chosen in order, from top to down.
[Solution:



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(b) as a consequence, say if the formula is satisfiable or not.
[Solution: The tableau search tree is closed, thus the formula is unsatisfiable.]

7

Consider the following simple SMT($\mathcal{EUF} \cup \mathcal{LIA}$) formula:

$$\varphi \stackrel{\text{def}}{=} (x_1 - x_2 \geq 0) \wedge (x_1 - x_2 \leq 0) \wedge (f(x_1) < f(x_2))$$

(a) Purify the formula φ. Call φ' the resulting formula.

(b) List the interface variables and interface equalities of φ'. (Order the variables as x_1, x_2, x_3, x_4.)

(c) Using Nelson-Oppen technique, decide if the formula φ' is $\mathcal{EUF} \cup \mathcal{LIA}$-satisfiable or not.

[Solution:

(a)

$$\varphi' = (x_1 - x_2 \geq 0) \wedge (x_1 - x_2 \leq 0) \wedge (x_3 \stackrel{\text{def}}{=} f(x_1)) \wedge (x_4 \stackrel{\text{def}}{=} f(x_2)) \wedge (x_3 < x_4)$$

(b) The interface variables are:
$\{x_1, x_2, x_3, x_4\}$,
hence the interface equalities are:
$\{(x_1 = x_2), (x_1 = x_3), (x_1 = x_4), (x_2 = x_3), (x_2 = x_4), (x_3 = x_4)\}$.

(c)

$$\begin{array}{rcl} (x_1 - x_2 \geq 0) \wedge (x_1 - x_2 \leq 0) & \models_{\mathcal{LIA}} & (x_1 = x_2) \\ (x_1 = x_2) \wedge (x_3 \stackrel{\text{def}}{=} f(x_1)) \wedge (x_4 \stackrel{\text{def}}{=} f(x_2)) & \models_{\mathcal{EUF}} & (x_3 = x_4) \\ (x_3 = x_4) \wedge (x_3 < x_4) & \models_{\mathcal{LIA}} & \bot \end{array}$$

from which φ' is $\mathcal{EUF} \cup \mathcal{LIA}$-unsatisfiable.

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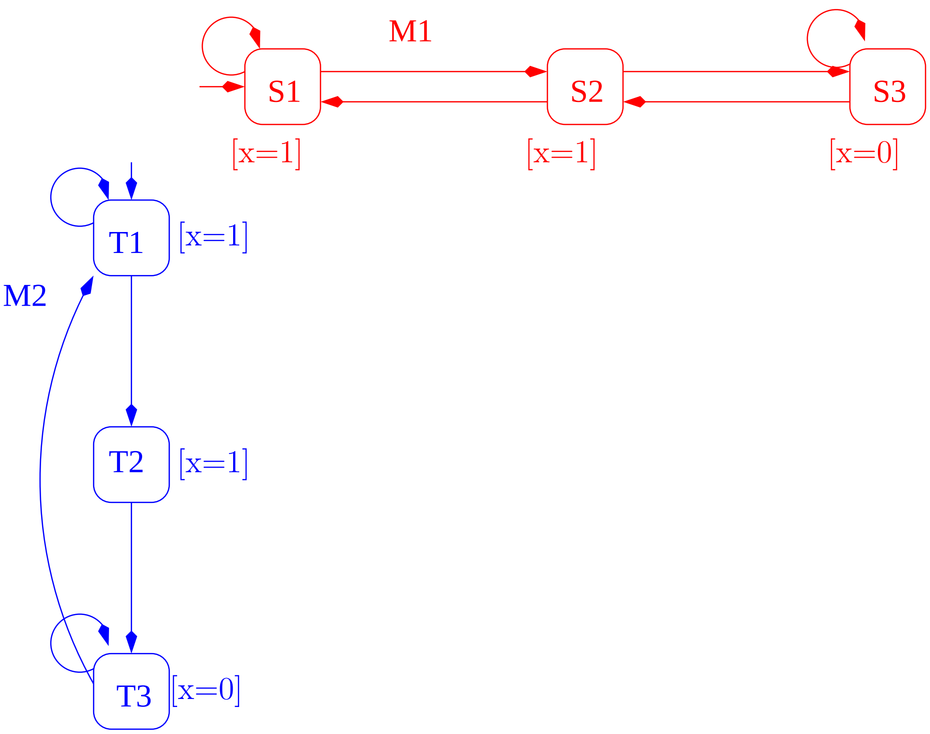
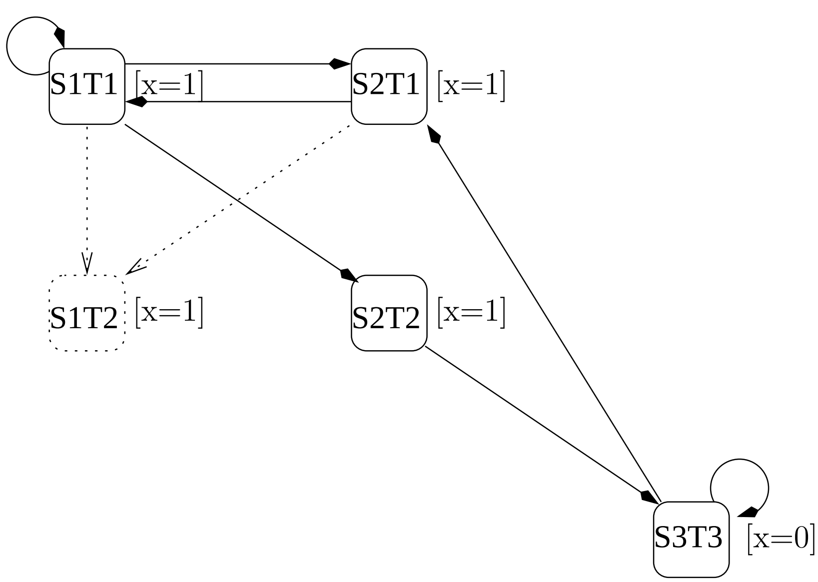
8

Consider the following two Kripke models $M1$ and $M2$, which share the variable x:



Compute and draw the graph of the synchronous product of $M1$ and $M2$.
Note: unreachable and deadend states should be removed.

[Solution:



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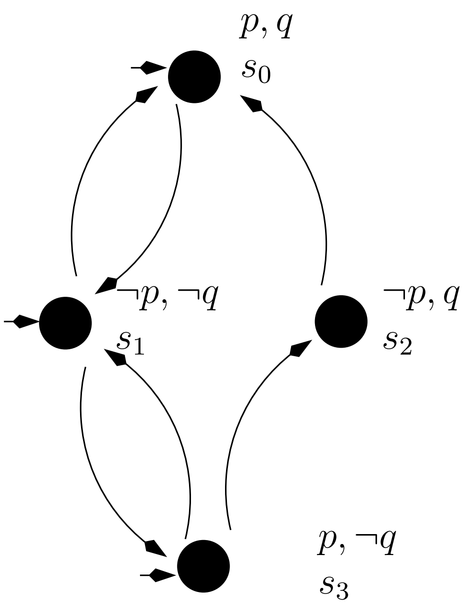
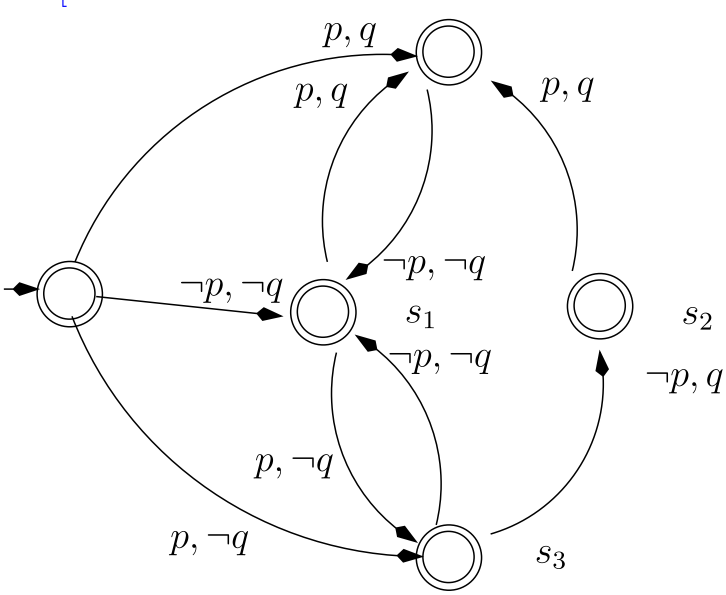


Consider the following Kripke model M:

Convert it into an equivalent Buchi automaton.

[Solution:



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10

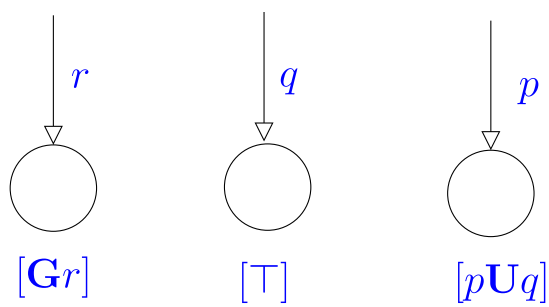
Consider the LTL formula $\varphi \stackrel{\text{def}}{=} (\neg p \mathbf{R} \neg q) \rightarrow \mathbf{G} r$

(a) rewrite φ into Negative Normal Form

[Solution: $(\neg p \mathbf{R} \neg q) \rightarrow \mathbf{G} r \Longrightarrow \neg(\neg p \mathbf{R} \neg q) \vee \mathbf{G} r \Longrightarrow (p \mathbf{U} q) \vee \mathbf{G} r$]

(b) find the initial states of a corresponding Generalized Büchi Automaton (for each state, define the labels of the incoming arcs and the "next" section.)

[Solution: Applying tableaux rules we obtain: $q \vee (p \wedge \mathbf{X}(p \mathbf{U} q)) \vee (r \wedge \mathbf{X} \mathbf{G} r)$, which is already in disjunctive normal form. This corresponds to the following three initial states:



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(c) How many distinct sets of accepting states will the final Generalized Büchi Automaton have?

[Solution: One, since there is one "**U**" subformulas occurring positively in φ.]