Course "Automated Reasoning" TEST

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[COPY WITH SOLUTIONS]

1

Let φ be a generic Boolean formula, and let $\varphi_1 \stackrel{\text{def}}{=} CNF(\varphi)$, s.c. $CNF()$ is the “classic” CNF conversion. Let $|\varphi|$ and $|\varphi_1|$ denote the size of φ and φ_1 respectively.

For each of the following sentences, say if it is true or false.

(a) If a DAG representation of formulas is used, then $|\varphi_1|$ is in worst-case polynomial in size wrt. $|\varphi|$. [Solution: False. $|\varphi_1|$ may grow exponentially wrt. $|\varphi|$, regardless the usage of DAG representations.]

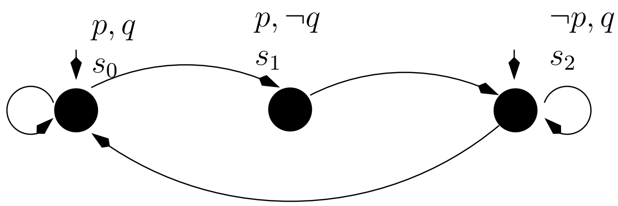
(b) If φ contains no $\leftrightarrow$'s, then $|\varphi_1|$ is in worst-case polynomial in size wrt. $|\varphi|$.
[Solution: False. $|\varphi_1|$ may grow exponentially wrt. $|\varphi|$, regardless the absence of $\leftrightarrow$'s.]

(c) If φ is valid, then φ_1 is valid.
[Solution: True.]

(d) If φ_1 is valid, then φ is valid.
[Solution: True]

2

Consider the following Kripke Model M:



For each of the following facts, say if it is true or false in LTL.

(a) $M \models \mathbf{F}p$
[Solution: false]

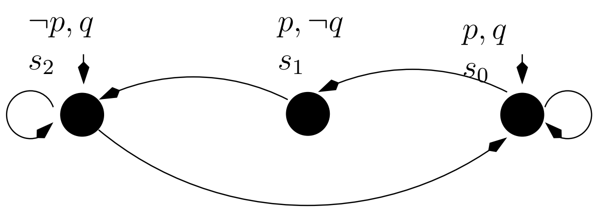
(b) $M \models \mathbf{G}\neg p$
[Solution: false]

(c) $M \models \mathbf{GF}\neg p$
[Solution: false]

(d) $M \models \mathbf{G}(p \vee q)$
[Solution: true]

3

Consider the following Kripke Model M:



For each of the following facts, say if it is true or false in CTL.

(a) $M \models \mathbf{EG}q$
[Solution: true]

(b) $M \models \mathbf{AF}p$
[Solution: false]

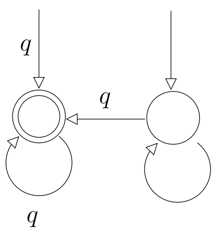
(c) $M \models \mathbf{AF}\neg q$
[Solution: false]

(d) $M \models (\mathbf{AGAF}\neg q)$
[Solution: false]

4

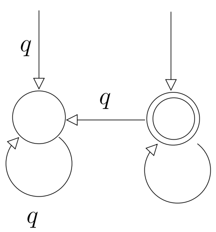
For each of the following fact regarding Buchi automata, say if it true or false.

(a) The following BA represents $\mathbf{FG}q$:



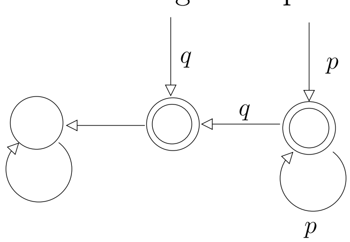
[Solution: True.]

(b) The following BA represents $\mathbf{FG}q$:



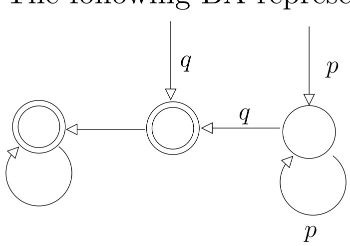
[Solution: False. It accepts every execution.]

(c) The following BA represents $p\mathbf{U}q$:



[Solution: No, it accepts $\mathbf{G}p$]

(d) The following BA represents $p\mathbf{U}q$:



[Solution: True]

5

Consider the following two $\mathcal{DL}$ formulas:
$\varphi_1 \stackrel{\text{def}}{=} (x_2 - x_1 \leq -6) \wedge (x_3 - x_2 \leq 5) \wedge (x_5 - x_4 \leq -4) \wedge (x_6 - x_5 \leq -7) \wedge (x_8 - x_7 \leq 4)$
$\varphi_2 \stackrel{\text{def}}{=} (x_4 - x_3 \leq 3) \wedge (x_7 - x_6 \leq -1) \wedge (x_1 - x_8 \leq 5)$

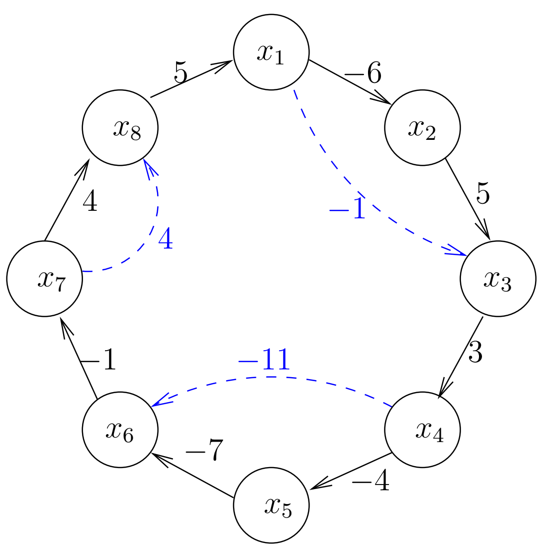
For each of the following facts, say if it is true or false

(a) The following is a $\mathcal{DL}$ interpolant of $\langle \varphi_1, \varphi_2 \rangle$
$(x_3 - x_1 \leq -1) \wedge (x_6 - x_4 \leq -11)$
[Solution: false]

(b) The following is a $\mathcal{LRA}$ interpolant of $\langle \varphi_1, \varphi_2 \rangle$:
$(x_3 - x_1 + x_6 - x_4 + x_8 - x_7 \leq -8)$
[Solution: true]

(c) The following is a $\mathcal{DL}$ interpolant of $\langle \varphi_1, \varphi_2 \rangle$:
$(x_3 - x_1 \leq -1) \wedge (x_6 - x_4 \leq -11) \wedge (x_8 - x_7 \leq 4)$
[Solution: true]

(d) The following is a $\mathcal{DL}$ interpolant of $\langle \varphi_1, \varphi_2 \rangle$
$(x_2 - x_1 \leq -6) \wedge (x_3 - x_2 \leq 5) \wedge (x_5 - x_4 \leq -4) \wedge (x_6 - x_5 \leq -7) \wedge$
$(x_4 - x_3 \leq 3) \wedge (x_7 - x_6 \leq -1) \wedge (x_1 - x_8 \leq 5) \wedge (x_8 - x_7 \leq 4)$
[Solution: false]

[Solution: ]

6

Consider the following Boolean formula φ:

$$\neg(((A_9 \to A_8) \wedge (\neg A_7 \to \neg A_4)) \vee ((\neg A_5 \to \neg A_6) \wedge (\neg A_7 \to A_8)))$$

1. Compute the Negative Normal Form of φ, called φ'.

   [Solution:

$$\begin{aligned}
& \varphi \\
\Longrightarrow\ & \neg(((A_9 \to A_8) \wedge (\neg A_7 \to \neg A_4)) \vee ((\neg A_5 \to \neg A_6) \wedge (\neg A_7 \to A_8))) \\
\Longrightarrow\ & (\neg((A_9 \to A_8) \wedge (\neg A_7 \to \neg A_4)) \wedge \neg((\neg A_5 \to \neg A_6) \wedge (\neg A_7 \to A_8))) \\
\Longrightarrow\ & ((\neg(A_9 \to A_8) \vee \neg(\neg A_7 \to \neg A_4)) \wedge (\neg(\neg A_5 \to \neg A_6) \vee \neg(\neg A_7 \to A_8))) \\
\Longrightarrow\ & (((A_9 \wedge \neg A_8) \vee (\neg A_7 \wedge A_4)) \wedge ((\neg A_5 \wedge A_6) \vee (\neg A_7 \wedge \neg A_8))) \\
=\ & \varphi'
\end{aligned}$$

   ]

2. For each of the following sentences, only one is true. Say which one.

   (a) φ and φ' are equivalent. [Solution: True]

   (b) φ and φ' are not necessarily equivalent. φ' has a model if and only φ has a model. [Solution: False]

   (c) There is no relation between the satisfiablity of φ and that of φ'. [Solution: False]

7

Let

$$\varphi \stackrel{\text{def}}{=} (\ A_2 \leftrightarrow \begin{pmatrix} (\ A_3 \vee \ A_6 \vee \ A_8) & \wedge \\ (\ A_5 \vee \ A_7 \vee \ A_8) & \wedge \\ (\neg A_4 \vee \neg A_6 \vee \neg A_8) & \wedge \\ (\neg A_6 \vee \ A_7 \vee \neg A_8) & \wedge \\ (\neg A_3 \vee \ A_6 \vee \ A_9) & \wedge \\ (\neg A_6 \vee \neg A_8 \vee \neg A_9) & \wedge \\ (\ A_3 \vee \ A_4 \vee \neg A_5) & \wedge \\ (\ A_5 \vee \ A_8 \vee \neg A_9) & \wedge \\ (\neg A_3 \vee \neg A_8 \vee \neg A_4) & \wedge \\ (\ A_6 \vee \ A_4 \vee \neg A_7) & \wedge \\ (\ A_5 \vee \ A_8 \vee \neg A_1) & \wedge \\ (\neg A_4 \vee \neg A_7 \vee \neg A_9) & \end{pmatrix}).$$

Using the variable ordering:

$$\text{"}\ A_1,\ \ A_3,\ \ A_4,\ \ A_5,\ \ A_6,\ \ A_7,\ \ A_8,\ \ A_9\text{"},$$

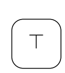
draw the OBDD corresponding to the formula φ' defined as:

$$\varphi' \stackrel{\text{def}}{=} \exists\ \ A_2.\varphi.$$

[Solution: Trivial, because φ is in the form "$(\ A_2 \leftrightarrow \psi)$", Thus:

$$\begin{array}{rlcl} \varphi' & \stackrel{\text{def}}{=} \exists\ A_2.(\ A_2 \leftrightarrow \psi) & & \\ & = ((\ A_2 \leftrightarrow \psi)[\ A_2 := \top]) & \vee & ((\ A_2 \leftrightarrow \psi)[\ A_2 := \bot]) \\ & = \psi & \vee & \neg\psi \\ & = \top & & \end{array}$$

which corresponds to the following OBDD:



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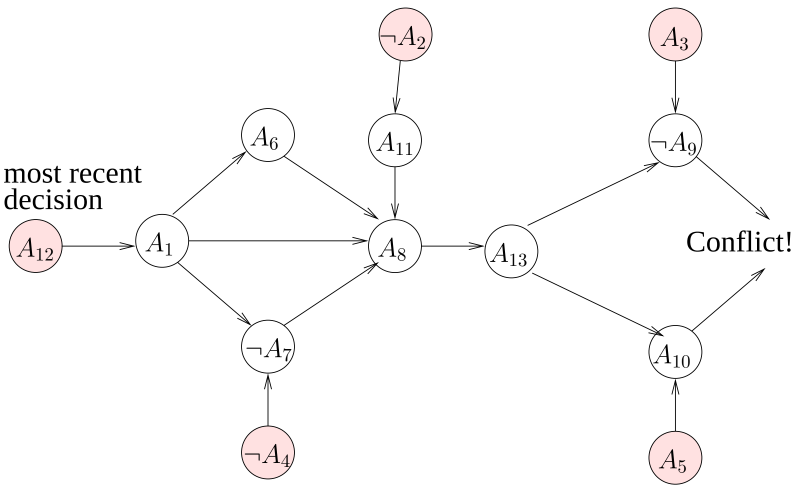
8

Consider the following implication graph:



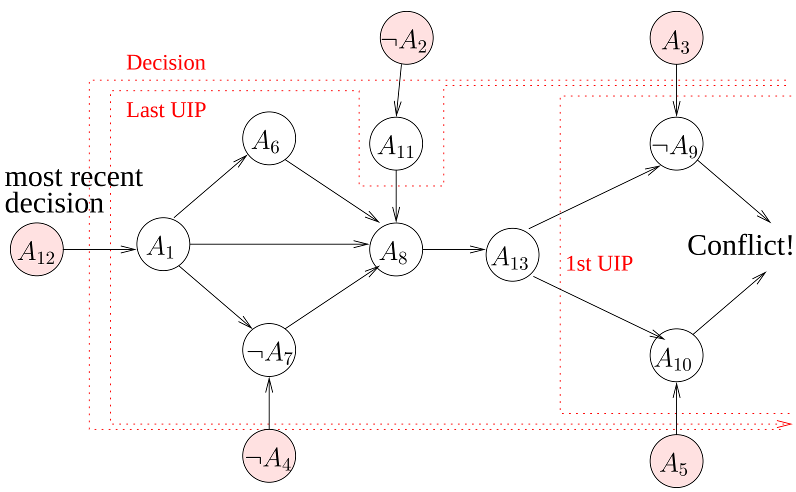
A_{12} being the most recent decision literal. Write the conflict clauses generated by

(a) the decision conflict analysis criterion

(b) the last UIP conflict analysis criterion

(c) the 1st UIP conflict analysis criterion

[Solution:



(a) Decision clause: $\neg A_{12} \vee A_2 \vee A_4 \vee \neg A_3 \vee \neg A_5$

(b) Last UIP clause: $\neg A_{12} \vee \neg A_{11} \vee A_4 \vee \neg A_3 \vee \neg A_5$

(c) 1st UIP clause: $\neg A_{13} \vee \neg A_3 \vee \neg A_5$

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9

Consider the following pair of SMT($\mathcal{LRA}$) sets of literals:

$$\begin{array}{lcl} A & \stackrel{\text{def}}{=} & \{(0 \leq -3x_1 - 5x_2 + 1), (0 \leq x_1 + x_2)\} \\ B & \stackrel{\text{def}}{=} & \{(0 \leq 3x_3 - 2x_1 - 3), (0 \leq x_1 - 2x_3 + 1)\}. \end{array}$$

(a) Write a proof P of $\mathcal{LRA}$-unsatisfiablity of $A \wedge B$

[Solution: A proof of unsatisfiability P for $A \wedge B$ is the following:

$$\dfrac{\dfrac{(0 \leq -3x_1 - 5x_2 + 1) \quad (0 \leq x_1 + x_2)}{\textsc{Comb}\ \ (0 \leq 2x_1 + 1)\ \textit{with c. 1 and 5}} \quad \dfrac{(0 \leq 3x_3 - 2x_1 - 3) \quad (0 \leq x_1 - 2x_3 + 1)}{\textsc{Comb}\ \ (0 \leq -x_1 - 3)\ \textit{with c. 2 and 3}}}{\textsc{Comb}\ \ (0 \leq -5)\ \textit{with c. 1 and 2}}$$

]

(b) From such a proof, compute a $\mathcal{LRA}$-interpolant for $\langle A, B\rangle$ using McMillan's technique.

[Solution: An interpolant $\langle A, B\rangle$ is the following:

$$\dfrac{\dfrac{(0 \leq -3x_1 - 5x_2 + 1) \quad (0 \leq x_1 + x_2)}{\textsc{Comb}\ \ (0 \leq 2x_1 + 1)\ \textit{with c. 1 and 5}} \quad \dfrac{(0 \leq 0) \quad (0 \leq 0)}{\textsc{Comb}\ \ (0 \leq 0)\ \textit{with c. 2 and 3}}}{\textsc{Comb}\ \ (0 \leq 2x_1 + 1)\ \textit{with c. 1 and 2}}$$

Thus, the interpolant obtained is $(0 \leq 2x_1 + 1)$.]

10

Consider the LTL formula $\varphi \stackrel{\text{def}}{=} p \vee q$, where p, q are atomic propositions. (Notice: LTL formula!) Compute the corresponding Generalized Büchi Automaton.

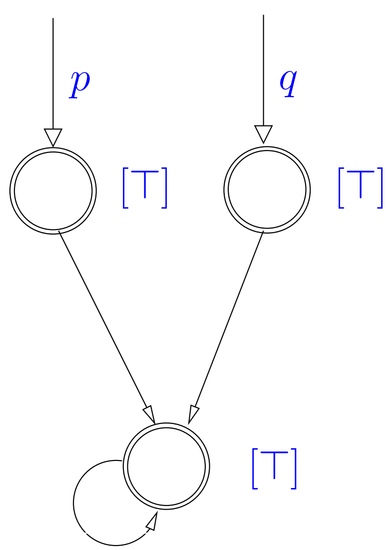
[Solution:

φ is already in DNF, the two disjuncts corresponding to two initial states:
$S_1 \stackrel{\text{def}}{=} \langle \{p\}, \{\top\}, \{p \vee q, p\} \rangle$
$S_2 \stackrel{\text{def}}{=} \langle \{q\}, \{\top\}, \{p \vee q, q\} \rangle$.

Then the expansion of their next part gives the "true state":
$s_3 \stackrel{\text{def}}{=} \langle \{\top\}, \{\top\}, \{\top\}.\rangle$

Since there is no until formula, there is only one group of accepting states including all states.

Thus, the resulting Büchi Automaton is the following:



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