Formal Methods:
Module II: Model Checking
Ch. 07: **SAT-Based Model Checking**

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Outline

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SAT-based Model Checking

- Key problems with BDD's:
  - they can explode in space
- A possible alternative:
  - Propositional Satisfiability Checking (SAT)
  - SAT technology is very advanced
- Advantages:
  - reduced memory requirements
  - limited sensitivity: one good setting, does not require expert users
  - much higher capacity (more variables) than BDD based techniques
- Various techniques: Bounded Model Checking (BMC), K-induction, Interpolant-based, IC3/PDR,...

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SAT-based Bounded Model Checking & K-Induction

Key Ideas:

- BMC: look for counter-example paths of increasing length k

  $\Longrightarrow$ oriented to finding bugs
- K-Induction: look for an induction proofs of increasing length k

  $\Longrightarrow$ oriented to prove correctness
- BMC [resp. K-induction]: for each k, build a Boolean formula that is satisfiable [resp. unsatisfiable] iff there is a counter-example [resp. proof] of length k
  - can be expressed using $k \cdot |\mathbf{s}|$ variables
  - formula construction is not subject to state explosion
- satisfiability of the Boolean formulas is checked by a SAT solver
  - can manage complex formulae on several 100K variables
  - returns satisfying assignment (i.e., a counter-example)
  - exploit incrementality

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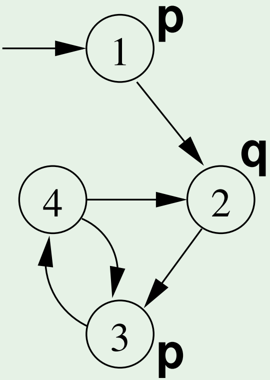
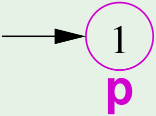
Bounded Model Checking: Example



- LTL Formula: $\mathbf{G}(p \rightarrow \mathbf{F}q)$
- Negated Formula (violation): $\mathbf{F}(p \wedge \mathbf{G}\neg q)$
- $k = 0$:



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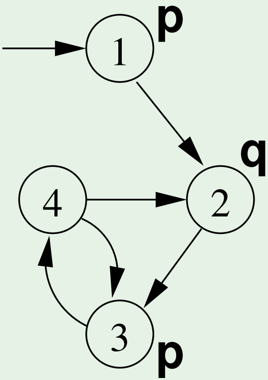

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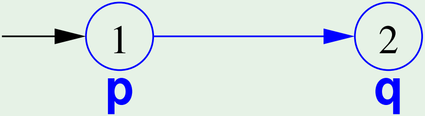

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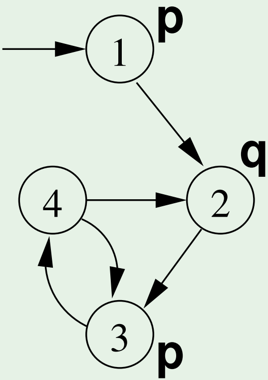

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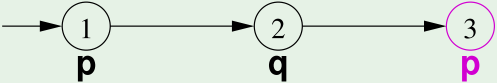
- $k = 2$:



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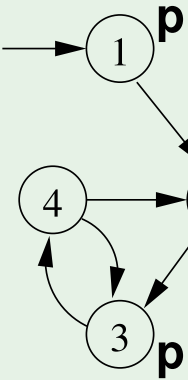
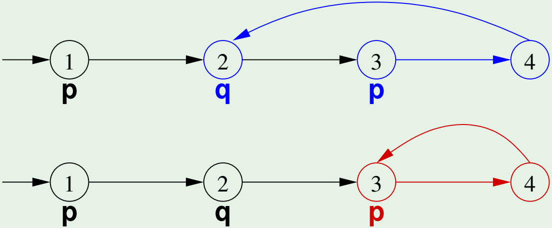


- LTL Formula: $\mathbf{G}(p \rightarrow \mathbf{F}q)$
- Negated Formula (violation): $\mathbf{F}(p \wedge \mathbf{G}\neg q)$
- $k = 3$:



- The 2nd trace is a counter-example!

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The problem [Biere et al, 1999]

Ingredients:

Assume states represented by an array s of n Boolean variables

- a system written as a Kripke structure $M := \langle I(s), R(s, s')\rangle$
- a property f written as a LTL formula
- an integer $k \geq 0$ (bound)

Problem

Is there an execution path π of M of length k satisfying the temporal property f?

$$M \models_k \mathbf{E}f$$

Note: f is the negation of the property in the LTL model checking problem $M \models \neg f$, and π is a counter-example of length k (bug).

- The check is repeated for increasing values of $k = 0, 1, 2, 3, ...$

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The encoding

Equivalent to the satisfiability problem of a Boolean formula $[[M, f]]_k$ defined as follows:

$$[[M, f]]_k \quad := \quad [[M]]_k \wedge [[f]]_k$$

$$[[M]]_k \quad := \quad I(s^0) \wedge \bigwedge_{i=0}^{k-1} R(s^i, s^{i+1}),$$

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- The vector s of propositional variables is replicated k+1 times s^0, s^1, ..., s^k
- $[\![M]\!]_k$ encodes the fact that the k-path is an execution of M
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The Encoding [cont.]

The encoding for a formula f with k steps, $[[f]]_k$ is the disjunction of

- The constraints needed to express a model without loopback:

$$(\neg(\bigvee_{l=0}^{k} R(s^k, s^l)) \wedge [[f]]_k^0)$$

  - $[[f]]_k^i,\ i \in [0, k]$: encodes the fact that f holds in s^i under the assumption that $s^0, ..., s^k$ is a no-loopback path
- The constraints needed to express a given loopback, for all possible points of loopback:
  - ${}_l[[f]]_k^i,\ i \in [0, k]$: encodes the fact that f holds in s^i under the assumption that $s^0, ..., s^k$ is a path with a loopback from s^k to s^l

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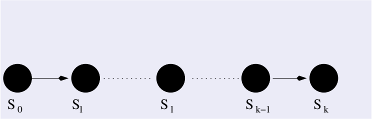

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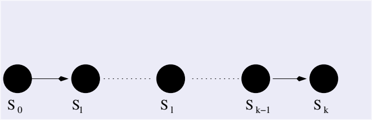

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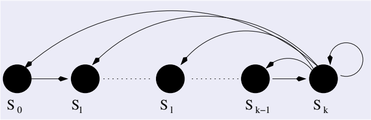

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The Encoding of $[[f]]_k^i$ and $_l[[f]]_k^i$

f	$[[f]]_k^i$	$_l[[f]]_k^i$
p	p_i	p_i
$\neg p$	$\neg p_i$	$\neg p_i$
$h \wedge g$	$[[h]]_k^i \wedge [[g]]_k^i$	$_l[[h]]_k^i \wedge {}_l[[g]]_k^i$
$h \vee g$	$[[h]]_k^i \vee [[g]]_k^i$	$_l[[h]]_k^i \vee {}_l[[g]]_k^i$
$\mathbf{X}g$	$[[g]]_k^{i+1}$ *if* $i < k$ $\bot$ *otherwise.*	$_l[[g]]_k^{i+1}$ *if* $i < k$ $_l[[g]]_k^l$ *otherwise.*
$\mathbf{G}g$	$\bot$	$\bigwedge_{j=min(i,l)}^{k} {}_l[[g]]_k^j$
$\mathbf{F}g$	$\bigvee_{j=i}^{k} [[g]]_k^j$	$\bigvee_{j=min(i,l)}^{k} {}_l[[g]]_k^j$
$h\mathbf{U}g$	$\bigvee_{j=i}^{k} \left([[g]]_k^j \wedge \bigwedge_{n=i}^{j-1} [[h]]_k^n \right)$	$\bigvee_{j=i}^{k} \left({}_l[[g]]_k^j \wedge \bigwedge_{n=i}^{j-1} {}_l[[h]]_k^n \right) \vee$ $\bigvee_{j=l}^{i-1} \left({}_l[[g]]_k^j \wedge \bigwedge_{n=i}^{k} {}_l[[h]]_k^n \wedge \bigwedge_{n=l}^{j-1} {}_l[[h]]_k^n \right)$
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Relevant Subcase: **F**p (reachability)

- $f := \mathbf{F}p$, s.t. p Boolean:
  is there a reachable state in which p holds?
- a finite path can show that the property holds
- $[[M, f]]_k$ is:

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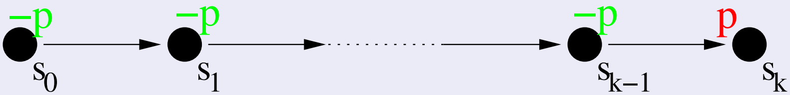

Important: incremental encoding

if done for increasing value of k, then it suffices that $[[M, f]]_k$ is:

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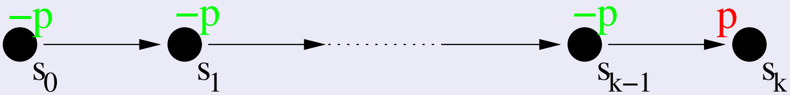

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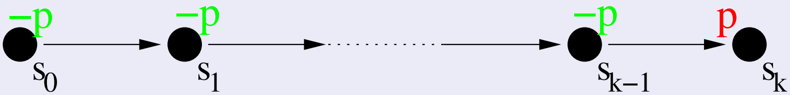

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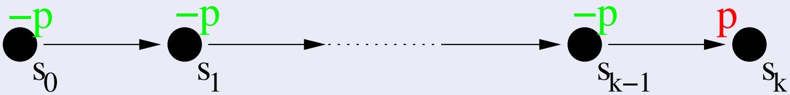

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Relevant Subcase: **G**p

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- We need to produce an infinite behaviour, with a finite number of transitions
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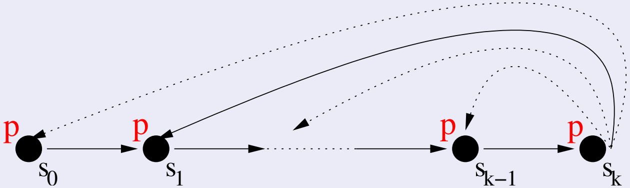

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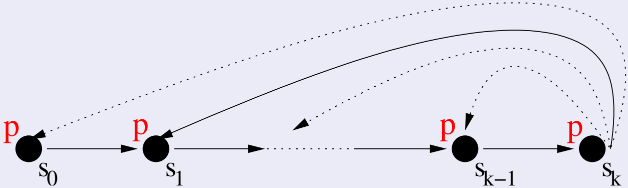

- $[[M, f]]_k$ is:

$$I(s^0) \wedge \bigwedge_{i=0}^{k-1} R(s^i, s^{i+1}) \wedge \bigvee_{l=0}^{k} R(s^k, s^l) \wedge \bigwedge_{j=0}^{k} p^j$$

Relevant Subcase: **GF**q (fair states)

- $f :=$ **GF**q, s.t. q Boolean: does q hold infinitely often?
- Again, we need to produce an infinite behaviour, with a finite number of transitions
- $[[M, f]]_k$ is:

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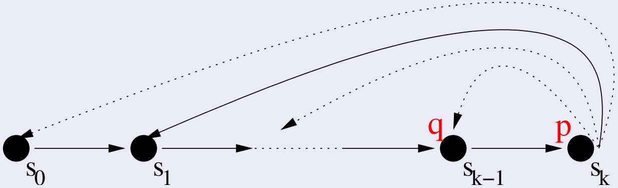

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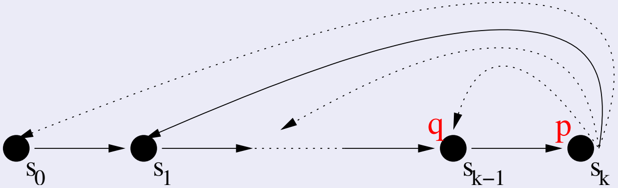

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Subcase Combination: $\mathbf{GF}q \wedge \mathbf{F}p$ (fair reachability)

- $f := \mathbf{GF}q \wedge \mathbf{F}p$, s.t. p, q Boolean: provided that q holds infinitely often, is there a reachable state in which p holds?
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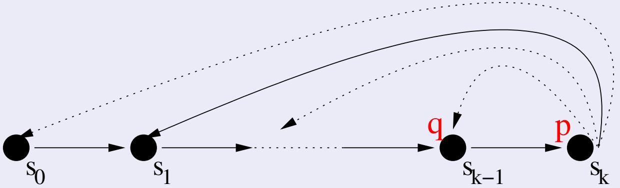

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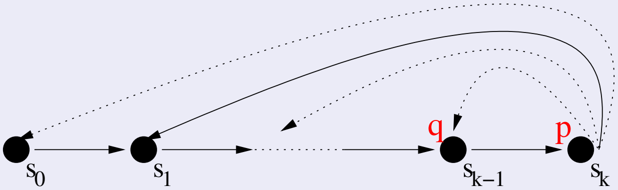

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Outline

Example: a bugged 3-bit shift register

- System M:
  - $I(x) := \neg x[0] \wedge \neg x[1] \wedge x[2]$
  - Correct R: $R(x, x') := (x'[0] \leftrightarrow x[1]) \wedge (x'[1] \leftrightarrow x[2]) \wedge (x'[2] \leftrightarrow 0)$
  - Bugged R: $R(x, x') := (x'[0] \leftrightarrow x[1]) \wedge (x'[1] \leftrightarrow x[2]) \wedge (x'[2] \leftrightarrow 1)$
- Property: $\mathbf{F}(\neg x[0] \wedge \neg x[1] \wedge \neg x[2])$
- BMC Problem: is there an execution π of $\mathcal{M}$ of length k s.t. $\pi \models \mathbf{G}((x[0] \vee x[1] \vee x[2]))$?

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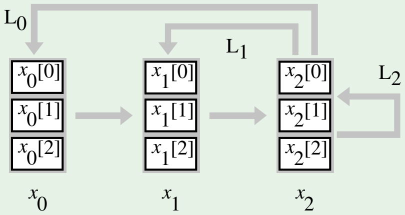
Example: a bugged 3-bit shift register [cont.]

$k = 0$:



I : $(\neg x_0[0] \wedge \neg x_0[1] \wedge x_0[2]) \wedge$

$\bigvee_{l=0}^{0} L_l$: $(\ ((x_0[0] \leftrightarrow x_0[1]) \ \wedge\ (x_0[1] \leftrightarrow x_0[2]) \ \wedge\ (x_0[2] \leftrightarrow 1))\ ) \ \wedge$

$\bigwedge_{i=0}^{0} (x \neq 0)$: $(\ (x_0[0] \vee x_0[1] \vee x_0[2])\ )$

$\Longrightarrow$ UNSAT: unit propagation:

$\neg x_0[0], \neg x_0[1], \quad x_0[2]$

$\Longrightarrow$ loop violated

Example: a bugged 3-bit shift register [cont.]

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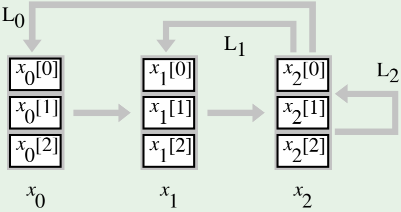

I : $(\neg x_0[0] \land \neg x_0[1] \land x_0[2]) \land$

$\bigvee_{l=0}^{0} L_l$: $(\ ((x_0[0] \leftrightarrow x_0[1]) \land (x_0[1] \leftrightarrow x_0[2]) \land (x_0[2] \leftrightarrow 1))\ ) \land$

$\bigwedge_{i=0}^{0}(x \neq 0)$: $(\ (x_0[0] \lor x_0[1] \lor x_0[2])\ )$

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Example: a bugged 3-bit shift register [cont.]

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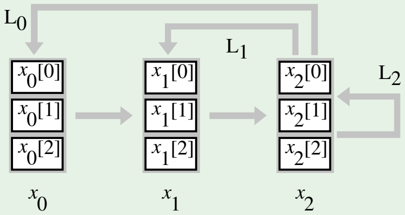

I : $(\neg x_0[0] \wedge \neg x_0[1] \wedge x_0[2]) \wedge$

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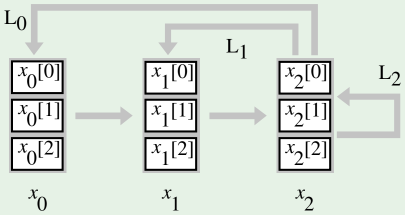
$\Longrightarrow$ loop violated

Example: a bugged 3-bit shift register [cont.]

$k = 1$:



I: $(\neg x_0[0] \wedge \neg x_0[1] \wedge x_0[2]) \wedge$

$[[M]]_1$: $(\ (x_1[0] \leftrightarrow x_0[1]) \ \wedge\ (x_1[1] \leftrightarrow x_0[2]) \ \wedge\ (x_1[2] \leftrightarrow 1)\ ) \ \wedge$

$\bigvee_{l=0}^{1} L_l$: $\left(\begin{array}{l} ((x_0[0] \leftrightarrow x_1[1]) \ \wedge\ (x_0[1] \leftrightarrow x_1[2]) \ \wedge\ (x_0[2] \leftrightarrow 1)) \vee \\ ((x_1[0] \leftrightarrow x_1[1]) \ \wedge\ (x_1[1] \leftrightarrow x_1[2]) \ \wedge\ (x_1[2] \leftrightarrow 1)) \end{array} \right) \wedge$

$\bigwedge_{i=0}^{1}(x \neq 0)$: $\left(\begin{array}{l} (x_0[0] \vee x_0[1] \vee x_0[2]) \wedge \\ (x_1[0] \vee x_1[1] \vee x_1[2]) \end{array} \right)$

$\Longrightarrow$ UNSAT: unit propagation:

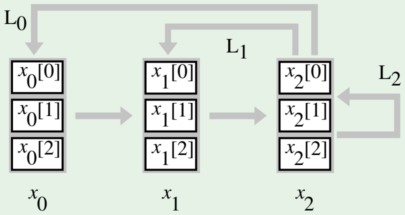
$\neg x_0[0], \neg x_0[1], \quad x_0[2]$

$\neg x_1[0], \quad x_1[1], \quad x_1[2]$

$\Longrightarrow$ both loop disjuncts violated

Example: a bugged 3-bit shift register [cont.]

$k = 1$:



I: $(\neg x_0[0] \wedge \neg x_0[1] \wedge x_0[2]) \wedge$

$[[M]]_1$: $(\ (x_1[0] \leftrightarrow x_0[1]) \wedge (x_1[1] \leftrightarrow x_0[2]) \wedge (x_1[2] \leftrightarrow 1)\ ) \wedge$

$\bigvee_{l=0}^{1} L_l$: $\left(\begin{array}{l} ((x_0[0] \leftrightarrow x_1[1]) \wedge (x_0[1] \leftrightarrow x_1[2]) \wedge (x_0[2] \leftrightarrow 1)) \vee \\ ((x_1[0] \leftrightarrow x_1[1]) \wedge (x_1[1] \leftrightarrow x_1[2]) \wedge (x_1[2] \leftrightarrow 1)) \end{array}\right) \wedge$

$\bigwedge_{i=0}^{1} (x \neq 0)$: $\left(\begin{array}{l} (x_0[0] \vee x_0[1] \vee x_0[2]) \wedge \\ (x_1[0] \vee x_1[1] \vee x_1[2]) \end{array}\right)$

$\Longrightarrow$ UNSAT: unit propagation:

$\neg x_0[0], \neg x_0[1], \quad x_0[2]$

$\neg x_1[0], \quad x_1[1], \quad x_1[2]$

$\Longrightarrow$ both loop disjuncts violated

Example: a bugged 3-bit shift register [cont.]

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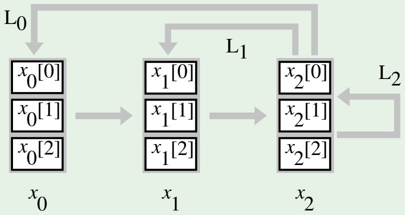

I: $(\neg x_0[0] \land \neg x_0[1] \land x_0[2]) \land$

$[[M]]_1$: $(\ (x_1[0] \leftrightarrow x_0[1]) \land (x_1[1] \leftrightarrow x_0[2]) \land (x_1[2] \leftrightarrow 1)\ ) \land$

$\bigvee_{l=0}^{1} L_l$: $\left(\begin{array}{l} ((x_0[0] \leftrightarrow x_1[1]) \land (x_0[1] \leftrightarrow x_1[2]) \land (x_0[2] \leftrightarrow 1)) \lor \\ ((x_1[0] \leftrightarrow x_1[1]) \land (x_1[1] \leftrightarrow x_1[2]) \land (x_1[2] \leftrightarrow 1)) \end{array} \right) \land$

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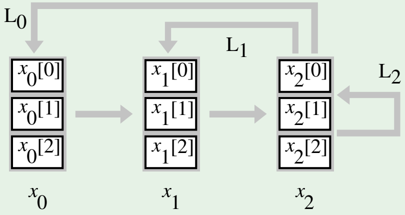
Example: a bugged 3-bit shift register [cont.]

$k = 2$:



I : $(\neg x_0[0] \land \neg x_0[1] \land x_0[2]) \land$

$[[M]]_2$: $\left(\begin{array}{l} (x_1[0] \leftrightarrow x_0[1]) \land (x_1[1] \leftrightarrow x_0[2]) \land (x_1[2] \leftrightarrow 1) \land \\ (x_2[0] \leftrightarrow x_1[1]) \land (x_2[1] \leftrightarrow x_1[2]) \land (x_2[2] \leftrightarrow 1) \end{array} \right) \land$

$\bigvee_{i=0}^{2} L_i$: $\left(\begin{array}{l} ((x_0[0] \leftrightarrow x_2[1]) \land (x_0[1] \leftrightarrow x_2[2]) \land (x_0[2] \leftrightarrow 1)) \lor \\ ((x_1[0] \leftrightarrow x_2[1]) \land (x_1[1] \leftrightarrow x_2[2]) \land (x_1[2] \leftrightarrow 1)) \lor \\ ((x_2[0] \leftrightarrow x_2[1]) \land (x_2[1] \leftrightarrow x_2[2]) \land (x_2[2] \leftrightarrow 1)) \end{array} \right) \land$

$\bigwedge_{i=0}^{2} (x \neq 0)$: $\left(\begin{array}{l} (x_0[0] \lor x_0[1] \lor x_0[2]) \land \\ (x_1[0] \lor x_1[1] \lor x_1[2]) \land \\ (x_2[0] \lor x_2[1] \lor x_2[2]) \end{array} \right)$

$\Longrightarrow$ SAT: $x_0[0] = x_0[1] = x_1[0] = 0;\ x_i[j] := 1\ \forall i, j$

Example: a bugged 3-bit shift register [cont.]

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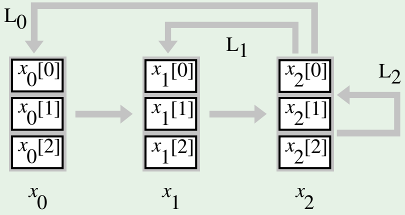

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$\bigwedge_{i=0}^{2}(x \neq 0)$: $\left(\begin{array}{l} (x_0[0] \vee x_0[1] \vee x_0[2]) \wedge \\ (x_1[0] \vee x_1[1] \vee x_1[2]) \wedge \\ (x_2[0] \vee x_2[1] \vee x_2[2]) \end{array}\right)$

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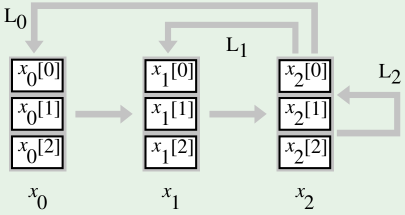

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$\bigwedge_{i=0}^{2} (x \neq 0)$: $\begin{pmatrix} (x_0[0] \vee x_0[1] \vee x_0[2]) \wedge \\ (x_1[0] \vee x_1[1] \vee x_1[2]) \wedge \\ (x_2[0] \vee x_2[1] \vee x_2[2]) \end{pmatrix}$

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Basic bounds for k

Theorem [Biere et al. TACAS 1999]

Let f be a LTL formula. $M \models \mathbf{E}f \iff M \models_k \mathbf{E}f$ for some $k \leq |M| \cdot 2^{|f|}$.

- $|M| \cdot 2^{|f|}$ is always a bound of k.
  - $|M|$ huge!

  $\Longrightarrow$ not so easy to compute in a symbolic setting.

$\Longrightarrow$ need to find better bounds!

Note: [Biere et al. TACAS 1999] use "$M \models \mathbf{E}f$" as "there exists a path of M verifying f", so that $M \not\models \neg f \iff M \models \mathbf{E}f$

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Other bounds for k

ACTL & ECTL

- ACTL is a subset of CTL in which "**A**..." (resp. "**E**...") sub-formulas occur only positively (resp. negatively) in each formula. (e.g. $\mathbf{AG}(p \rightarrow \mathbf{AGAF}q)$)
- Many frequently-used LTL properties $\neg f$ have equivalent ACTL representations $\mathbf{A}\neg f'$
  - e.g. $\mathbf{X}q \iff \mathbf{AX}q$, $\mathbf{G}q \iff \mathbf{AG}q$, $\mathbf{F}q \iff \mathbf{AF}q$, $p\mathbf{U}q \iff \mathbf{A}(p\mathbf{U}q)$, $\mathbf{GF}q \iff \mathbf{AGAF}q$, $\mathbf{G}(p \rightarrow \mathbf{GF}q) \iff \mathbf{AG}(p \rightarrow \mathbf{AGAF}q)$
- ECTL is a subset of CTL in which "**E**..." (resp. "**A**...") sub-formulas occur only positively (resp. negatively) in each formula. (e.g. $\mathbf{EF}(p \wedge \mathbf{EFEG}\neg q)$)
- ECTL is the dual subset of ACTL: $\phi \in ECTL \iff \neg\phi \in ACTL$.

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Let f be an ECTL formula. $M \models \mathbf{E}f \iff M \models_k \mathbf{E}f$ for some $k \leq |M|$.

Other bounds for k

ACTL & ECTL

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Given M, the diameter of M is the smallest integer d s.t. for every path $s_0, ..., s_{d+1}$ there exist a path $t_0, ..., t_l$ s.t. $l \leq d$, $t_0 = s_0$ and $t_l = s_{d+1}$.

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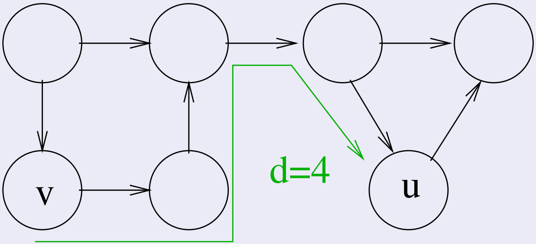

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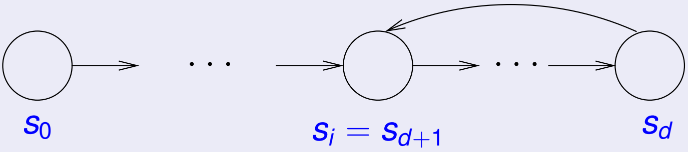

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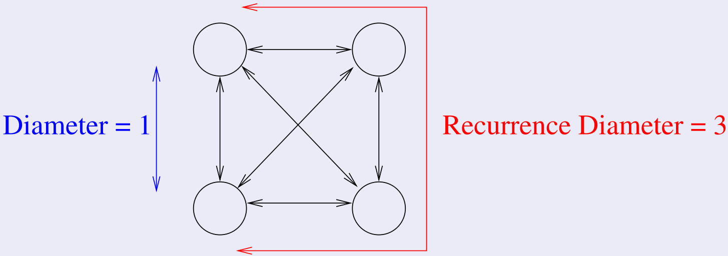

Outline

Bounded Model Checking: summary

- Incomplete technique:
  - if you find all formulas unsatisfiable, it tells you nothing
  - computing the maximum k (diameter) possible but extremely hard
- Very efficient for some problems (typically debugging)
- Lots of enhancements
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Efficiency Issues in Bounded Model Checking

- Incrementality:
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  - Reduced Boolean Circuits (RBC)
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Other Successful SAT-based MC Techniques

- Inductive reasoning on invariants (aka "K-Induction")
- Counter-example guided abstraction refinement (CEGAR)
  [Clarke et al. CAV 2002]
- Interpolant-based MC
  [Mc Millan, TACAS 2005]
- IC3/PDR
  [Bradley, VMCAI 2011]
- ...

For a survey see e.g.
[Amla et al., CHARME 2005, Prasad et al. STTT 2005].

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Invariant: "**G***Good*", *Good* being a Boolean formula

(i) If all the initial states are good,

(ii) and if from good states we only go to good states

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- $I(s^0) \rightarrow Good(s^0)$ is valid (i.e. its negation is unsatisfiable)

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"$(I(s^0) \wedge \neg Good(s^0))$" is step-0 incremental BMC encoding for $\mathbf{F}\neg Good$.

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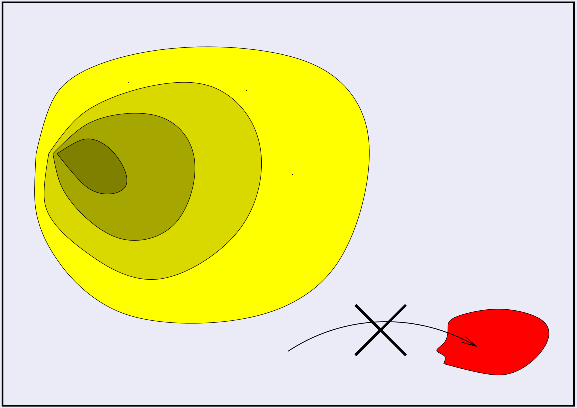

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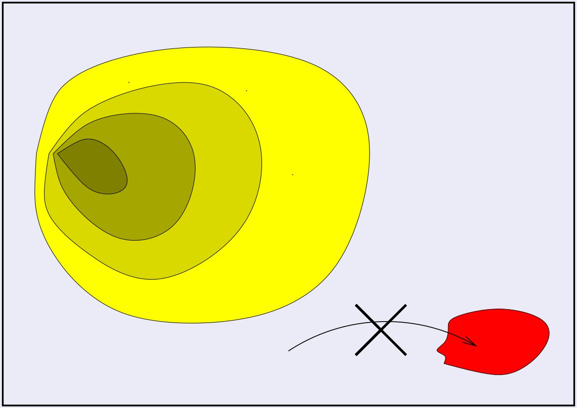

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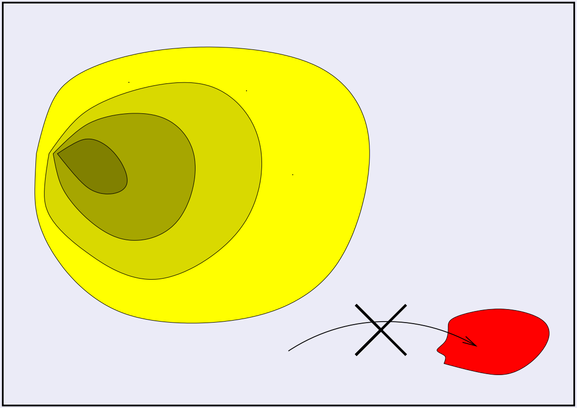

Strengthening of Invariants [cont.]

Solution (once you know you cannot reach $\neg Good$ in up to 1 step):

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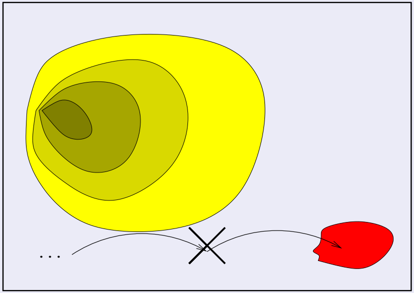

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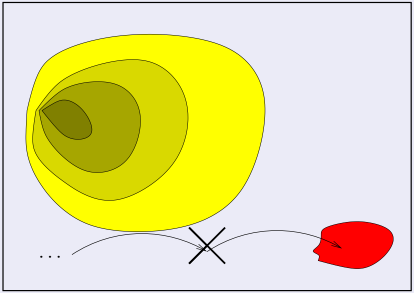

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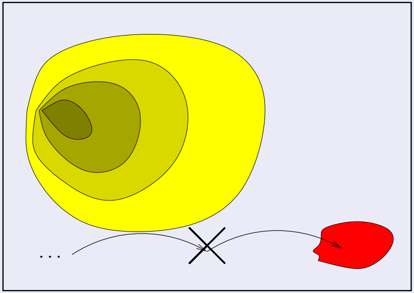

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- repeat for increasing values of the gap $1, 2, 3, 4, ...$
- intuition: increasingly tighten the constraint for "spurious" counterexamples: a spurious counterexample must be a chain $s_{k-n}, ..., s_k$ of unreachable and different states s.t. $\neg Good(s_k)$ and $R(s_i, s_{i+1}), \forall i$.
- dual to –and interleaved with– bounded model checking steps
- K-Induction steps can be shifted ($k \stackrel{\text{def}}{=} 0$) to share the subformulas: $\bigwedge_{i=0}^{k-1}(R(s^i, s^{i+1}) \wedge Good(s^i)) \wedge \neg Good(s^{k-2})$

Strengthening of Invariants [cont.]

$\Longrightarrow$ Check for the [un]satisfiability of the Boolean formulas:

$I(s^0) \wedge \neg Good(s^0);\ \ [BMC_0]$

$(Good(s^0) \wedge R(s^0, s^1)) \wedge \neg Good(s^1);\ \ [Kind_0]$

$I(s^0) \wedge (R(s^0, s^1) \wedge Good(s^0)) \wedge \neg Good(s^1);\ \ [BMC_1]$

$(Good(s^0) \wedge R(s^0, s^1) \wedge Good(s^1) \wedge R(s^1, s^2)) \wedge \neg Good(s^2)$
$\wedge \neg(s^0 = s^1);\ \ [Kind_1]$

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K-Induction Algorithm [Sheeran et al. 2000]

Algorithm

Given:

$$
\begin{aligned}
Base_n &:= I(\mathbf{s}_0) \wedge \bigwedge_{i=0}^{n-1} (R(\mathbf{s}_i, \mathbf{s}_{i+1}) \wedge \varphi(\mathbf{s}_i)) \wedge \neg\varphi(\mathbf{s}_n) \\
Step_n &:= \bigwedge_{i=0}^{n} (R(\mathbf{s}_i, \mathbf{s}_{i+1}) \wedge \varphi(\mathbf{s}_i)) \wedge \neg\varphi(\mathbf{s}_{n+1}) \\
Unique_n &:= \bigwedge_{0 \le i \le j \le n} \neg(\mathbf{s}_i = \mathbf{s}_{j+1})
\end{aligned}
$$

```
function CHECK_PROPERTY (I, R, φ)
    for n := 0, 1, 2, 3, .... do
        if (DPLL(Base_n) == SAT)
            then return PROPERTY_VIOLATED;
        else if (DPLL(Step_n ∧ Unique_n) == UNSAT)
            then return PROPERTY_VERIFIED;
    end for;
```

$\Longrightarrow$ reuses previous search if DPLL is incremental!!

K-Induction Algorithm [Sheeran et al. 2000]

Algorithm

Given:

$$Base_n := I(\mathbf{s}_0) \wedge \bigwedge_{i=0}^{n-1} (R(\mathbf{s}_i, \mathbf{s}_{i+1}) \wedge \varphi(\mathbf{s}_i)) \wedge \neg\varphi(\mathbf{s}_n)$$

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1. **function** CHECK_PROPERTY (I, R, φ)
2. **for** $n := 0, 1, 2, 3, ...$ **do**
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Step_n &:= \bigwedge_{i=0}^{n} \left(R(\mathbf{s}_i, \mathbf{s}_{i+1}) \wedge \varphi(\mathbf{s}_i)\right) \wedge \neg\varphi(\mathbf{s}_{n+1}) \\
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Outline

Example: a correct 3-bit shift register

- System M:
  - $I(x) := (\neg x[0] \wedge \neg x[1] \wedge \neg x[2])$
  - $R(x, x') := ((x'[0] \leftrightarrow x[1]) \wedge (x'[1] \leftrightarrow x[2]) \wedge (x'[2] \leftrightarrow 0))$
- Property: $\mathbf{G}\neg x[0]$

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Example: a correct 3-bit shift register [cont.]

- Init (BMC Step 0): $((\neg x^0[0] \land \neg x^0[1] \land \neg x^0[2]) \land x^0[0]) \Longrightarrow$ unsat
- K-Induction Step 1:

$$\left(\begin{array}{l} (\neg x^0[0] \land ((x^1[0] \leftrightarrow x^0[1]) \land (x^1[1] \leftrightarrow x^0[2]) \land (x^1[2] \leftrightarrow 0))) \\ \quad \land x^1[0] \end{array} \right)$$

$\Longrightarrow$ (partly by unit-propagation)

sat: $\left\{ \begin{array}{ccc} \neg x^0[0], & x^0[1], & x^0[2], \\ x^1[0], & x^1[1], & \neg x^1[2] \end{array} \right\}$

$\Longrightarrow$ not proved

Remark

Both $\{\neg x^0[0], \ x^0[1], \ x^0[2])\}$ and $\{ \ x^1[0], \ x^1[1], \neg x^1[2]\}$ are non-reachable.

Example: a correct 3-bit shift register [cont.]

- Init (BMC Step 0): $((\neg x^0[0] \land \neg x^0[1] \land \neg x^0[2]) \land x^0[0]) \Longrightarrow$ unsat
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Both $\{\neg x^0[0], \ x^0[1], \ x^0[2]])\}$ and $\{\ x^1[0], \ x^1[1], \neg x^1[2]\}$ are non-reachable.

Example: a correct 3-bit shift register [cont.]

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- K-Induction Step 1:

$$\left(\begin{array}{l} (\neg x^0[0] \land ((x^1[0] \leftrightarrow x^0[1]) \land (x^1[1] \leftrightarrow x^0[2]) \land (x^1[2] \leftrightarrow 0))) \\ \quad \land\, x^1[0] \end{array} \right)$$

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Remark

Both $\{\neg x^0[0], \ x^0[1], \ x^0[2])\}$ and $\{ \ x^1[0], \ x^1[1], \neg x^1[2]\}$ are non-reachable.

Example: a correct 3-bit shift register [cont.]

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$\Longrightarrow$ not proved

Remark

Both $\{\neg x^0[0], \ \ x^0[1], \ \ x^0[2])\}$ and $\{\ \ x^1[0], \ \ x^1[1], \neg x^1[2]\}$ are non-reachable.

Example: a correct 3-bit shift register [cont.]

- BMC Step 1: (...)$\Longrightarrow$ unsat
- K-Induction Step 2:

$$\left(\begin{array}{l} (\neg x^0[0] \wedge ((x^1[0] \leftrightarrow x^0[1]) \wedge (x^1[1] \leftrightarrow x^0[2]) \wedge (x^1[2] \leftrightarrow 0)) \wedge \\ \quad \neg x^1[0] \wedge ((x^2[0] \leftrightarrow x^1[1]) \wedge (x^2[1] \leftrightarrow x^1[2]) \wedge (x^2[2] \leftrightarrow 0)) \\ \ ) \wedge x^2[0] \end{array} \right)$$

$$\wedge \neg((x^1[0] \leftrightarrow x^0[0]) \wedge (x^1[1] \leftrightarrow x^0[1]) \wedge (x^1[2] \leftrightarrow x^0[2]))$$

$$\Longrightarrow \text{ sat: } \left\{ \begin{array}{lll} \neg x^0[0], & \neg x^0[1], & x^0[2] \\ \neg x^1[0], & x^1[1], & \neg x^1[2] \\ x^2[0], & \neg x^2[1], & \neg x^2[2] \end{array} \right\} \Longrightarrow \text{ not proved}$$

Remark

$\{\neg x^0[0], \neg x^0[1], x^0[2]\}$, $\{\neg x^1[0], x^1[1], \neg x^1[2]\}$, and $\{x^2[0], \neg x^2[1], \neg x^2[2]\}$ are non-reachable.

Example: a correct 3-bit shift register [cont.]

- BMC Step 1: (...)$\Longrightarrow$ unsat
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$\{\neg x^0[0], \neg x^0[1], \; x^0[2]\}$, $\{\neg x^1[0], \; x^1[1], \neg x^1[2]\}$, and $\{\; x^2[0], \neg x^2[1], \neg x^2[2]\}$ are non-reachable.

Example: a correct 3-bit shift register [cont.]

- BMC Step 2: (...) $\Longrightarrow$ unsat
- K-Induction Step 3:

$$\left(\begin{array}{l} (\neg x^0[0] \wedge ((x^1[0] \leftrightarrow x^0[1]) \wedge (x^1[1] \leftrightarrow x^0[2]) \wedge (x^1[2] \leftrightarrow 0))\wedge \\ \quad \neg x^1[0] \wedge ((x^2[0] \leftrightarrow x^1[1]) \wedge (x^2[1] \leftrightarrow x^1[2]) \wedge (x^2[2] \leftrightarrow 0))\wedge \\ \quad \neg x^2[0] \wedge ((x^3[0] \leftrightarrow x^2[1]) \wedge (x^3[1] \leftrightarrow x^2[2]) \wedge (x^3[2] \leftrightarrow 0)) \\ \ ) \wedge x^3[0] \end{array}\right)$$
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$\Longrightarrow$ (unit-propagation) $\{x^3[0], x^2[1], x^1[2]\}$

$\Longrightarrow$ unsat

$\Longrightarrow$ proved!

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Example: a correct 3-bit shift register [cont.]

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- K-Induction Step 3:

$$
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$$
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$\Longrightarrow$ unsat

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Outline

Ex: Bounded Model Checking

Given the symbolic representation of a FSM M, expressed in terms of the two Boolean formulas: $I(x, y) \stackrel{\text{def}}{=} \neg x \wedge y$,
$T(x, y, x', y') \stackrel{\text{def}}{=} (x' \leftrightarrow (x \leftrightarrow \neg y)) \wedge (y' \leftrightarrow \neg y)$, and the LTL property:
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   [Solution: The question corresponds to the Bounded Model Checking problem $M \models_2 \mathbf{E}\ \mathbf{F} f$, s.t. $f(x, y) \stackrel{\text{def}}{=} (x \wedge y)$. Thus we have:

$$
\begin{array}{lcl}
\neg x_0 \wedge y_0 & \wedge & // \ I(x_0, y_0) \wedge \\
(x_1 \leftrightarrow (x_0 \leftrightarrow \neg y_0)) \wedge (y_1 \leftrightarrow \neg y_0) & \wedge & // \ T(x_0, y_0, x_1, y_1) \wedge \\
(x_2 \leftrightarrow (x_1 \leftrightarrow \neg y_1)) \wedge (y_2 \leftrightarrow \neg y_1) & \wedge & // \ T(x_1, y_1, x_2, y_2) \wedge \\
((x_0 \wedge y_0) & \vee & // \ (f(x_0, y_0) \vee \\
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(x_2 \leftrightarrow (x_1 \leftrightarrow \neg y_1)) \wedge (y_2 \leftrightarrow \neg y_1) & \wedge & //\ T(x_1, y_1, x_2, y_2) \wedge \\
((x_0 \wedge y_0) & \vee & //\ (f(x_0, y_0) \vee \\
(x_1 \wedge y_1) & \vee & //\ \ f(x_1, y_1) \vee \\
(x_2 \wedge y_2)) & & //\ \ f(x_2, y_2))
\end{array}
$$

   ]

2. Is there a solution? If yes, find the corresponding execution; if no, show why.

Ex: Bounded Model Checking

Given the symbolic representation of a FSM M, expressed in terms of the two Boolean formulas: $I(x,y) \stackrel{\text{def}}{=} \neg x \wedge y$, $T(x,y,x',y') \stackrel{\text{def}}{=} (x' \leftrightarrow (x \leftrightarrow \neg y)) \wedge (y' \leftrightarrow \neg y)$, and the LTL property: $\varphi \stackrel{\text{def}}{=} \neg \mathbf{F}(x \wedge y)$,

1. Write a Boolean formula whose solutions (if any) represent executions of M of length 2 which violate φ.

   [Solution: The question corresponds to the Bounded Model Checking problem $M \models_2 \mathbf{E}\,\mathbf{F} f$, s.t. $f(x,y) \stackrel{\text{def}}{=} (x \wedge y)$. Thus we have:

$$
\begin{array}{lcl}
\neg x_0 \wedge y_0 & \wedge & // \ I(x_0,y_0) \wedge \\
(x_1 \leftrightarrow (x_0 \leftrightarrow \neg y_0)) \wedge (y_1 \leftrightarrow \neg y_0) & \wedge & // \ T(x_0,y_0,x_1,y_1) \wedge \\
(x_2 \leftrightarrow (x_1 \leftrightarrow \neg y_1)) \wedge (y_2 \leftrightarrow \neg y_1) & \wedge & // \ T(x_1,y_1,x_2,y_2) \wedge \\
((x_0 \wedge y_0) & \vee & // \ (f(x_0,y_0) \vee \\
(x_1 \wedge y_1) & \vee & // \ \ f(x_1,y_1) \vee \\
(x_2 \wedge y_2)) & & // \ \ f(x_2,y_2))
\end{array}
$$

   ]

2. Is there a solution? If yes, find the corresponding execution; if no, show why.

   [Solution: Yes: $\{\neg x_0, y_0, x_1, \neg y_1, x_2, y_2\}$, corresponding to the execution: $(0,1) \rightarrow (1,0) \rightarrow (1,1)$]

Ex: Bounded Model Checking

Ex: Bounded Model Checking

3. From the solutions to question #1 and #2 we can conclude that:

   (a) $M \models \varphi$

   (b) $M \not\models \varphi$

   (c) we can conclude nothing.

Ex: Bounded Model Checking

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   [Solution: b)]

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   [Solution: b)]
4. What are the diameter and the recurrence diameter of this system?

Ex: Bounded Model Checking

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   [Solution: b)]
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   [Solution:

   ]

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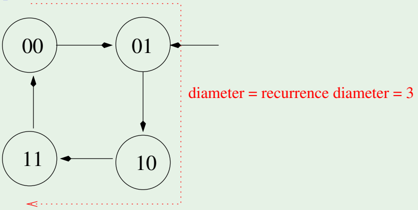
   [Solution: b)]
4. What are the diameter and the recurrence diameter of this system?

   [Solution:



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