Course “Automated Reasoning”
TEST

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[COPY WITH SOLUTIONS]

1

Consider the propositional semantics of "$\alpha \to \beta$" ("α implies β").

For each of the following facts, say whether it is true or false under the standard interpretation of "even", "odd", "Rome", "Florence", "Tunisi", "Bangkok", "Asia", "Africa", "is", "is in", and of the propositions "[...]" built on top of them.

(a) [5 is even] $\to$ [Rome is in Asia].
[Solution: true]

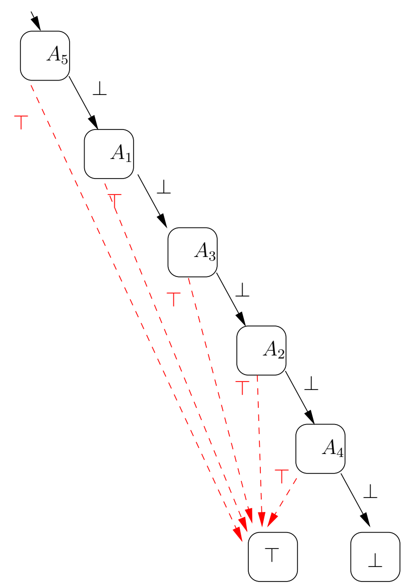
(b) [5 is odd] $\to$ [Florence is in Italy].
[Solution: true]

(c) [5 is odd] $\to$ [Rome is in Asia].
[Solution: false]

(d) [Rome is in Asia] $\to$ [5 is odd]
[Solution: true]

2

Given the following OBDD, with the ordering $\{\ A_5,\ \ A_1,\ \ A_3,\ \ A_2,\ \ A_4\}$,



for each of the following Boolean formulas, say whether the OBDD represents it or not.

(a) $(\neg A_5 \rightarrow (\neg A_1 \rightarrow (\neg A_3 \rightarrow (\neg A_2 \rightarrow \ \ A_4))))$
[Solution: true]

(b) $(\ \ A_2 \vee \ \ A_1 \vee \ \ A_5 \vee \ \ A_3 \vee \ \ A_4)$
[Solution: true]

(c) $(\ \ A_3 \wedge \ \ A_5 \wedge \ \ A_4 \wedge \ \ A_1 \wedge \ \ A_2)$
[Solution: false]

(d) $(\ \ A_5 \rightarrow (\ \ A_1 \rightarrow (\ \ A_3 \rightarrow (\ \ A_2 \rightarrow \neg A_4))))$
[Solution: false]

3

Consider the following two $\mathcal{DL}$ formulas:
$\varphi_1 \stackrel{\text{def}}{=} (x_2 - x_1 \leq 5) \wedge (x_3 - x_2 \leq -6) \wedge (x_5 - x_4 \leq -4) \wedge (x_6 - x_5 \leq -7) \wedge (x_8 - x_7 \leq 4)$
$\varphi_2 \stackrel{\text{def}}{=} (x_4 - x_3 \leq 3) \wedge (x_7 - x_6 \leq -1) \wedge (x_1 - x_8 \leq 5)$

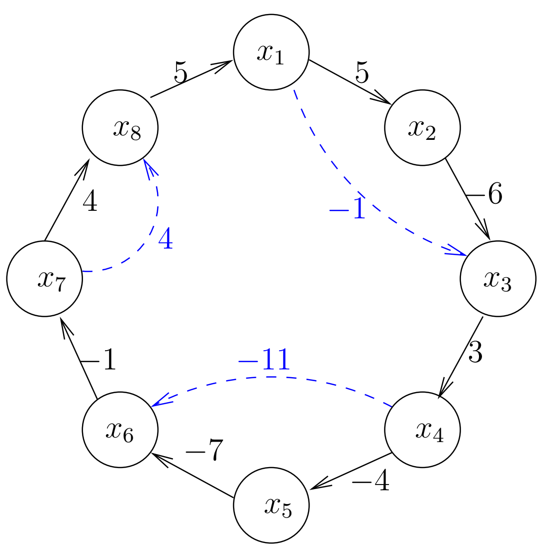
For each of the following facts, say if it is true or false

(a) The following is a $\mathcal{DL}$ interpolant of $\langle \varphi_1, \varphi_2 \rangle$:
$(x_3 - x_1 \leq -1) \wedge (x_6 - x_4 \leq -11) \wedge (x_8 - x_7 \leq 4)$
[Solution: true]

(b) The following is a $\mathcal{LRA}$ interpolant of $\langle \varphi_1, \varphi_2 \rangle$:
$(x_3 - x_1 + x_6 - x_4 + x_8 - x_7 \leq -8)$
[Solution: true]

(c) The following is a $\mathcal{DL}$ interpolant of $\langle \varphi_1, \varphi_2 \rangle$
$(x_2 - x_1 \leq 5) \wedge (x_3 - x_2 \leq -6) \wedge (x_5 - x_4 \leq -4) \wedge (x_6 - x_5 \leq -7) \wedge$
$(x_4 - x_3 \leq 3) \wedge (x_7 - x_6 \leq -1) \wedge (x_1 - x_8 \leq 5)(x_8 - x_7 \leq 4)$
[Solution: false]

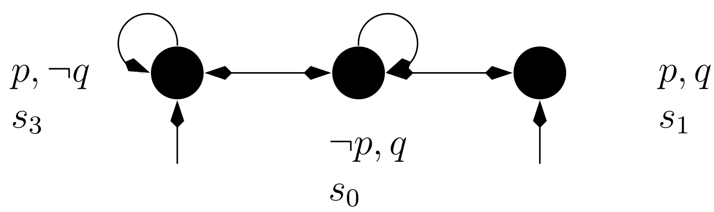
(d) The following is a $\mathcal{DL}$ interpolant of $\langle \varphi_1, \varphi_2 \rangle$
$(x_3 - x_1 \leq -1) \wedge (x_6 - x_4 \leq -11)$
[Solution: false]



[Solution:]

4

Consider the following Kripke Model M:



For each of the following facts, say if it is true or false in LTL.

(*a*) $M \models \mathbf{GF}p$
[Solution: false]

(*b*) $M \models \mathbf{FG}\neg p$
[Solution: false]

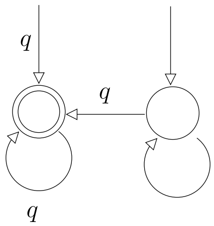
(*c*) $M \models p\mathbf{U}q$
[Solution: false]

(*d*) $M \models (\mathbf{GF}\neg p \wedge \mathbf{GF}\neg q) \rightarrow p$
[Solution: true]

5

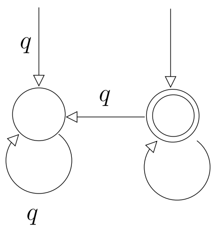
For each of the following fact regarding Buchi automata, say if it true or false.

(a) The following BA represents $\mathbf{FG}q$:



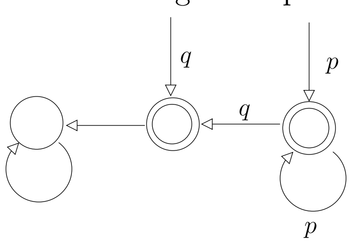
[Solution: Yes.]

(b) The following BA represents $\mathbf{FG}q$:



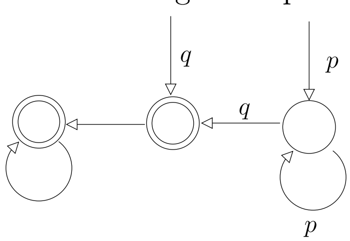
[Solution: No, it accepts every execution.]

(c) The following BA represents $p\mathbf{U}q$:



[Solution: No, it accepts $\mathbf{G}p$]

(d) The following BA represents $p\mathbf{U}q$:



[Solution: Yes]

6

Consider the following propositional formula φ:

$$((\neg A_5 \wedge \ \ A_1) \vee (\ \ A_7 \wedge \ \ A_2) \vee (\neg A_3 \wedge \neg A_1) \vee (\ \ A_4 \wedge \neg A_2))$$

1. Using the <u>*improved*</u> CNF_{label} conversion, produce the CNF formula $CNF_{label}(\varphi)$.

   [Solution: We introduce fresh Boolean variables naming the subformulas of φ:

$$(\overbrace{\overbrace{(\neg A_5 \wedge \ \ A_1)}^{B_1} \vee \overbrace{(\ \ A_7 \wedge \ \ A_2)}^{B_2} \vee \overbrace{(\neg A_3 \wedge \neg A_1)}^{B_3} \vee \overbrace{(\ \ A_4 \wedge \neg A_2)}^{B_4}}^{B})$$

   from which we obtain:

$$\begin{array}{ll} (B) & \wedge \\ (\neg B \vee B_1 \vee B_2 \vee B_3 \vee B_4) & \wedge \\ (\neg B_1 \vee \neg A_5) \wedge (\neg B_1 \vee \ \ A_1) & \wedge \\ (\neg B_2 \vee \ \ A_7) \wedge (\neg B_2 \vee \ \ A_2) & \wedge \\ (\neg B_3 \vee \neg A_3) \wedge (\neg B_3 \vee \neg A_1) & \wedge \\ (\neg B_4 \vee \ \ A_4) \wedge (\neg B_4 \vee \neg A_2) & \end{array} .$$

   ]

2. For each of the following sentences, only one is true. Say which one.

   (a) φ and $CNF_{label}(\varphi)$ are equivalent. [Solution: False]

   (b) φ and $CNF_{label}(\varphi)$ are not necessarily equivalent. $CNF_{label}(\varphi)$ has a model if and only φ has a model. [Solution: true]

   (c) There is no relation between the satisfiablity of φ and that of $CNF_{label}(\varphi)$. [Solution: False]

7

Consider the following piece of a much bigger formula, which has been fed to a CDCL SAT solver:

$$\begin{array}{ll}
c_1: & \neg A_9 \vee \quad A_{12} \vee \neg A_1 \\
c_2: & \quad A_9 \vee \neg A_7 \vee \neg A_3 \\
c_3: & \neg A_{11} \vee \quad A_5 \vee \quad A_2 \\
c_4: & \neg A_{10} \vee \neg A_{12} \vee \quad A_{11} \\
c_5: & \neg A_{11} \vee \quad A_6 \vee \quad A_4 \\
c_6: & \neg A_9 \vee \quad A_{10} \vee \neg A_1 \\
c_7: & \quad A_9 \vee \quad A_8 \vee \neg A_3 \\
c_8: & \neg A_5 \vee \neg A_6 \\
c_9: & \quad A_7 \vee \neg A_8 \vee \quad A_{13} \\
\ldots &
\end{array}$$

Suppose the solver has decided, in order, the following literals (possibly interleaved by others not occurring in the above clauses):

$\{\ldots, \quad A_1, \ldots \neg A_2, \ldots \neg A_4, \ldots \quad A_3, \ldots \neg A_{13}, \ldots, \quad A_9\}$

(*a*) List the sequence of unit-propagations following after the last decision, each literal tagged (in square brackets) by its antecedent clause

[Solution:

$$\begin{array}{ll}
A_{12} & [c_1] \\
A_{10} & [c_6] \\
A_{11} & [c_4] \\
A_5 & [c_3] \\
A_6 & [c_5] \\
\multicolumn{2}{l}{conflict\ on\ c_8}
\end{array}$$

]

(*b*) Derive the conflict clause via conflict analysis by means of the 1st-UIP technique

[Solution:

$$\dfrac{\neg A_{11} \vee \quad A_5 \vee \quad A_2 \qquad \dfrac{\neg A_{11} \vee \quad A_6 \vee \quad A_4 \qquad \overbrace{\neg A_5 \vee \neg A_6}^{Conflicting\ cl.}}{\neg A_{11} \vee \neg A_5 \vee \quad A_4}\ (\quad A_6)}{\underbrace{\neg A_{11}}_{1st\ UIP} \vee \quad A_2 \vee \quad A_4}\ (\quad A_5)$$

]

(*c*) Using the 1st-UIP backjumping strategy, update the list of literals above after the backjumping step and the unit-propagation of the UIP

[Solution: $\{\ldots, \quad A_1, \ldots \neg A_2, \ldots \neg A_4, \neg A_{11}\}$]

8

Consider the following SMT formula in the theory of linear arithmetic on the rationals ($\mathcal{LRA}$).

$$
\begin{array}{ll}
\varphi = & \{(v_1 - v_2 \leq 3) \ \vee A_2\} \quad \wedge \\
& \{\neg(2v_3 + v_4 \geq 5) \ \vee \underline{\neg(v_1 - v_3 \leq 6)} \ \vee \neg A_1\} \ \wedge \\
& \{A_1 \vee (v_1 - v_2 \leq 3)\} \quad \wedge \\
& \{\underline{(v_2 - v_4 \leq 6)} \ \vee (v_5 = 5 - 3v_4) \vee \neg A_1\} \ \wedge \\
& \{\underline{\neg(v_2 - v_3 > 2)} \ \vee A_1\} \wedge \\
& \{\underline{\neg A_2} \vee (v_1 - v_5 \leq 1)\} \wedge \\
& \{A_1 \vee \underline{(v_3 = v_5 + 6)} \ \vee A_2\}
\end{array}
$$

and consider the partial truth assignment μ given by the underlined literals above:

$$\{\neg(v_2 - v_3 > 2), \neg A_2, \neg(v_1 - v_3 \leq 6), (v_2 - v_4 \leq 6), (v_3 = v_5 + 6)\}.$$

1. Does (the Boolean abstraction of) μ propositionally satisfy (the Boolean abstraction of) φ?
2. Is μ satisfiable in $\mathcal{LRA}$?
   (a) If no, find a minimal conflict set for μ and the corresponding conflict clause C.
   (b) If yes, show one unassigned literal which can be deduced from μ, and show the corresponding deduction clause C.

[Solution:

1. No, since there are two clauses which are not satisfied.
2. Yes (there is no cycle among the constraints). $v_2 = v_3 = v_4 = 0.0$, $v_1 = 7.0$, $v_5 = -6.0$ is a solution.
   (a) ...
   (b) One possible deduction is:

   $$\{\neg(v_2 - v_3 > 2), \neg(v_1 - v_3 \leq 6)\} \models_{\mathcal{T}} \neg(v_1 - v_2 \leq 3).$$

   which corresponds to learning the deduction clause:

   $$(v_2 - v_3 > 2) \vee (v_1 - v_3 \leq 6) \vee \neg(v_1 - v_2 \leq 3).$$

   Another possible deduction is:

   $$\{(v_3 = v_5 + 6), \neg(v_1 - v_3 \leq 6)\} \models_{\mathcal{T}} \neg(v_1 - v_5 \leq 1).$$

   which corresponds to learning the deduction clause:

   $$\neg(v_3 = v_5 + 6) \vee (v_1 - v_3 \leq 6) \vee \neg(v_1 - v_5 \leq 1).$$

]

9

For each of the following FOL formulas, compute its CNF-Ization.
Use symbols $C_1, C_2, C_3, ...$ for Skolem constants and symbols $F_1, F_2, F_3, ...$ for Skolem functions.

(a) $\exists x.\forall y.\exists z.RepairsWith(x, y, z)$
[Solution:
$RepairsWith(C_1, y, F_1(x))$]

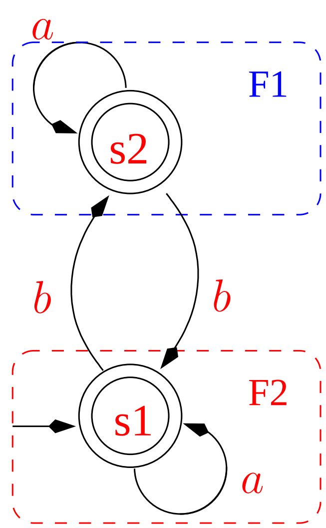
(b) $(\exists x.\forall y.\exists z.RepairsWith(x, y, z)) \to (\exists x.\forall y.\exists z.AskToRepair(x, y, z))$
[Solution:
$(\exists x.\forall y.\exists z.RepairsWith(x, y, z)) \to (\exists x.\forall y.\exists z.AskToRepair(x, y, z))$
$(\neg\exists x.\forall y.\exists z.RepairsWith(x, y, z)) \vee (\exists x.\forall y.\exists z.AskToRepair(x, y, z))$
$(\forall x.\exists y.\forall z.\neg RepairsWith(x, y, z)) \vee (\exists x.\forall y.\exists z.AskToRepair(x, y, z))$
$\neg RepairsWith(x, F_1(x), z) \;\vee\; AskToRepair(C_2, y, F_2(y))$]

(c) $\forall x.(\forall y.Cares(x, y) \to \exists z.IsLovedBy(x, z))$
[Solution:
$\forall x.(\forall y.Cares(x, y) \to \exists z.IsLovedBy(x, z))$
$\forall x.(\neg\forall y.Cares(x, y) \vee \exists z.IsLovedBy(x, z))$
$\forall x.(\exists y.\neg Cares(x, y) \vee \exists z.IsLovedBy(x, z))$
$\neg Cares(x, F_1(x)) \vee IsLovedBy(x, F_2(x))$]

(d) $\forall x.(\exists y.Cares(x, y) \to \forall z.IsLovedBy(x, z))$
[Solution:
$\forall x.(\exists y.Cares(x, y) \to \forall z.IsLovedBy(x, z))$
$\forall x.(\neg\exists y.Cares(x, y) \vee \forall z.IsLovedBy(x, z))$
$\forall x.(\forall y.\neg Cares(x, y) \vee \forall z.IsLovedBy(x, z))$
$\neg Cares(x, y) \vee IsLovedBy(x, z)$]

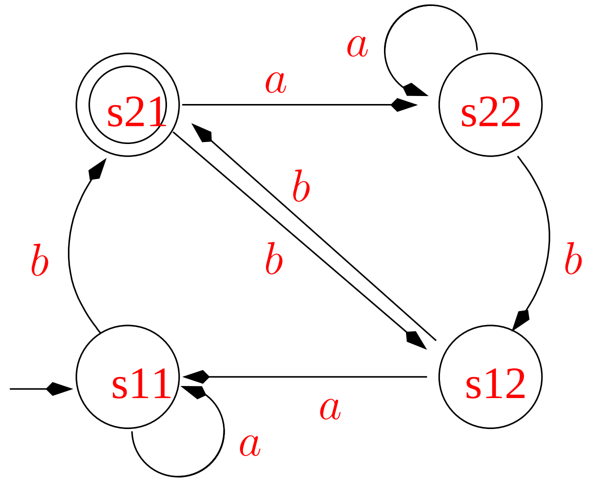
10

Given the following generalized Büchi automaton $A \stackrel{\text{def}}{=} \langle Q, \Sigma, \delta, I, FT \rangle$, $\{a, b\}$ being labels, with two sets of accepting states $FT \stackrel{\text{def}}{=} \{F1, F2\}$ s.t. $F1 \stackrel{\text{def}}{=} \{s2\}, F2 \stackrel{\text{def}}{=} \{s1\}$:



convert it into an equivalent plain Büchi automaton.

[Solution: The result is:



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